Electromagnetic response of a Weyl semimetal with coexisting density waves

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Outline

- Anomaly as (non-)conservation law - *the ABJ anomaly*
- Low energy theory and implications - *our model*
- Lattice model and numerics - *current response*
- Summary

The axial anomaly in brief

$$\mathcal{L} = i\bar{\psi}\left(\partial\!\!\!/ - ieA\!\!\!/\right)\psi$$

- The axial symmetry of the classical Dirac Lagrangian is violated at the quantum level

$$\psi \to e^{i\theta(x)\gamma^5}\psi$$
$$\bar{\psi} \to \bar{\psi}e^{i\theta(x)\gamma^5}$$

Chiral phase rotation

- Axial charge is no longer conserved

$$\langle \partial_\mu j_5^\mu \rangle = \frac{e^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B} \qquad \textbf{(ABJ)}$$

- In the presence of an axial vector potential anomalous currents are present

$$\mathcal{L}_b = \bar{\psi}b\!\!\!/\gamma^5\psi \quad \Rightarrow \quad \mathbf{j}_{\text{AHE}} = \frac{e^2}{2\pi^2}\mathbf{b}\times\mathbf{E}$$

$$\mathbf{j}_{\text{CME}} = \frac{e^2}{2\pi^2}b_0\mathbf{B} \quad \star$$

Adler, Phys. Rev. 177, (1969)
Bell, Jackiw, Il Nuovo Cimento A 60, 1969
(Lattice) Nielsen, Ninomiya, Physics Letters 130 (1983)
(Lattice) Son, Spivak, PRB 88 (2013)
Burkov, J. Phys. Condens. Matter 27 (2015)
...

Anomalous (non-)conservation relations

- In path integral language — families of theories related by a transformation generate a hierarchy of (non)-conservation laws

- Variation of the action leads to **classical** (non)-conservation equations

- Non-invariance of the **measure** leads to quantum **anomaly** terms

Fujikawa, PRD 29 (1984)

$$Z[\alpha] = \int \mathcal{D}[\bar{\psi}, \psi] e^{-S[\bar{\psi}\tilde{U}[\alpha], U[\alpha]\psi]}$$

$$= \det\left(J[\alpha]\right) Z[0]$$

$$\downarrow$$

$$\frac{1}{Z[0]} \frac{\delta Z[\alpha]}{\delta \alpha}\bigg|_{\alpha=0} = \frac{\delta \det J[\alpha]}{\delta \alpha}\bigg|_{\alpha=0}$$

$$\downarrow$$

$$\partial_\mu \langle j_5^\mu \rangle + \langle \text{classical} \rangle = \mathcal{A}(x)$$

Anomalies without symmetry

$$\mathcal{L} = i\bar{\psi}\left(\not{\partial} - ie\not{A} + im\right)\psi$$

- The classical symmetry is broken but *an anomaly is still present*
- This can most easily be seen by looking at the divergence of the associated Noether current
- For the massive Dirac, theory the anomaly function is the same as the massless case

$$\mathcal{A}(x) = \frac{e^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B} \quad \textbf{(ABJ)}$$

Classical (Noether)

$$\partial_\mu j_5^\mu = -2im\bar{\psi}\gamma^5\psi$$

Quantum

$$\partial_\mu \langle j_5^\mu \rangle = -2im\langle\bar{\psi}\gamma^5\psi\rangle + \mathcal{A}(x)$$

Zyuzin, Burkov, PRB 86 (2012)

Low energy model

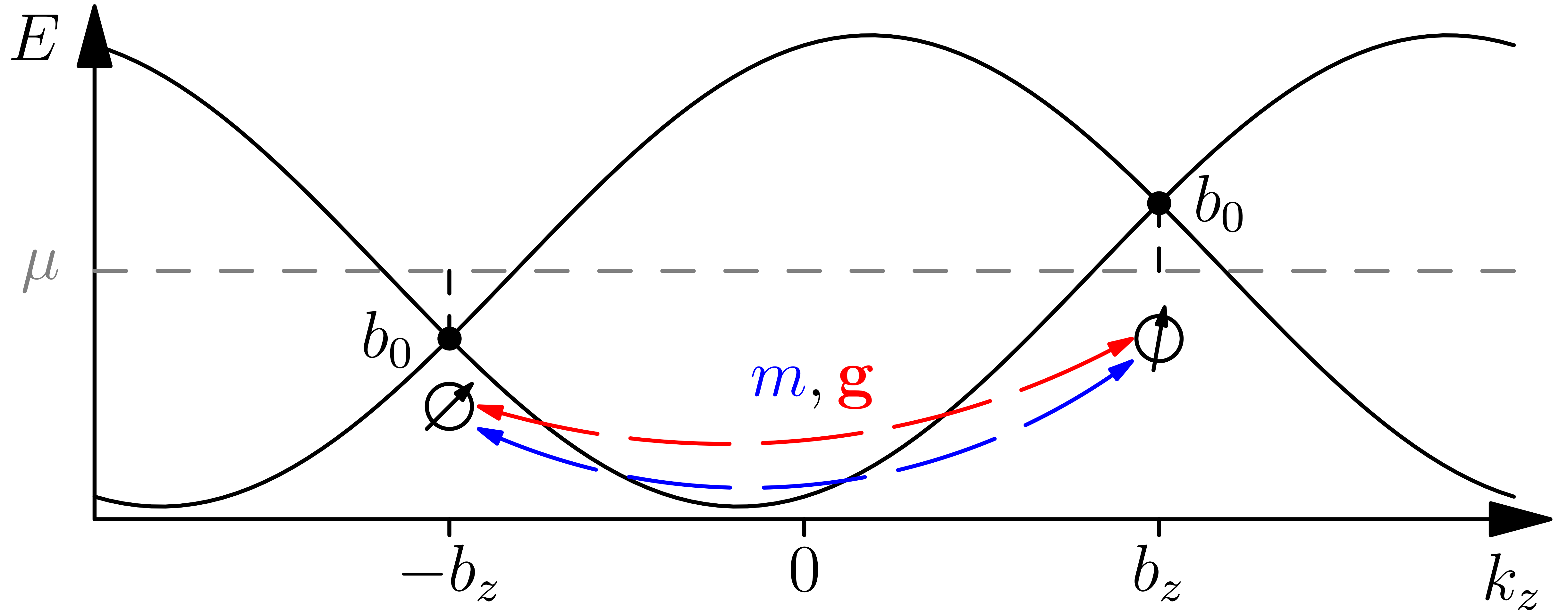
$$\mathcal{L} = \bar{\psi}\left[i\not{D} - \not{b}\gamma^5 - |m|e^{i\alpha\gamma^5} - \Delta_{\mu\nu}\sigma^{\mu\nu}\right]\psi$$ Dirac + other 4x4

Spin
↓

$$H = \begin{bmatrix} (\mathbf{k}-\mathbf{b})\cdot\boldsymbol{\sigma} - b_0 & me^{i\alpha} + \mathbf{g}\cdot\boldsymbol{\sigma} \\ me^{-i\alpha} + \mathbf{g}^*\cdot\boldsymbol{\sigma} & -(\mathbf{k}+\mathbf{b})\cdot\boldsymbol{\sigma} + b_0 \end{bmatrix}$$ Valley

The new terms admit a simple interpretation in a condensed matter system:

- m is internode charge mixing
- $\mathbf{g}$ is internode spin mixing



Low energy model

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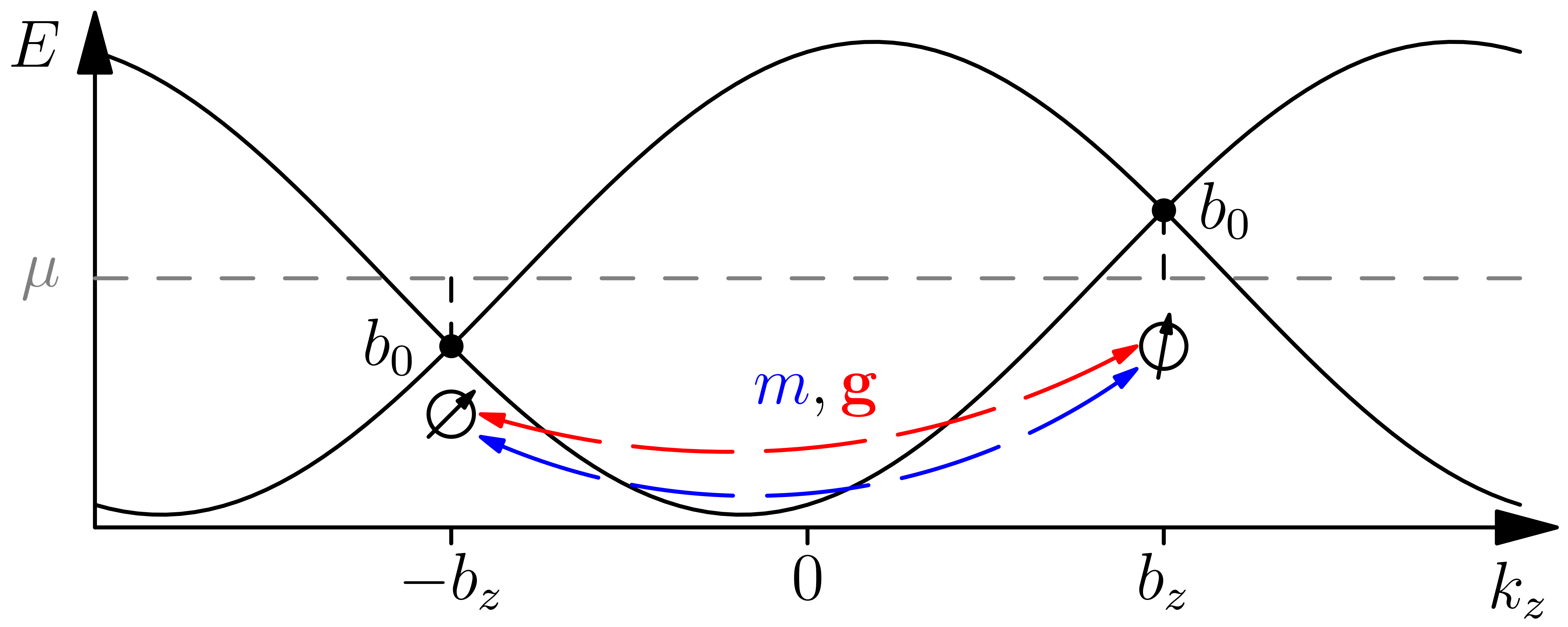
Such terms could be realized

- As mean-field decoupling of interactions
- Proximity induced couplings
- Dynamically within a Floquet Hamiltonian



Modification of the anomaly function

$$\mathcal{A}(x) = \frac{e^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B} - \frac{e}{\pi^2}\mathbf{E}\cdot\mathrm{Re}\left[\mathbf{g}m^*\right]$$

$$\partial_\mu \langle j_5^\mu \rangle + \langle \text{classical} \rangle = \mathcal{A}(x)$$

(ABJ)

New terms act in place of the magnetic field

- The presence of the combined terms m and $\mathbf{g}$ leads to a term in the anomaly function
- This term is zero without both m and $\mathbf{g}$ present

Removing the axial vector

- We may perform a change of variables to remove the axial vector b^μ from the Fermionic Lagrangian

$$\psi = e^{-2ib\cdot x\gamma^5}\psi'$$

$$\bar{\psi} = \bar{\psi}' e^{-2ib\cdot x\gamma^5}$$

$$\mathcal{L}' = \bar{\psi}' \left[i\not{D} - |m| e^{i(\alpha - 2b\cdot x)\gamma^5} - \Delta_{\mu\nu} e^{-2ib\cdot x\gamma^5} \right] \psi'$$

- This introduces a new term in the action through the non-invariance of the measure — *the anomaly*

$$\mathcal{L}_J = ib\cdot x\mathcal{A}(x)$$

$$\mathcal{A}(x) = \frac{e^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B} - \frac{e}{\pi^2}\mathbf{E}\cdot\mathrm{Re}\left[\mathbf{g}m^*\right]$$

Induced current

- The coupling of the added terms allows for new contributions to the current
- This current must be understood in a manner similar to the Chiral Magnetic Effect

$$\mathcal{L}_J = ib \cdot x\mathcal{A}(x)$$

$$= ib \cdot x \left[\frac{e^2}{2\pi^2} \mathbf{E} \cdot \mathbf{B} - \frac{e}{\pi^2} \mathbf{E} \cdot \mathrm{Re}\left[\mathbf{g} m^*\right] \right]$$

$$\Rightarrow$$

$$\mathbf{j}_J = -\frac{\delta S}{\delta \mathbf{A}} = \frac{e}{\pi^2} b_0 \mathrm{Re}\left[\mathbf{g} m^*\right]$$

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$$\Rightarrow$$

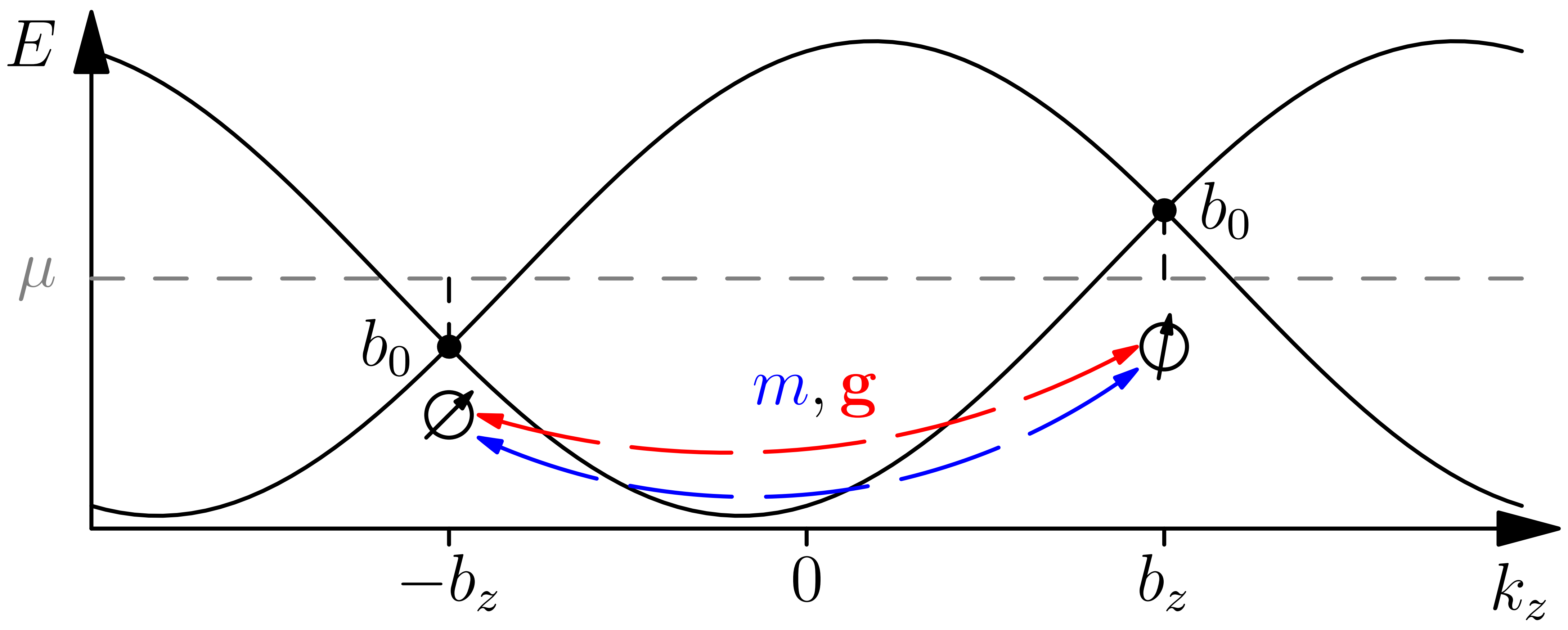
$$\mathbf{g} = g_z \hat{\mathbf{z}}$$

$$\mathbf{j}_J = -\frac{\delta S}{\delta \mathbf{A}} = \frac{e}{\pi^2} b_0 |m| g_z \cos\alpha .$$

Lattice model

- The connection between the low-energy theory and more realistic models of the solid state can be subtle

- We consider a lattice model of Weyl fermions to show that the predicted current is indeed physical



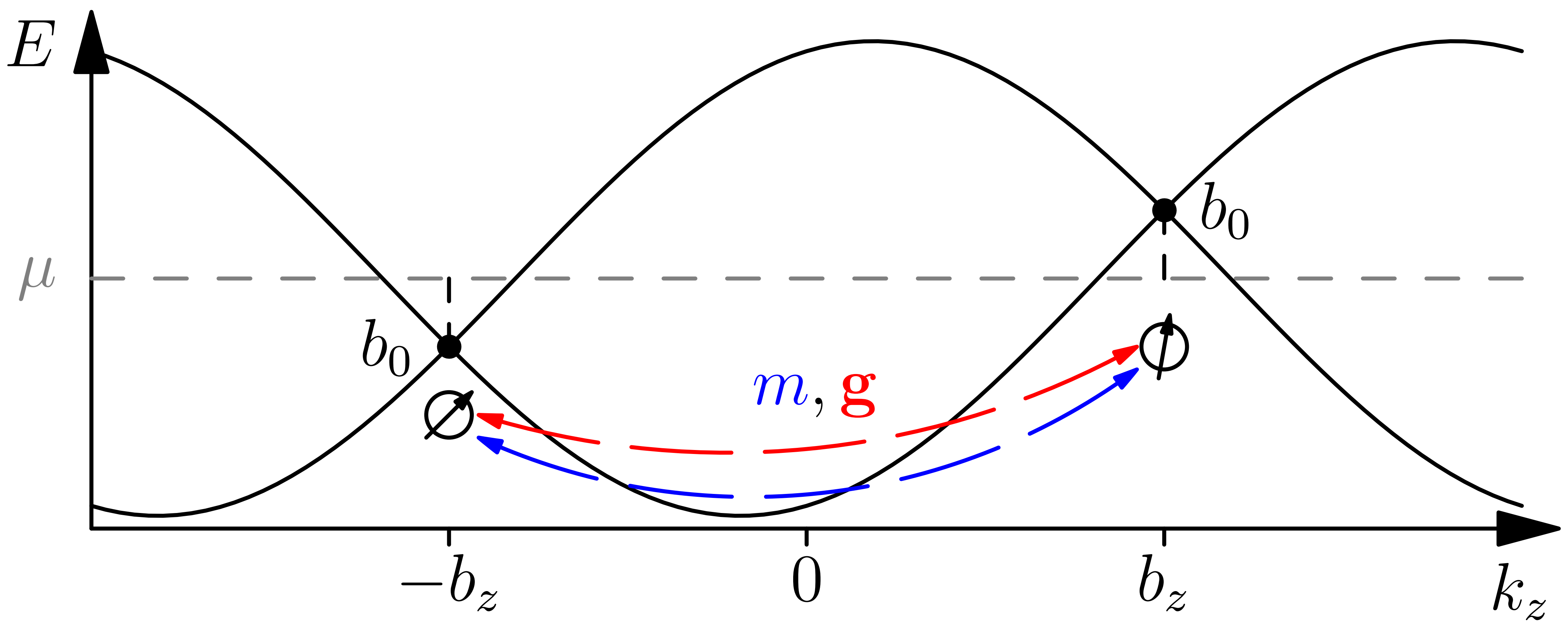
$$H_0 = \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}} \left[\epsilon(\mathbf{k}) + \mathbf{d}(\mathbf{k}) \cdot \boldsymbol{\sigma}\right]$$

$$\epsilon(\mathbf{k}) = t_1 \sin k_z$$

$$\mathbf{d}(\mathbf{k}) = \begin{pmatrix} \sin k_x \\ \sin k_y \\ 2 + \cos b_z - \sum_i \cos k_i \end{pmatrix}$$

Lattice model

- H_0 describes massless Weyl particles in the presence of an axial vector b
- H_m adds a mass term
- H_g will provide a coupling to a space and time dependent field $\mathbf{g}$



$$H_0 = \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}} \left[\epsilon(\mathbf{k}) + \mathbf{d}(\mathbf{k}) \cdot \boldsymbol{\sigma}\right]$$

$$H_m = m \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}+\mathbf{b}} e^{-i\alpha} \sigma_z c_{\mathbf{k}-\mathbf{b}} + h.c.$$

$$H_g = \sum_{\mathbf{k}} c^\dagger_{\mathbf{k}+\mathbf{b}} \sigma_z \mathbf{g}(\tau) \cdot \boldsymbol{\sigma} c_{\mathbf{k}-\mathbf{b}} + h.c.$$

‘Linear response’

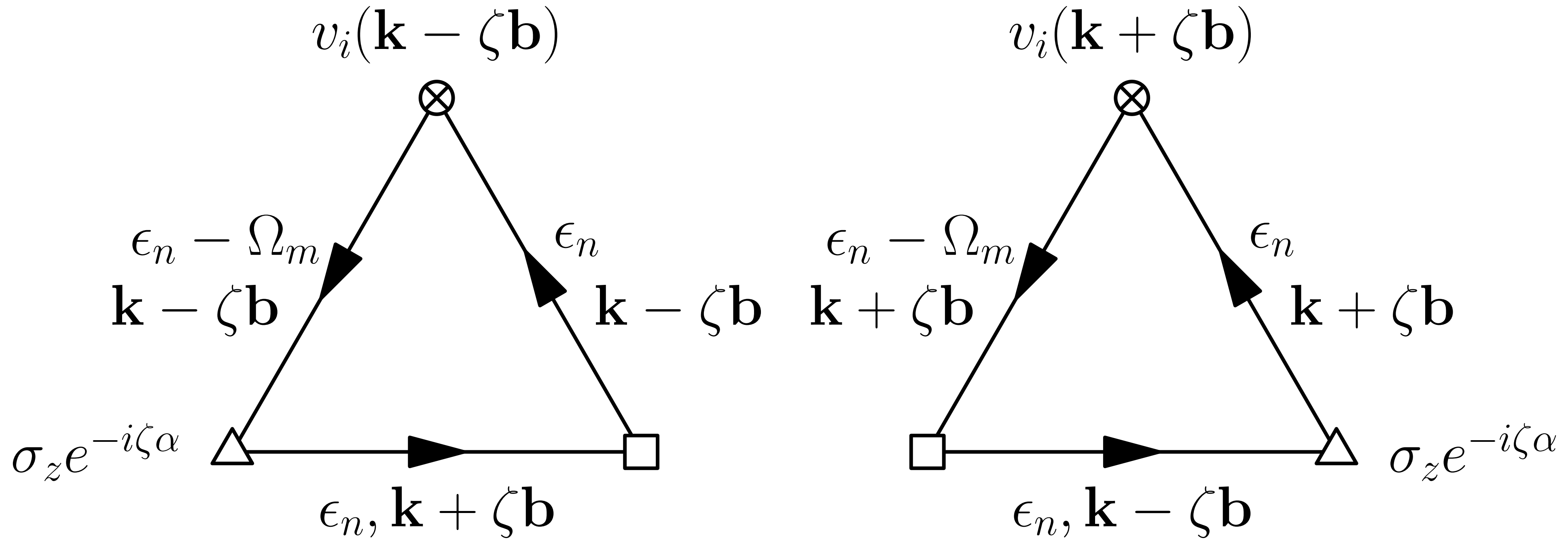
- We calculate current as ‘linear response’ to the spacetime-dependent vector $\mathbf{g}(\tau)$ in the presence of m

$$\chi_i(i\Omega_m, \mathbf{q}) = \frac{\delta j_i(i\Omega_m, \mathbf{q})}{\delta m \delta g(-i\Omega_m, -\mathbf{q})}$$

- In particular we are interested in the **DC** current response — $\mathbf{q} \to 0$ before $\omega \to 0$

$$\lim_{\omega \to 0} \lim_{\mathbf{q} \to 0} \chi_i^{\mathrm{R}}(\omega, \mathbf{q})$$

  - **N.B.** Current response must vanish in the opposite limit ($\omega \to 0$ first)



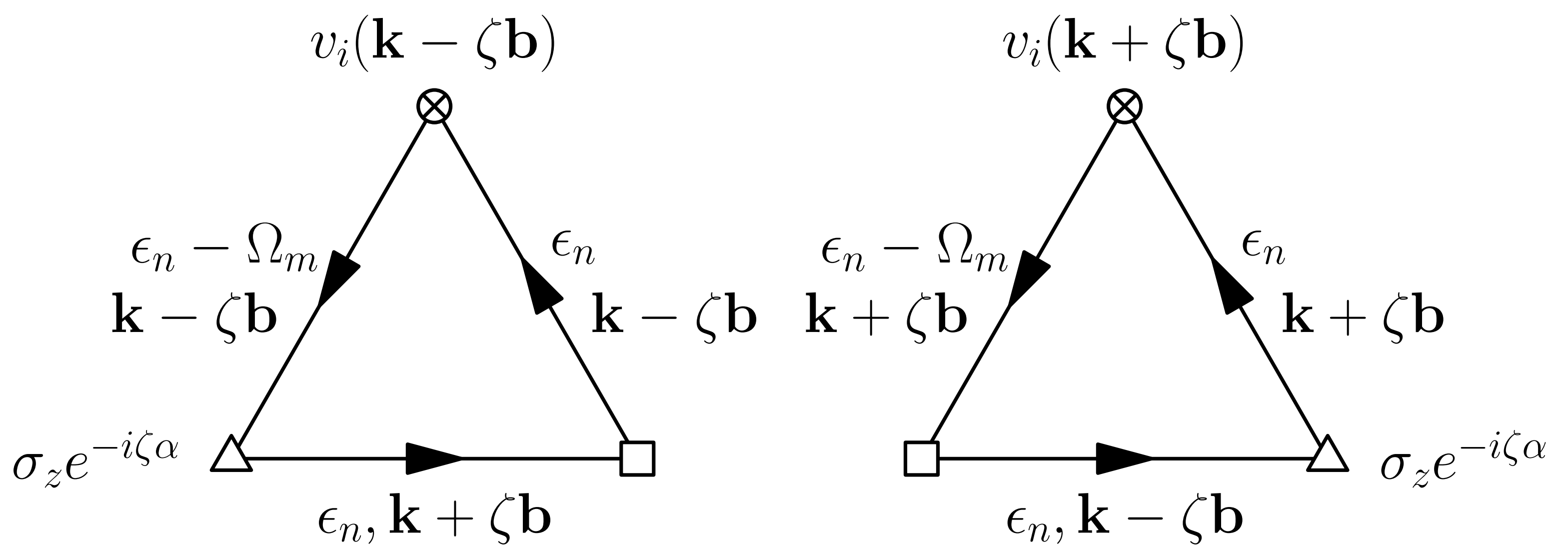
$$\mathbf{j}_J \propto b_0 |m| g_z \cos\alpha$$

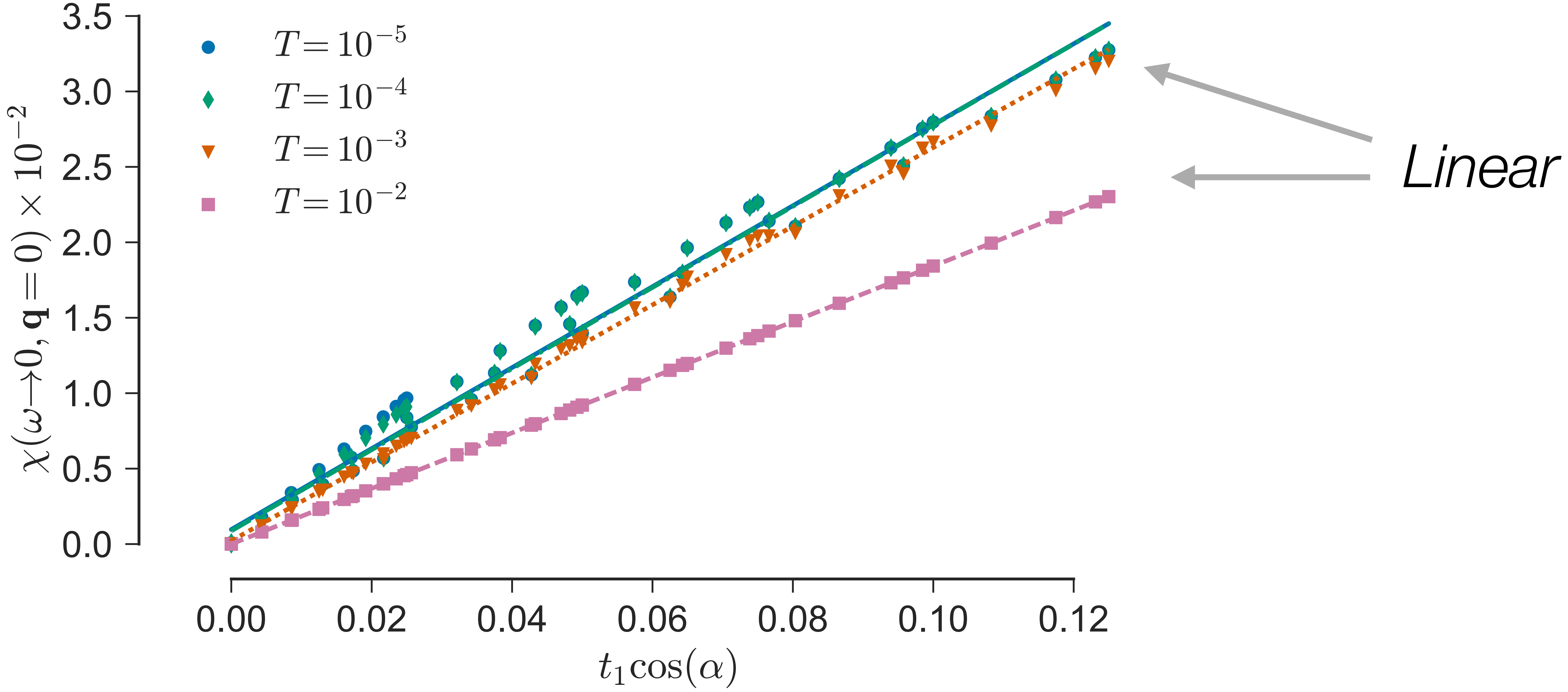
$$\frac{\delta j}{\delta |m| \delta g} \propto b_0 \cos\alpha$$

Low energy

$$\mathcal{L}_J \ni ib \cdot x \frac{e}{\pi^2} \mathbf{E} \cdot \mathrm{Re}\left[\mathbf{g} m^*\right]$$

Lattice



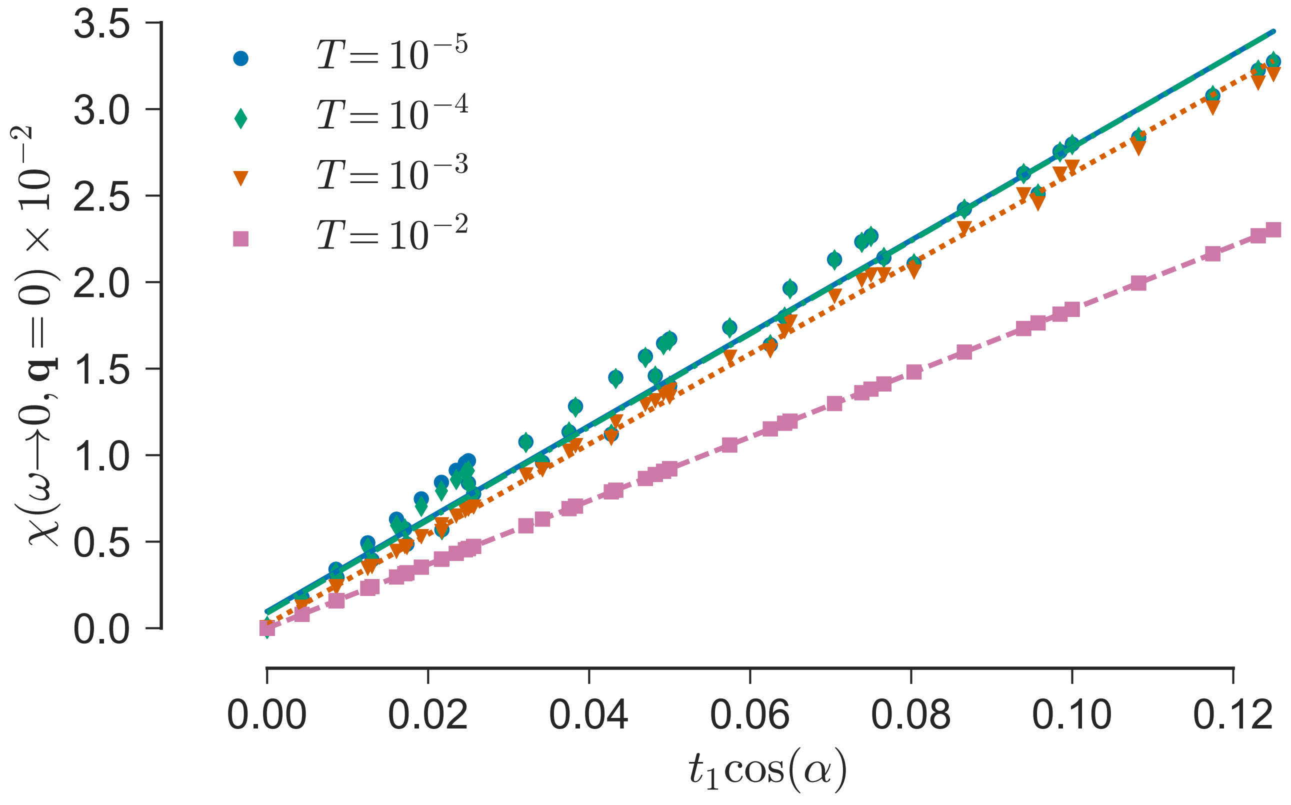


Summary

- We considered the axial anomaly in the presence of additional symmetry breaking terms
  - We found a new contribution to the divergence of the axial current indicating an *additional contribution to the anomaly*
- The low-energy theory predicted a *DC current response* to introduced terms which was reproduced in a lattice model

$$\partial_\mu \langle j_5^\mu \rangle + \langle \text{classical} \rangle = \mathcal{A}(x)$$

$$\mathcal{A}(x) = \frac{e^2}{2\pi^2}\mathbf{E}\cdot\mathbf{B} - \frac{e}{\pi^2}\mathbf{E}\cdot \mathrm{Re}\left[\mathbf{g}m^*\right]$$



$$\mathbf{j}_J \propto b_0 |m| g_z \cos\alpha$$

$$\frac{\delta j}{\delta |m| \delta g} \propto b_0 \cos\alpha$$

Thank you

Extra Slides

$$W = \mathcal{D}^\dagger \mathcal{D}$$

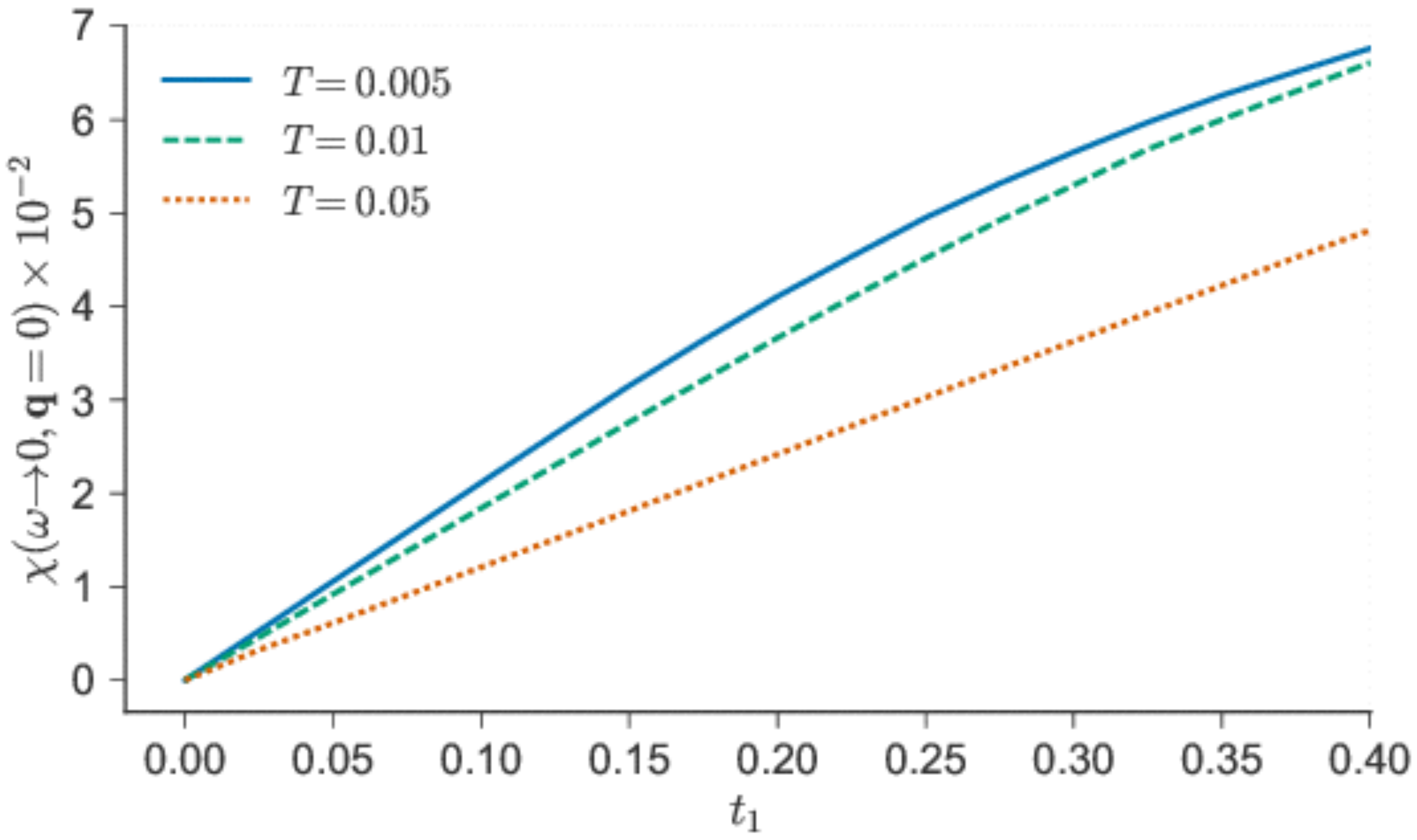
$$\widetilde{W} = \mathcal{D}\mathcal{D}^\dagger$$

$$W \phi_n(x) = \lambda_n^2 \phi_n(x)$$

$$\tilde{\phi}_n^\dagger(x)\, \widetilde{W} = \tilde{\phi}_n^\dagger(x) \lambda_n^2$$

$$\mathcal{A}(x) = I(x) + \tilde{I}(x)$$

$$I(x) = \lim_{M\to\infty} \lim_{y\to x} \int_k \mathrm{tr}\, e^{ik\cdot y} \gamma^5 e^{-W/M^2} e^{-ik\cdot x}.$$



$T = 0.005$
$T = 0.01$
$T = 0.05$
$\chi(\omega \to 0, \mathbf{q} = 0) \times 10^{-2}$
t_1
0
1
2
3
4
5
6
7
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40

CME

1. Alavirad, Y. & Sau, J. D. Role of boundary conditions, topology, and disorder in the chiral magnetic effect in Weyl semimetals. Phys. Rev. B 94, 115160 (2016).
1. Chang, M. & Yang, M. Chiral magnetic effect in a two-band lattice model of Weyl semimetal. Phys. Rev. B 91, 115203 (2015).
1. Landsteiner, K. Anomalous transport of Weyl fermions in Weyl semimetals. Phys. Rev. B - Condens. Matter Mater. Phys. 89, 1–11 (2014).
1. Burkov, A. A. Chiral anomaly and transport in Weyl metals. J. Phys. Condens. Matter 27, 113201 (2015).
1. Burkov, A. A. & Balents, L. Weyl semimetal in a topological insulator multilayer. Phys. Rev. Lett. 107, 1–4 (2011).