Spin-valley collective modes of the electron liquid in graphene

Yale

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Collaborators

Part I

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Outline

Part I - Spin rotation invariant graphene

(Absence of) neutral zero sound in graphene

(Absence of) neutral first sound in graphene

Part II - Magnetic fields and extrinsic SOC

Spin-valley Silin modes

Signatures in the optical conductivity

10.1103/PhysRevB.103.075422

ZMR, Fal’ko, Glazman, PRB 103, 075422, 2020

In prep.

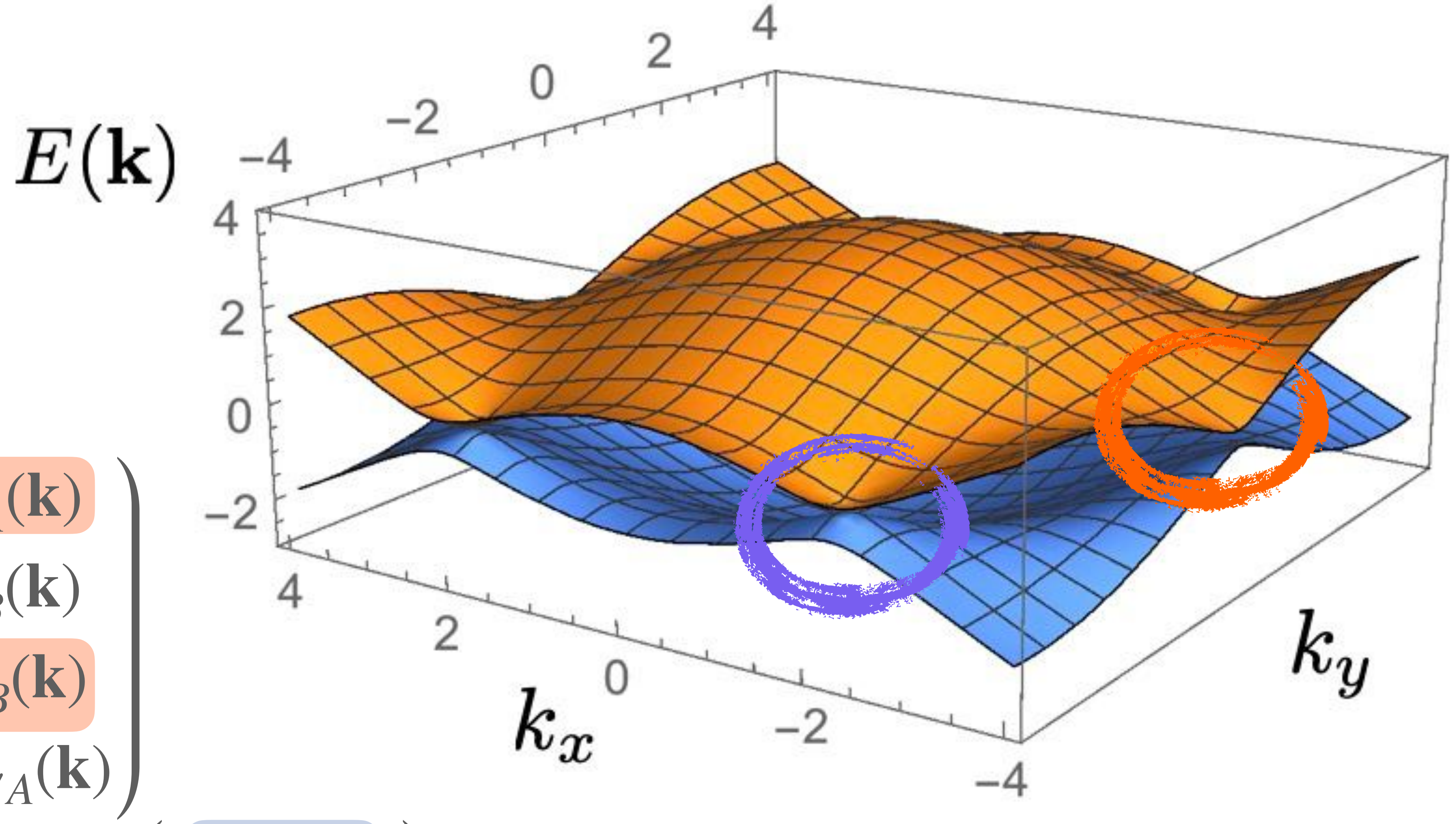
ZMR, Maslov, Glazman

$$\Psi_{\mathbf{k}} = \begin{pmatrix} \psi_{KA}(\mathbf{k}) \\ \psi_{KB}(\mathbf{k}) \\ \psi_{K'B}(\mathbf{k}) \\ -\psi_{K'A}(\mathbf{k}) \end{pmatrix}$$

A quick note on notation

Pauli matrices

We will be largely concerned with physics of the conduction band



- $\psi_{\zeta\Sigma}$ is a spinor
- Spin matrices σ act on $\begin{pmatrix} \psi_{KA}(\mathbf{k}) \\ \psi_{KB}(\mathbf{k}) \\ \psi_{K'B}(\mathbf{k}) \\ -\psi_{K'A}(\mathbf{k}) \end{pmatrix}$
- Valley matrices τ act on $\begin{pmatrix} \psi_{KA}(\mathbf{k}) \\ \psi_{KB}(\mathbf{k}) \\ \psi_{K'B}(\mathbf{k}) \\ -\psi_{K'A}(\mathbf{k}) \end{pmatrix}$
- Sublattice matrix Σ acts on $\begin{pmatrix} \psi_{KA}(\mathbf{k}) \\ \psi_{KB}(\mathbf{k}) \\ \psi_{K'B}(\mathbf{k}) \\ -\psi_{K'A}(\mathbf{k}) \end{pmatrix}$

Part I - Absence of neutral first and second sound (spin rotation invariant case)

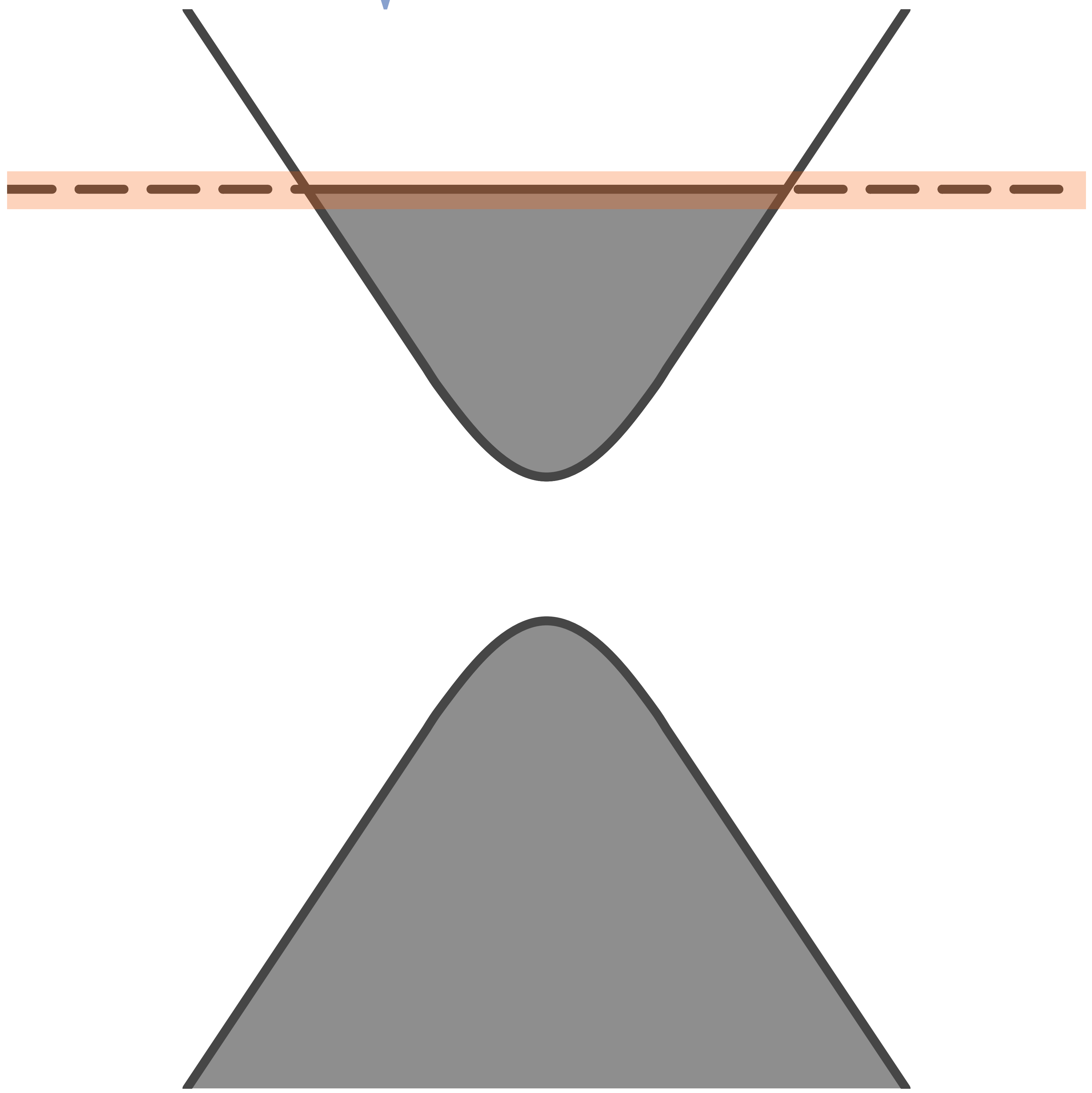
Fermi Liquid graphene

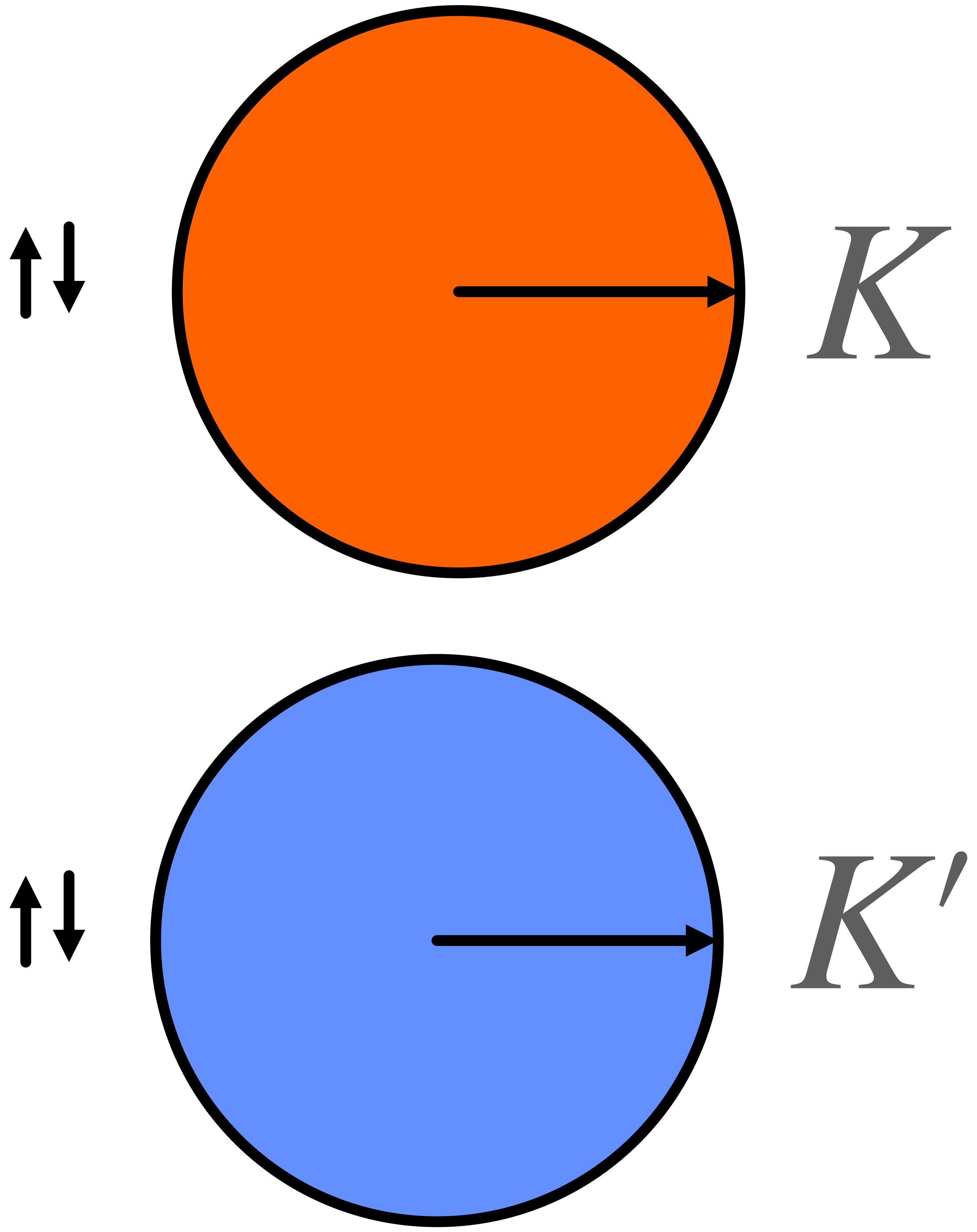
A multicomponent Fermi liquid

$$\epsilon_{ij}(\mathbf{p},\mathbf{r}) = \xi(\mathbf{p}) + \sum_{\mathbf{p}',lm} f_{ij;lm}(\mathbf{p}\cdot\mathbf{p}')\hat{\rho}_{lm}(\mathbf{r},\mathbf{p}'),$$

- We want to construct a Fermi liquid theory of graphene without sub lattice symmetry
- We need to construct the quasiparticle excitation energy functional ϵ (in terms of the density matrix ρ)

$$\xi(\mathbf{p}) = \sqrt{v^2p^2 + \Delta^2} - \mu$$





10.1103/PhysRevB.103.075422

Symmetry of gapped graphene

What short ranged interactions are symmetry allowed?

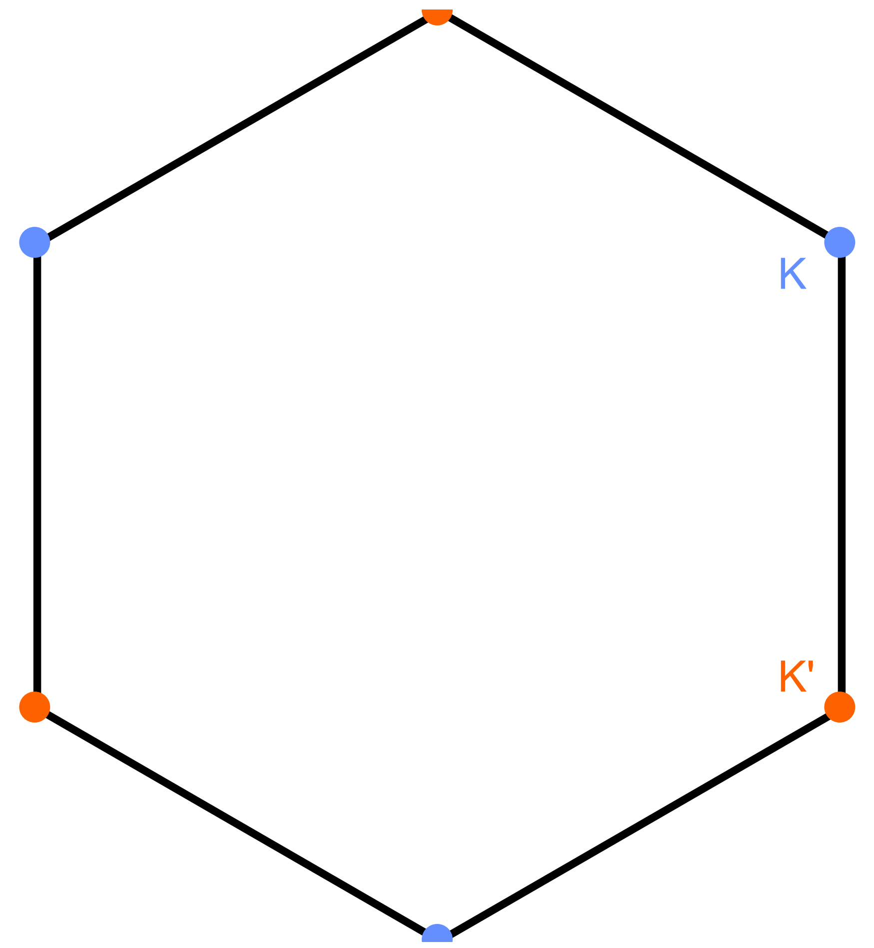
$$\hat{\Psi}_\sigma(\mathbf{r}) = \begin{pmatrix} u_{KA}(\mathbf{r}) & u_{KB}(\mathbf{r}) & u_{K'B}(\mathbf{r}) & -u_{K'A}(\mathbf{r}) \end{pmatrix} \cdot \hat{\vec{\psi}}_\sigma(\mathbf{r})$$

- For the low energy theory we expand in terms of the Bloch wave functions at the Dirac points and slowly varying envelope functions
- These Bloch wave functions have well defined symmetry properties under lattice transformations

Low energy theory

Expand around K and K' points

$$\hat{\vec{\psi}}_\mathbf{k} = \begin{pmatrix} \psi_{KA}(\mathbf{k}) \\ \psi_{KB}(\mathbf{k}) \\ \psi_{K'B}(\mathbf{k}) \\ -\psi_{K'A}(\mathbf{k}) \end{pmatrix}$$



Aleiner, Kharzeev, Tsvelik, PRB 76, 195415 (2007)

Kharitonov, PRB 85, 155439 (2012)

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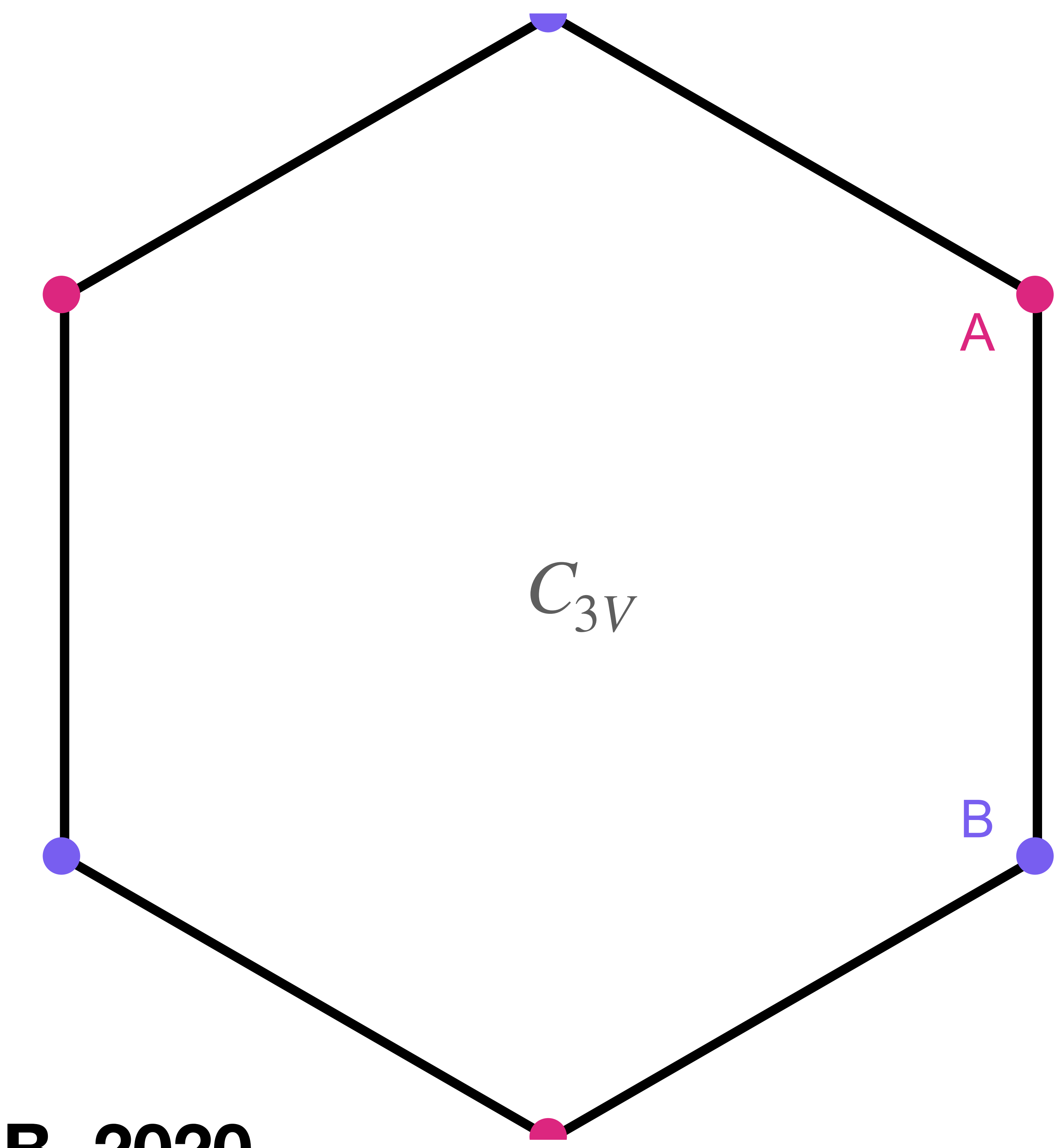
Low energy theory

Rotation $C_3 : e^{i\frac{\pi}{3}\hat{\Sigma}_3}$

Mirror plane $\sigma_V : \hat{\Sigma}_2\hat{\tau}_2$

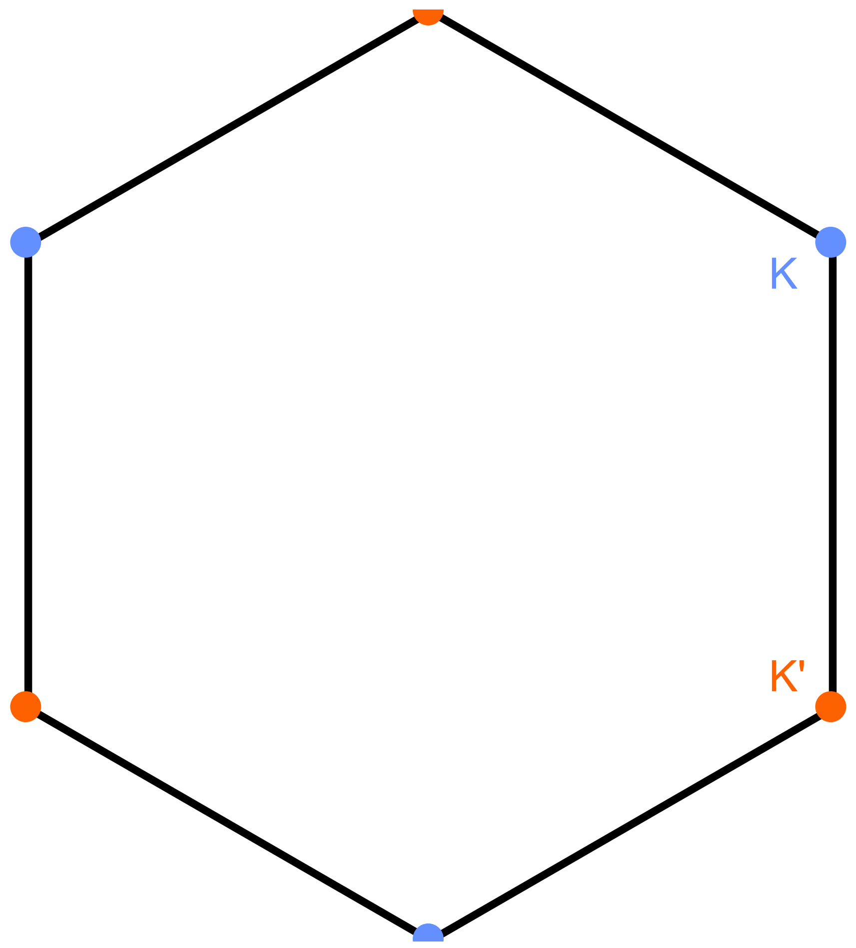
Translation $T : e^{\frac{2\pi i}{3}\hat{\tau}^z}$

Time Reversal $\Theta : \sigma_V K$



Expand around K and K' points

$$\Psi_{\mathbf{k}} = \begin{pmatrix} \psi_{KA}(\mathbf{k}) \\ \psi_{KB}(\mathbf{k}) \\ \psi_{K'B}(\mathbf{k}) \\ -\psi_{K'A}(\mathbf{k}) \end{pmatrix}$$



ZMR, Fal'ko, Glazman, PRB, 2020

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Interactions from symmetry

- We can approximate the interaction constants from matrix elements of the Coulomb interaction
- Due to symmetry there are 3 independent short ranged coupling constants + long ranged part of Coulomb
- Interactions form a natural hierarchy of scales related to their characteristic length scale

$$g \propto \int u^*_{\zeta\Sigma}(r) V(|r - r'|) u_{\zeta'\Sigma'}(r')$$

Long Range

Strongest

$$V(\mathbf{q})$$

$$g_{\perp\perp} \sim V(|\mathbf{K} - \mathbf{K}'|)$$

$$g_{zz} \sim V(|\mathbf{b}|)$$

$$\tilde{g}_{00} \sim \frac{g_{zz}\Delta}{v_F |\mathbf{K} - \mathbf{K}'|}$$

Weakest

Short Range

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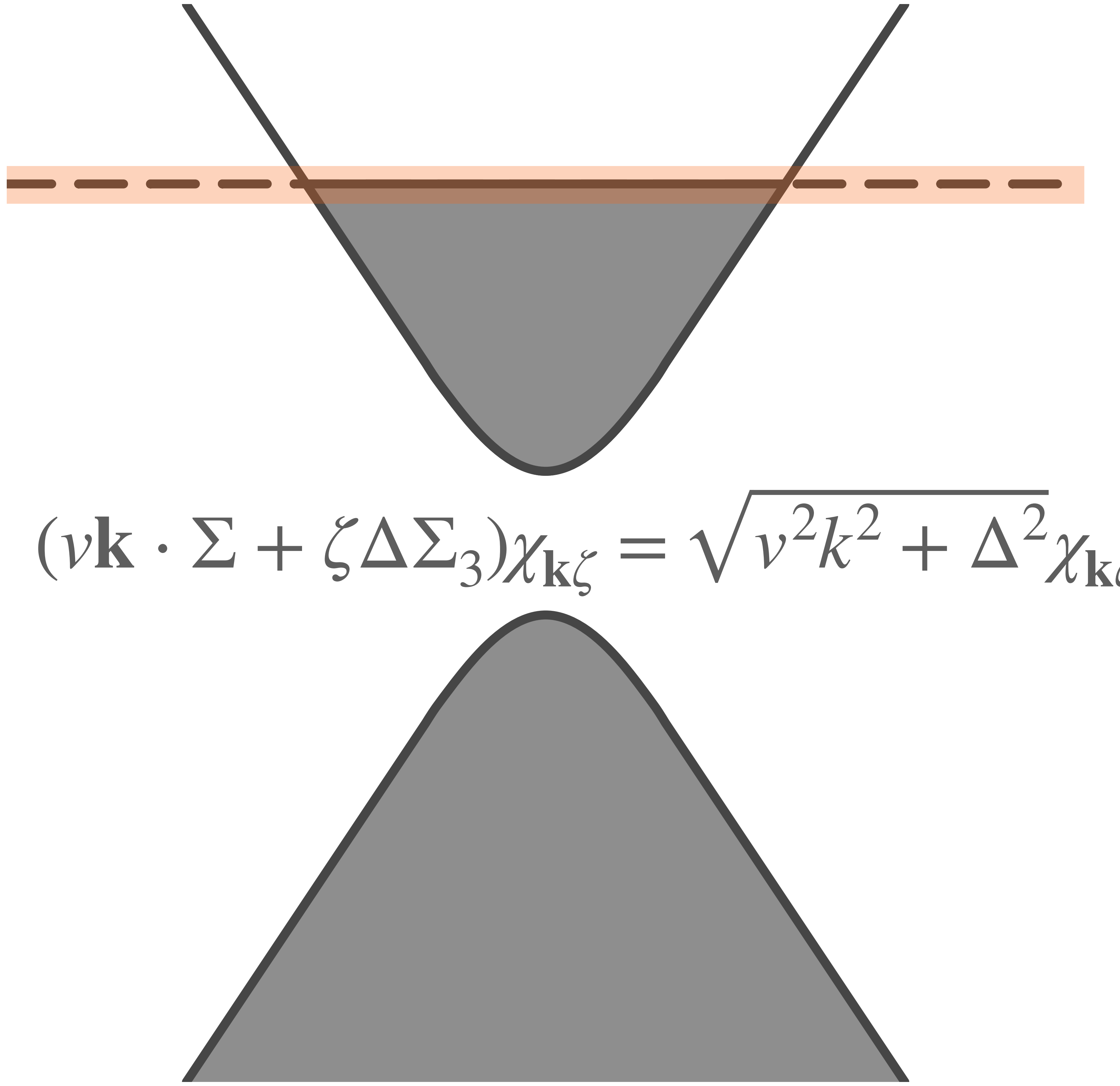
Upper band description



$$(v\mathbf{k}\cdot\Sigma+\zeta\Delta\Sigma_3)\chi_{\mathbf{k}\zeta}=\sqrt{v^2k^2+\Delta^2}\chi_{\mathbf{k}\zeta}$$

$$U(\mathbf{p},\mathbf{p}',\mathbf{q})=U^{d}_{\mathbf{p},\mathbf{p}',\mathbf{q}}+U^{s}_{\mathbf{p},\mathbf{p}',\mathbf{q}}\sigma\cdot\sigma+U^{v\|}_{\mathbf{p},\mathbf{p}',\mathbf{q}}\tau^{\|}\cdot\tau^{\|}$$
$$+U^{vz}_{\mathbf{p},\mathbf{p}',\mathbf{q}}\tau^3\tau^3+U^{m\|}_{\mathbf{p},\mathbf{p}',\mathbf{q}}\tau^{\|}\cdot\tau^{\|}\sigma\cdot\sigma+U^{mz}_{\mathbf{p},\mathbf{p}',\mathbf{q}}\tau^3\tau^3\sigma\cdot\sigma,$$

- Six possibly distinct short ranged interaction functions + long ranged Coulomb
    - These could be considered inputs of the theory
- Momentum dependence comes from the spinor matrix elements

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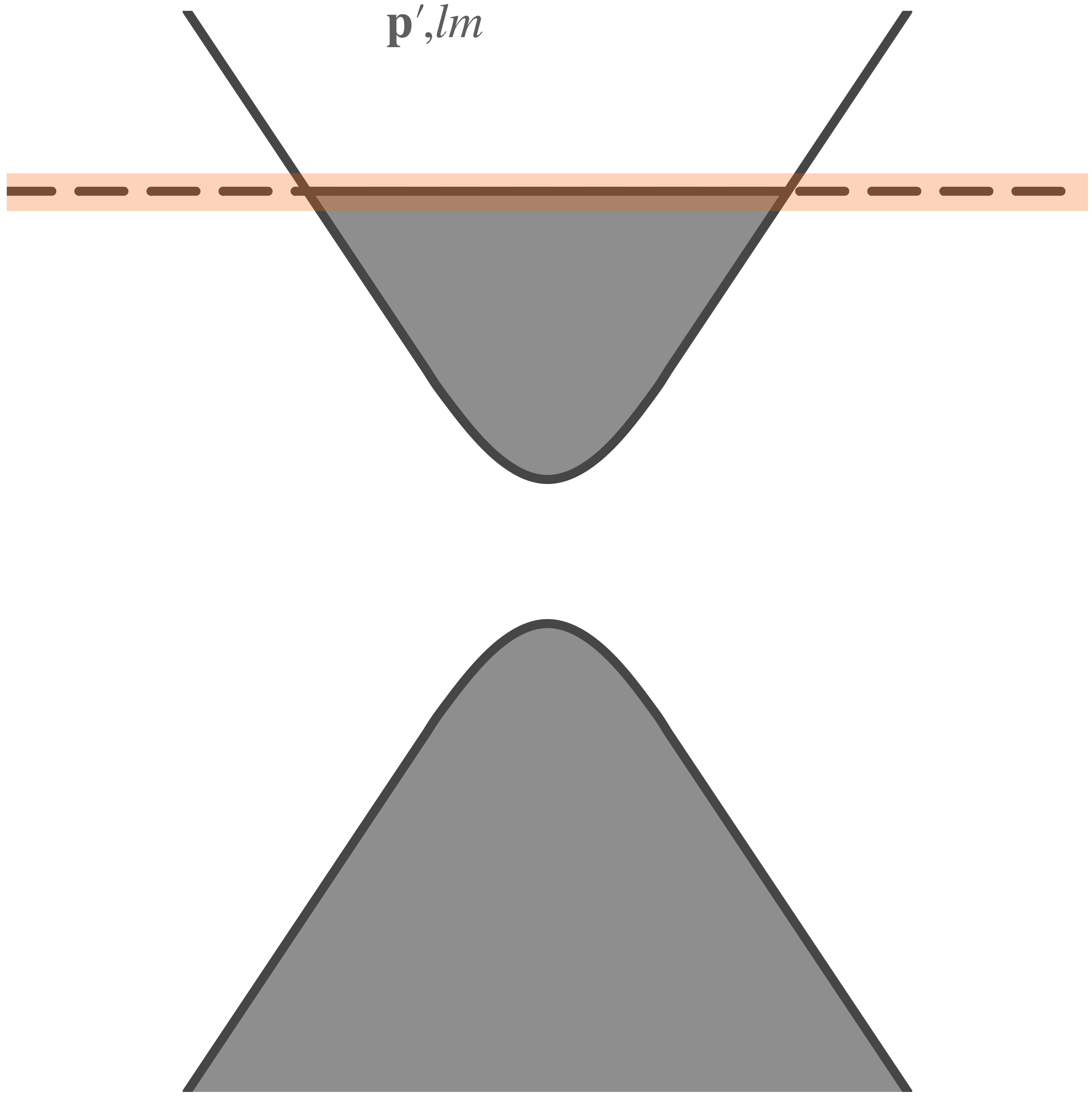
Fermi Liquid graphene

A multicomponent Fermi liquid

$$\epsilon_{ij}(\mathbf{p},\mathbf{r}) = \xi(\mathbf{p}) + \sum_{\mathbf{p}',lm} f_{ij;lm}(\mathbf{p}\cdot\mathbf{p}')\hat{\rho}_{lm}(\mathbf{r},\mathbf{p}'),$$



$$f(\mathbf{p},\mathbf{p}',\mathbf{q}) = f^{d}_{\mathbf{p},\mathbf{p}',\mathbf{q}} + f^{s}_{\mathbf{p},\mathbf{p}',\mathbf{q}}\sigma\cdot\sigma + f^{v\|}_{\mathbf{p},\mathbf{p}',\mathbf{q}}\tau^{\|}\cdot\tau^{\|} + f^{vz}_{\mathbf{p},\mathbf{p}',\mathbf{q}}\tau^3\tau^3 + f^{m\|}_{\mathbf{p},\mathbf{p}',\mathbf{q}}\tau^{\|}\cdot\tau^{\|}\sigma\cdot\sigma + f^{mz}_{\mathbf{p},\mathbf{p}',\mathbf{q}}\tau^3\tau^3\sigma\cdot\sigma,$$

- The Fermi liquid interaction functions inherit the invariant structure
- We now have all the pieces of the Fermi liquid theory
- Symmetry suggests a particular parametrization

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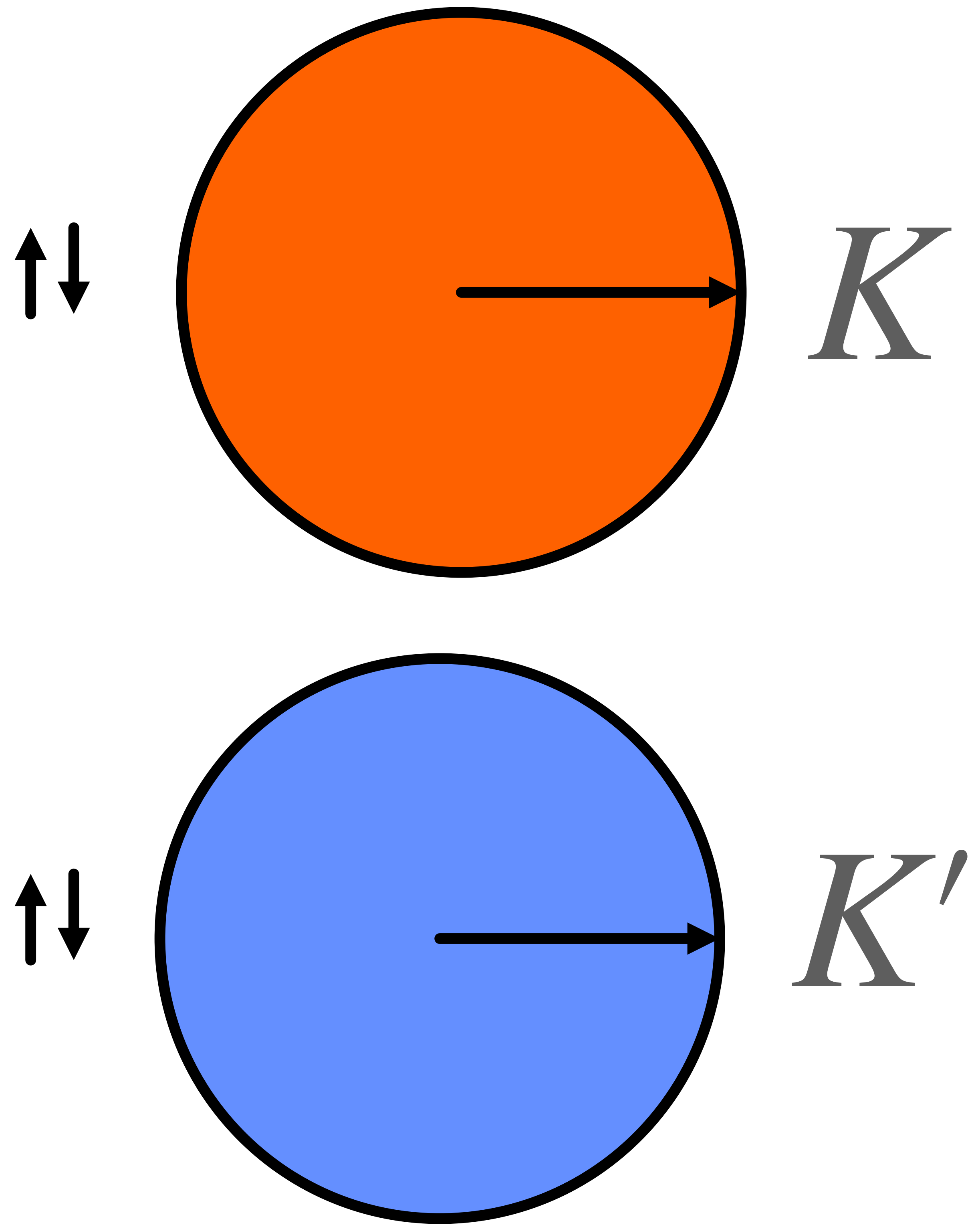
Fermi Liquid graphene

A multicomponent Fermi liquid



- Instead of spin and valley indices let's talk about symmetry distinct channels

$$n(\mathbf{r},\mathbf{p}) = \frac{1}{G_s G_v}\,\mathrm{tr}\,\hat{\sigma}_0\hat{\tau}_0\hat{\rho}(\mathbf{r},\mathbf{p})$$

$$\mathbf{s}(\mathbf{r},\mathbf{p}) = \frac{1}{G_s G_v}\,\mathrm{tr}\,\hat{\sigma}\hat{\rho}(\mathbf{r},\mathbf{p})$$

$$\mathbf{Y}(\mathbf{r},\mathbf{p}) = \frac{1}{G_s G_v}\,\mathrm{tr}\,\hat{\tau}\hat{\rho}(\mathbf{r},\mathbf{p})$$

$$M_i^j(\mathbf{r},\mathbf{p}) = \frac{1}{G_s G_v}\,\mathrm{tr}\,\hat{\tau}_i\hat{\sigma}_j\hat{\rho}(\mathbf{r},\mathbf{p})$$

$\parallel\,,\,\perp$

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Landau-Silin kinetic theory

Dynamics

$$\frac{\partial \delta\rho^\mu(\mathbf{k},\mathbf{r})}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\delta\bar{\rho}^\mu(\mathbf{k},\mathbf{r}) + \frac{\partial n}{\partial \epsilon}\bigg|_{\bar{\epsilon}} \mathbf{v}\cdot\mathscr{F}^\mu = \frac{1}{G_s G_v}\operatorname{tr}\hat{X}^\mu \hat{I}[\delta\hat{\rho}]$$

- Collective modes associated with oscillatory fluctuations of the density matrix $\delta\rho^\mu$
- Dynamics governed by the linearized Landau-Silin kinetic equation
- Equations for the symmetry distinct components decouple
  - e.g. $\mathscr{L}\mathbf{Y} = I_Y[\mathbf{Y}]$

$$\hat{\rho} = n_F + \sum \hat{X}^\mu \delta\rho^\mu$$

$$n(\mathbf{r},\mathbf{p}) = \frac{1}{G_s G_v}\operatorname{tr}\hat{\sigma}_0\hat{\tau}_0\hat{\rho}(\mathbf{r},\mathbf{p})$$

$$\mathbf{s}(\mathbf{r},\mathbf{p}) = \frac{1}{G_s G_v}\operatorname{tr}\hat{\sigma}\hat{\rho}(\mathbf{r},\mathbf{p})$$

$$\mathbf{Y}(\mathbf{r},\mathbf{p}) = \frac{1}{G_s G_v}\operatorname{tr}\hat{\tau}\hat{\rho}(\mathbf{r},\mathbf{p})$$

$$M_i^j(\mathbf{r},\mathbf{p}) = \frac{1}{G_s G_v}\operatorname{tr}\hat{\tau}_i\hat{\sigma}_j\hat{\rho}(\mathbf{r},\mathbf{p})$$

$\parallel\,,\ \perp$

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Conventional Sound

$$\omega \propto v_F q$$

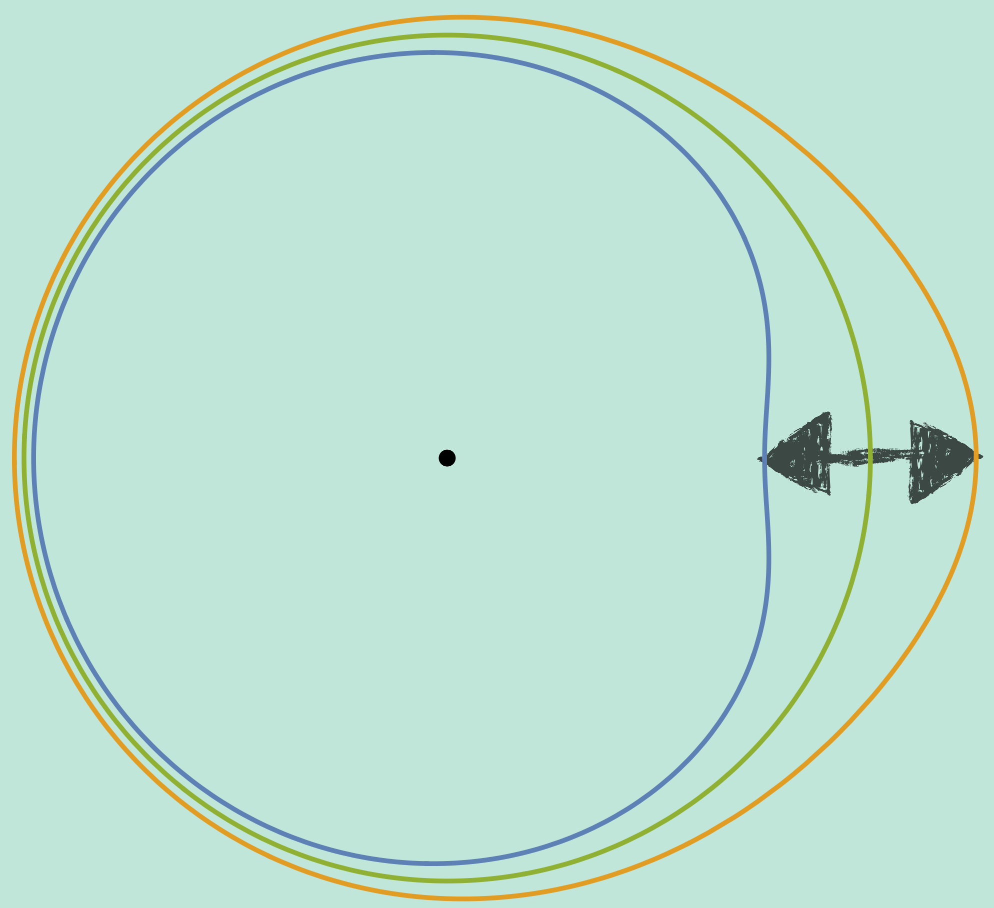
(uncharged) 2D FL

$$\frac{\partial \delta\rho^\mu(\mathbf{k},\mathbf{r})}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\delta\bar{\rho}^\mu(\mathbf{k},\mathbf{r}) + \cancel{\left.\frac{\partial n}{\partial \epsilon}\right|_{\bar{\epsilon}} \mathbf{v}\cdot\mathscr{F}^\mu} = \boxed{\frac{1}{G_s G_v}\operatorname{tr}\hat{X}^\mu \hat{I}[\delta\hat{\rho}]}$$

Zero Sound

$$\omega \gg \frac{1}{\tau}$$

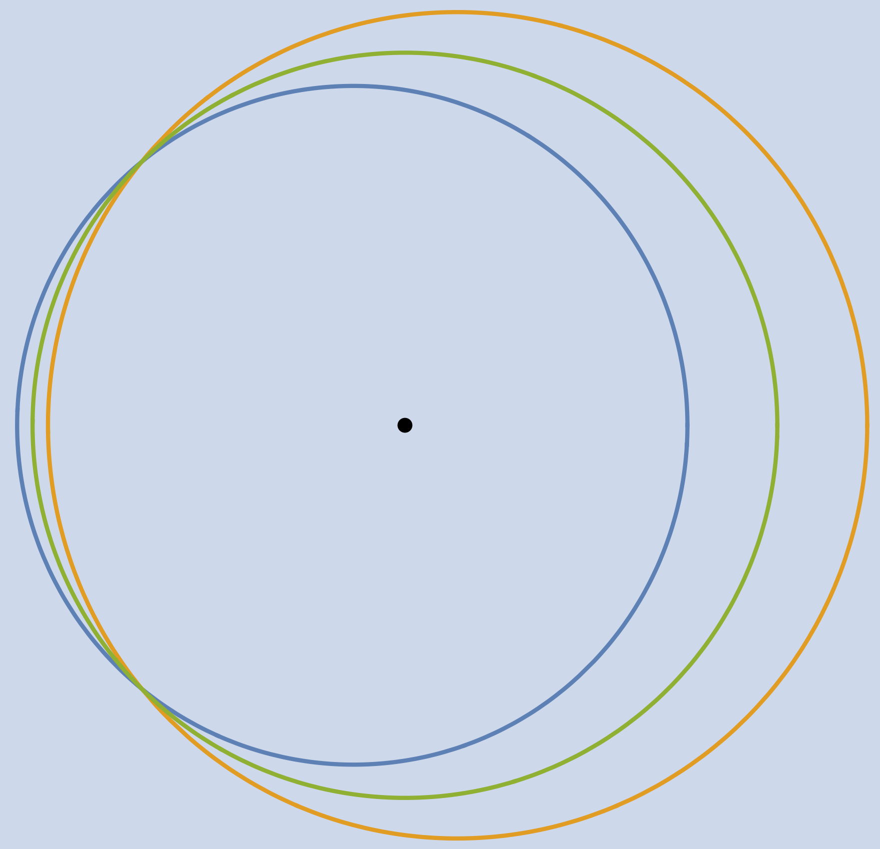
Collisionless



First Sound

$$\frac{1}{\tau_1} \ll \omega \ll \frac{1}{\tau_2}$$

Collisional



- For charged liquids the Coulomb potential turns both into the plasmon

$$\omega \approx \sqrt{\frac{4\pi e^2 n}{m_*}q}$$

Plasmon

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Other spin-valley channels

Well studied and generic to 2D FL

Plasmon $n(\mathbf{r},\mathbf{p}) = \frac{1}{G_s G_v} \operatorname{tr} \hat{\sigma}_0 \hat{\tau}_0 \hat{\rho}(\mathbf{r},\mathbf{p})$

What analogues of first and zero sound exist here?

$$\mathbf{s}(\mathbf{r},\mathbf{p}) = \frac{1}{G_s G_v} \operatorname{tr} \hat{\boldsymbol{\sigma}} \hat{\rho}(\mathbf{r},\mathbf{p})$$

Multi valley materials

$$\mathbf{Y}(\mathbf{r},\mathbf{p}) = \frac{1}{G_s G_v} \operatorname{tr} \hat{\boldsymbol{\tau}} \hat{\rho}(\mathbf{r},\mathbf{p})$$

$$M_i^j(\mathbf{r},\mathbf{p}) = \frac{1}{G_s G_v} \operatorname{tr} \hat{\tau}_i \hat{\sigma}_j \hat{\rho}(\mathbf{r},\mathbf{p})$$

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Neutral sound modes

What kills first and second sound

$$\frac{\partial \delta\rho^{\mu}(\mathbf{k},\mathbf{r})}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\delta\bar{\rho}^{\mu}(\mathbf{k},\mathbf{r}) + \cancel{\left.\frac{\partial n}{\partial \epsilon}\right|_{\bar{\epsilon}} \mathbf{v}\cdot\mathcal{F}^{\mu}} = \boxed{\frac{1}{G_s G_v}\operatorname{tr}\hat{X}^{\mu}\hat{I}[\delta\hat{\rho}]}$$

Zero Sound

$$\omega \gg \frac{1}{\tau}$$

Collisionless

Can be killed by Landau damping

Regime generically exists at low enough temperature

Not guaranteed to exist

First Sound

$$\frac{1}{\tau_1} \ll \omega \ll \frac{1}{\tau_2}$$

Collisional

Can be killed by collisional damping

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Zero sound

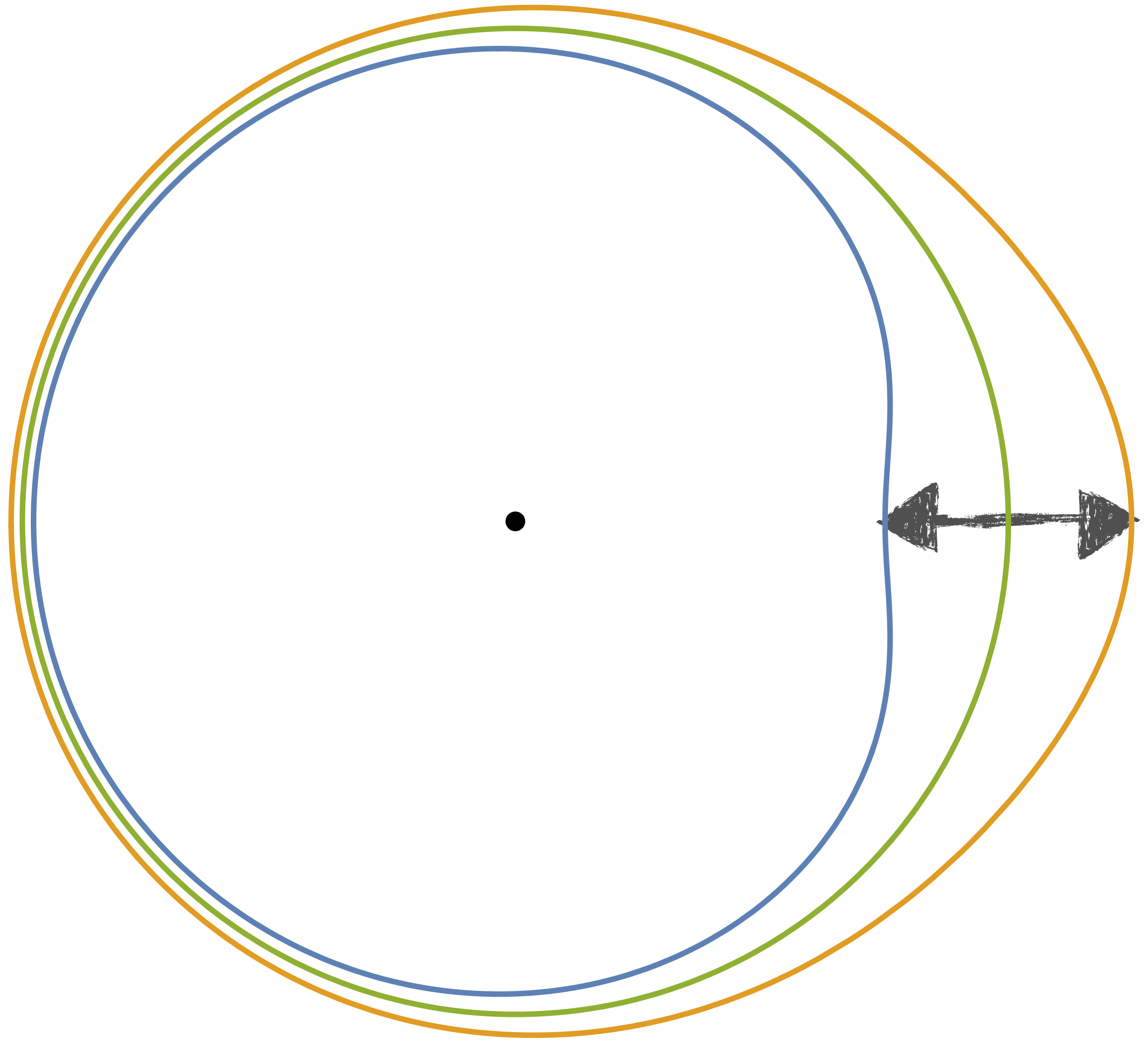
Collisionless equations for the uncharged channels

$$\frac{\partial \delta\rho^\mu(\mathbf{k},\mathbf{r})}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\delta\bar{\rho}^\mu(\mathbf{k},\mathbf{r}) + \left.\frac{\partial n}{\partial \epsilon}\right|_{\bar{\epsilon}} \mathbf{v}\cdot\mathscr{F}^\mu = \cancel{\frac{1}{G_s G_v}\,\mathrm{tr}\,\hat{X}^\mu \hat{I}[\delta\hat{\rho}]}$$

$$\omega \gg \frac{1}{\tau}$$

occurs when

- Zero sound occurs in the collisionless limit
  - Sound oscillations are much faster than relaxation
  - e.g $T \to 0$ since $I \propto (T/E_F)^2$
- Relaxes through **Landau damping**



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Zero sound

Is it damped?

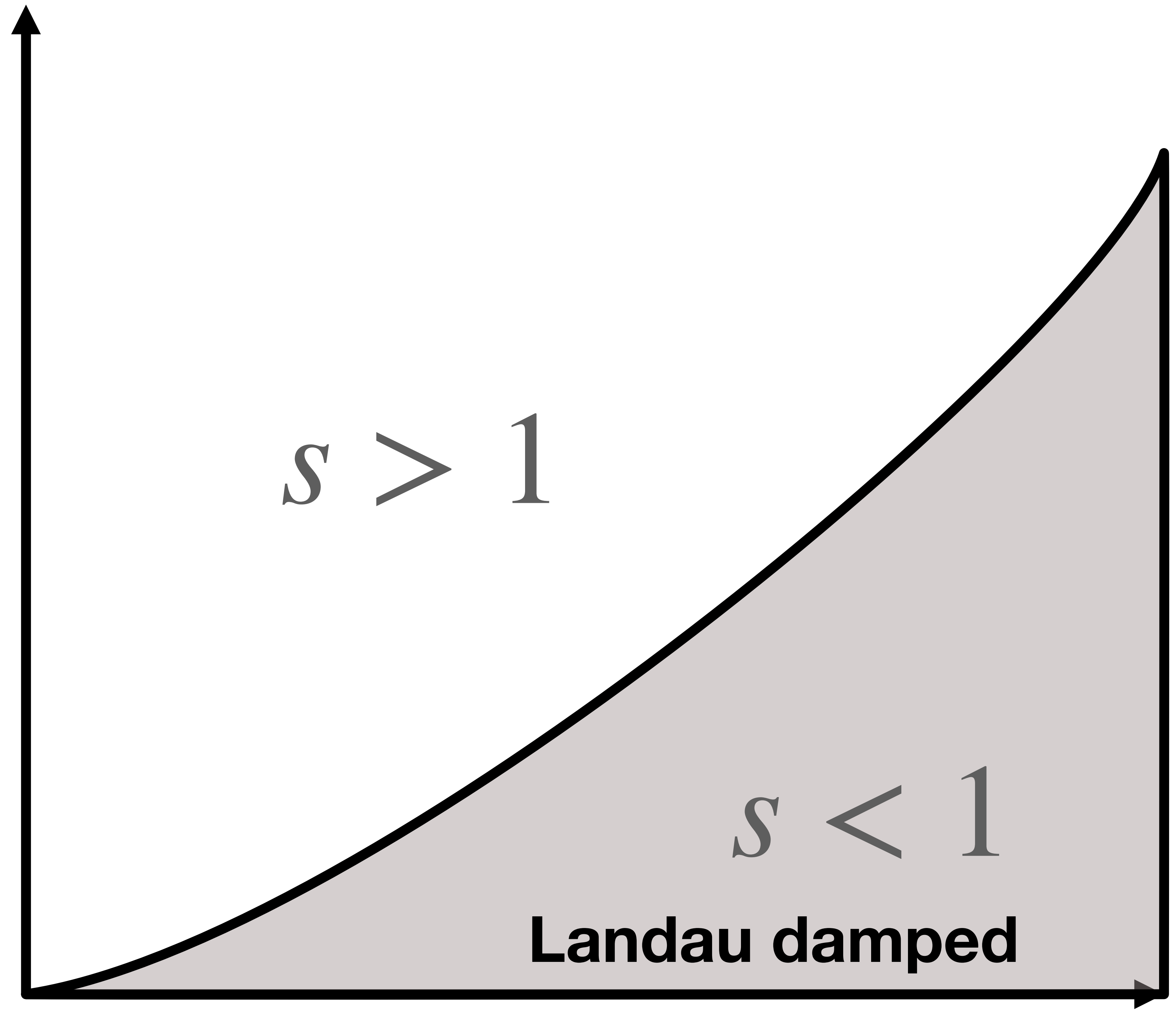
$\omega \gg \frac{1}{\tau}$

$$-i\omega\rho^{\mu}(\mathbf{k},\mathbf{q}) + i\mathbf{v}\cdot\mathbf{q}\delta\bar{\rho}^{\mu}(\mathbf{k},\mathbf{r}) = -\left.\frac{\partial n}{\partial \epsilon}\right|_{\bar{\epsilon}} \mathbf{v}\cdot\mathscr{F}^{\mu}$$

- Natural independent variable

$$|s| \equiv \left|\frac{\omega}{v_F q}\right|$$

- Solutions for $s > 1$ undamped
- Solutions for $s < 1$ Landau damped



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Absence of zero sound

Simplest model

- For model of an attractive constant interaction only model it can be shown there is no zero sound

- Landau damped, $\omega < v_F q$

$$F_0^\mu < 0 \implies |s| \equiv \left| \frac{\omega}{v_F q} \right| < 1$$

Klein, Maslov, Pitaevskii, Chubukov, PRR 1, 033134 (2019)
Klein, Maslov, Chubukov, Npj Quantum Materials 5, 55 (2020)

10.1103/PhysRevB.103.075422

Absence of zero sound

Generic Landau damping

- At low temperature deviations of the occupation function are restricted to the Fermi surface

- This allows us to rephrase the zero sound equation a self consistent integral expression

$$\delta\rho^\mu(\mathbf{p},\mathbf{r}) \equiv -\left.\frac{\partial n}{\partial \epsilon}\right|_{\bar{\epsilon}} \nu^\mu(\phi,\mathbf{r})$$

What is f^μ?

$$\oint \frac{d\phi}{2\pi}(s-\cos\phi')[\nu^\mu(\phi)]^2 = \frac{G_s G_v p_F}{v_F}\oint\oint \frac{d\phi d\phi'}{2\pi}\nu^\mu(\phi) f^\mu(\phi-\phi')\nu^\mu(\phi')$$

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Absence of zero sound

What is f?

- We recall that we estimated the interaction functions by taking matrix elements of the Coulomb potential

- The leading contribution to the interaction functions comes from the Coulomb potential

Long Range

Strongest

$$V(\mathbf{q})$$

$$g_{\perp\perp} \sim V(|\mathbf{K}-\mathbf{K}'|)$$

$$g_{zz} \sim V(|\mathbf{b}|)$$

$$\tilde{g}_{00} \sim \frac{g_{zz}\Delta}{v_F|\mathbf{K}-\mathbf{K}'|}$$

Weakest

Short Range

$$f^{\mu}(\theta) \approx -\frac{1}{2}V\left[2k_F \sin\frac{\theta}{2}\right]\left[\cos^2\left(\frac{\theta}{2}\right) + \frac{\Delta^2}{(\Delta+E_F)^2}\sin^2\left(\frac{\theta}{2}\right)\right]$$

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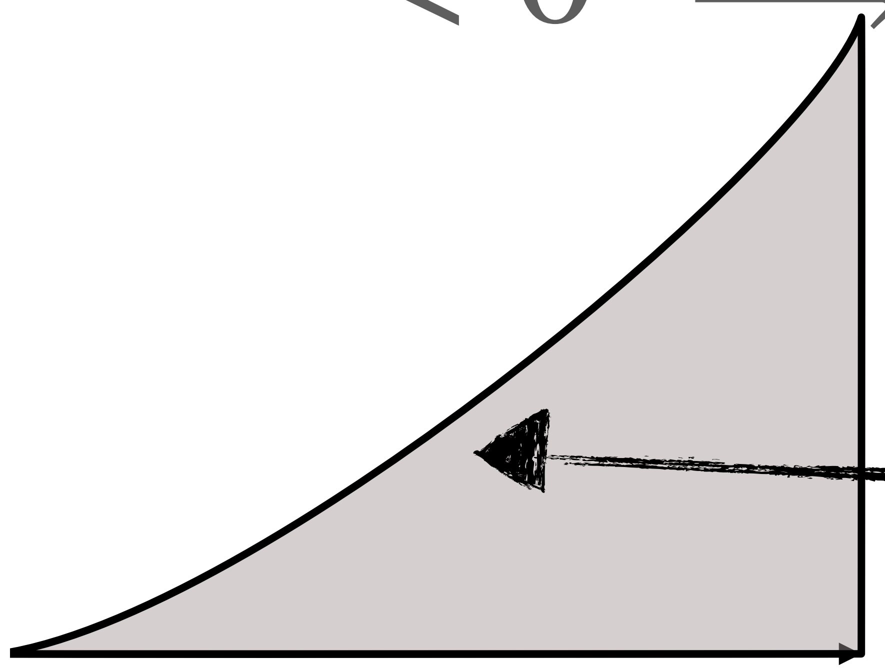
Absence of zero sound

Generic Landau damping

Due to the properties of the Coulomb interaction f^μ is negative definite

$$\oint \frac{d\phi}{2\pi}(s-\cos\phi')[\nu^\mu(\phi)]^2 = \frac{G_s G_v p_F}{v_F}\oint\oint \frac{d\phi d\phi'}{2\pi}\nu^\mu(\phi) f^\mu(\phi-\phi')\nu^\mu(\phi')$$

$< 0 \implies s < 1$ $\qquad$ < 0



Landau damped, $\omega < v_F q$

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What about first sound?

collisional kinetics

$$\frac{\partial \delta\rho^\mu(\mathbf{k},\mathbf{r})}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\delta\bar{\rho}^\mu(\mathbf{k},\mathbf{r}) + \frac{\partial n}{\partial \epsilon}\bigg|_{\bar{\epsilon}} \mathbf{v}\cdot\mathscr{F}^\mu = \frac{1}{G_s G_v}\,\mathrm{tr}\,\hat{X}^\mu \hat{I}[\delta\hat{\rho}]$$

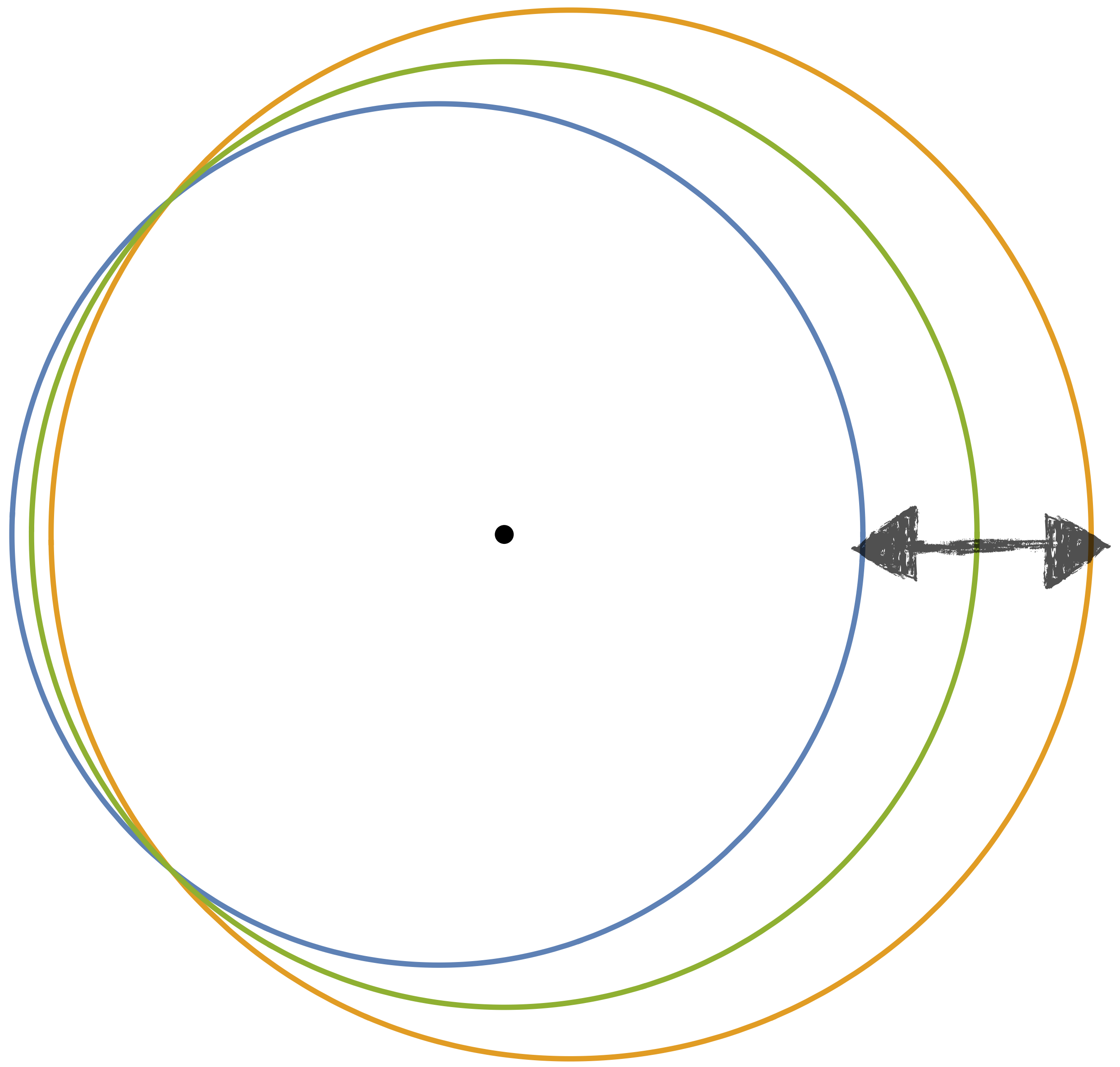
occurs when

$$\frac{1}{\tau_1} \ll \omega \ll \frac{1}{\tau_2}$$

$$\delta\rho^\mu(\mathbf{k},\mathbf{r}) = -\frac{\partial n}{\partial \epsilon}\sum_m e^{im\phi_{\mathbf{k}_F}}\nu_m^\mu(\mathbf{r})$$

$$\frac{\partial n}{\partial \epsilon}\sum_m \frac{\nu_m^\mu}{\tau_m^\mu}$$

?



- Existence of first sound rests on the behavior of the collision integral
- Specifically the relation between scattering time for different angular harmonics on the Fermi surface

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What about first sound

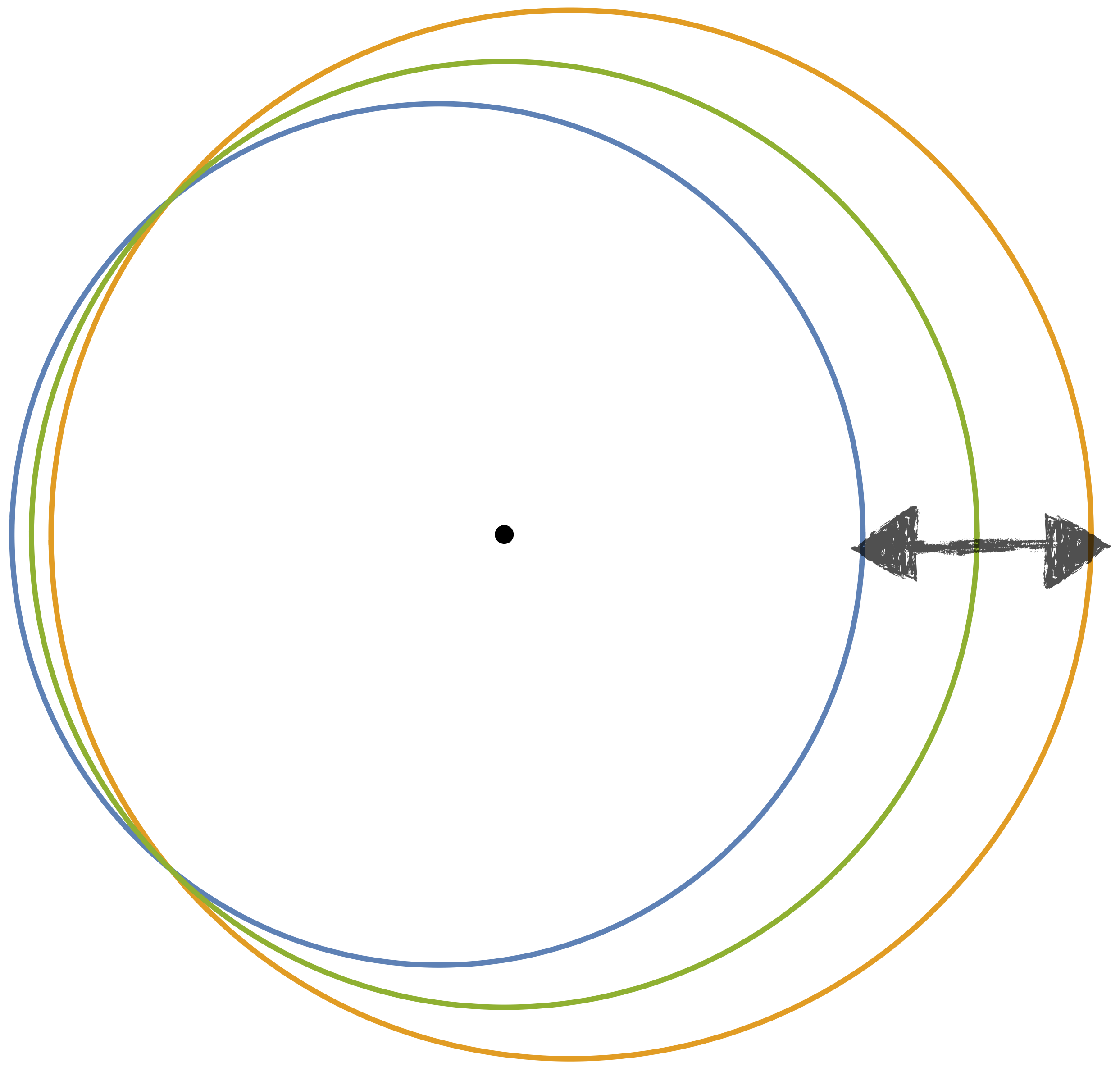
is there a hydrodynamic regime in neutral channels

$m = 0$	$\sum_p \delta\rho_p^\mu$	**Density**	**Conserved**
$m = \lvert 1 \rvert$	$\sum_p \cos\phi_{p_F}\delta\rho_p^\mu$	**Current**	
$m = \lvert 2 \rvert$	$\sum_p \cos^2\phi_{p_F}\delta\rho_p^\mu$		**Not**
	$\vdots$		

occurs when

$$\frac{1}{\tau_1} \ll \omega \ll \frac{1}{\tau_2}$$

?



- Existence of first sound rests on the behavior of the collision integral
- Specifically the relation between scattering time for different angular harmonics on the Fermi surface

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What about first sound

is there a hydrodynamic regime in neutral channels

$$\frac{1}{\tau_1} \ll \omega \ll \frac{1}{\tau_2} \propto T^2$$

Behaves similar to the charge channel

?

We have to evaluate the collision integral to know

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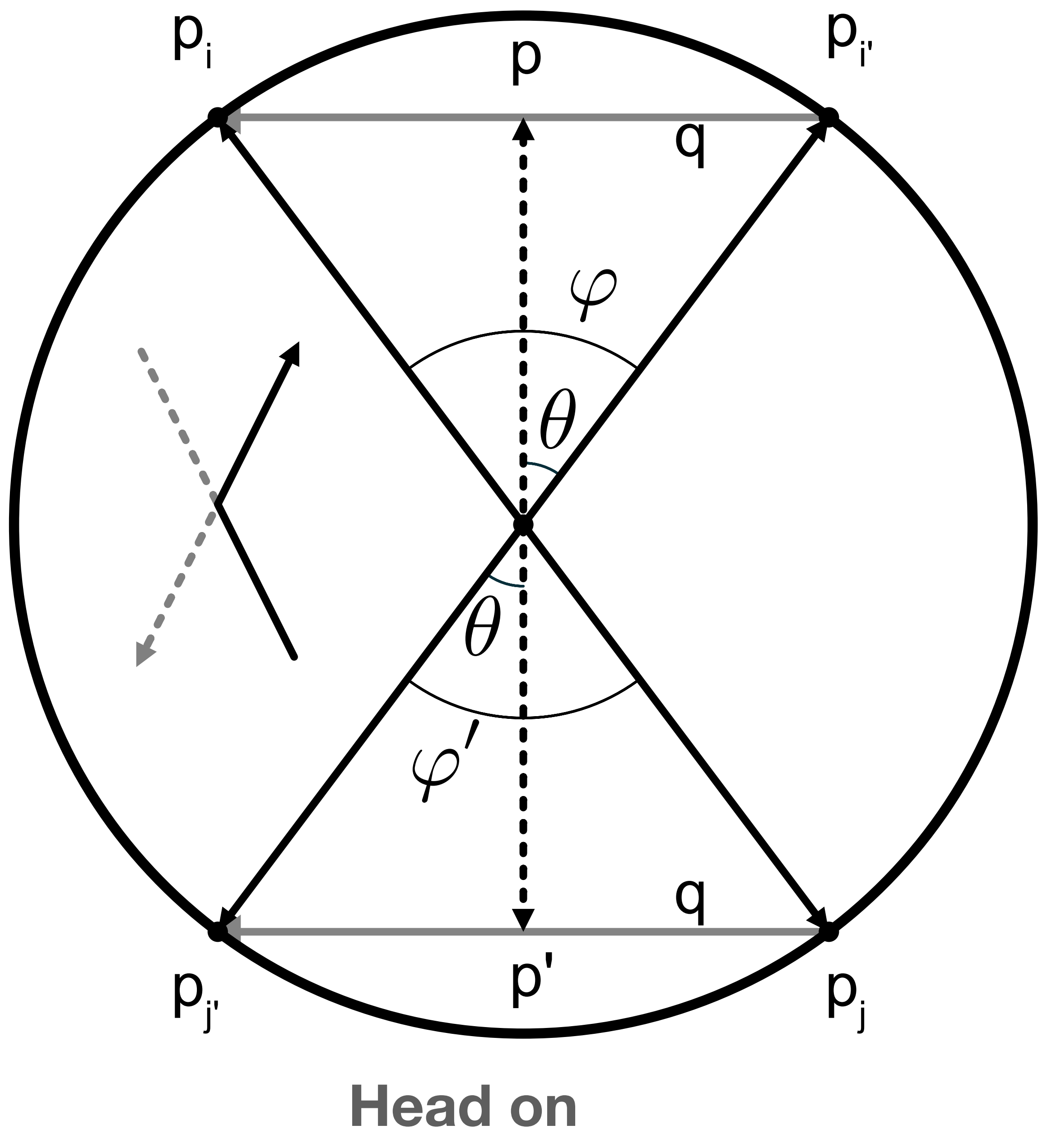
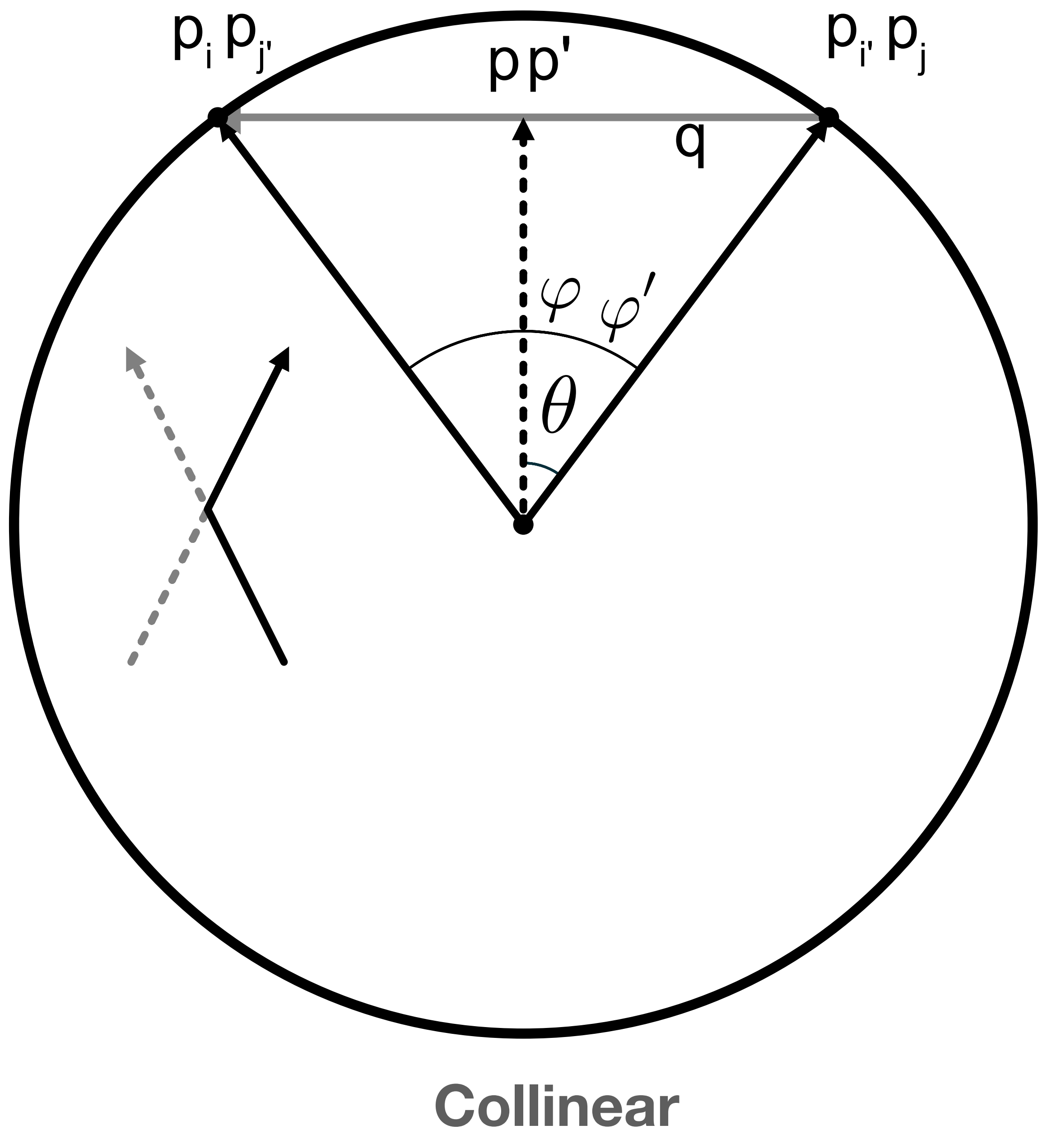
Collision integral in 2D

Allowed scattering processes

- At low temperatures collision are restricted to the Fermi surface
- There are two types of allowed scattering processes





Laikhtman, PRB 45, 1259 (1992)

Ledwith, Guo, Levitov, Ann. of Phys. 411, 167913 (2019)

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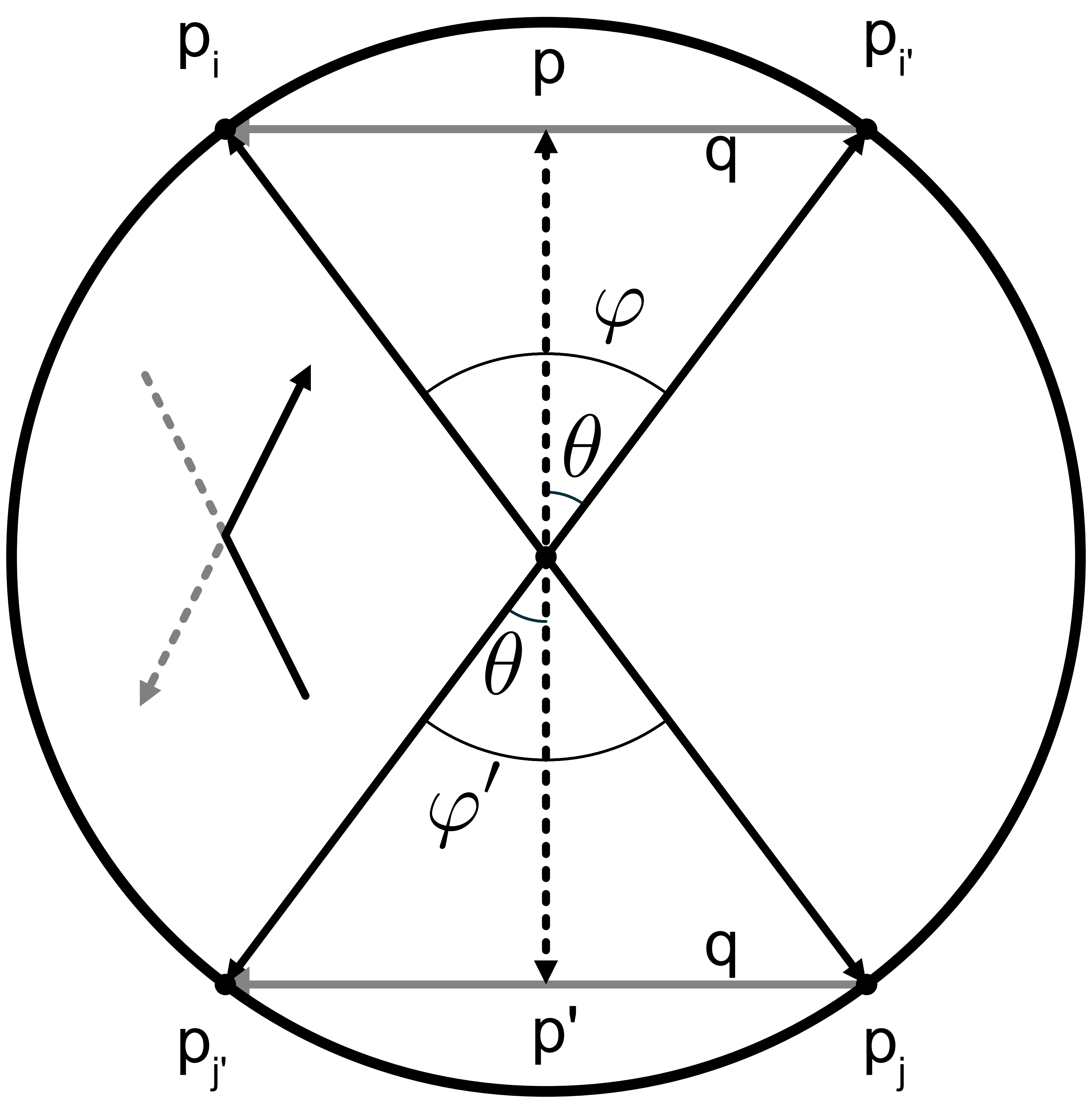
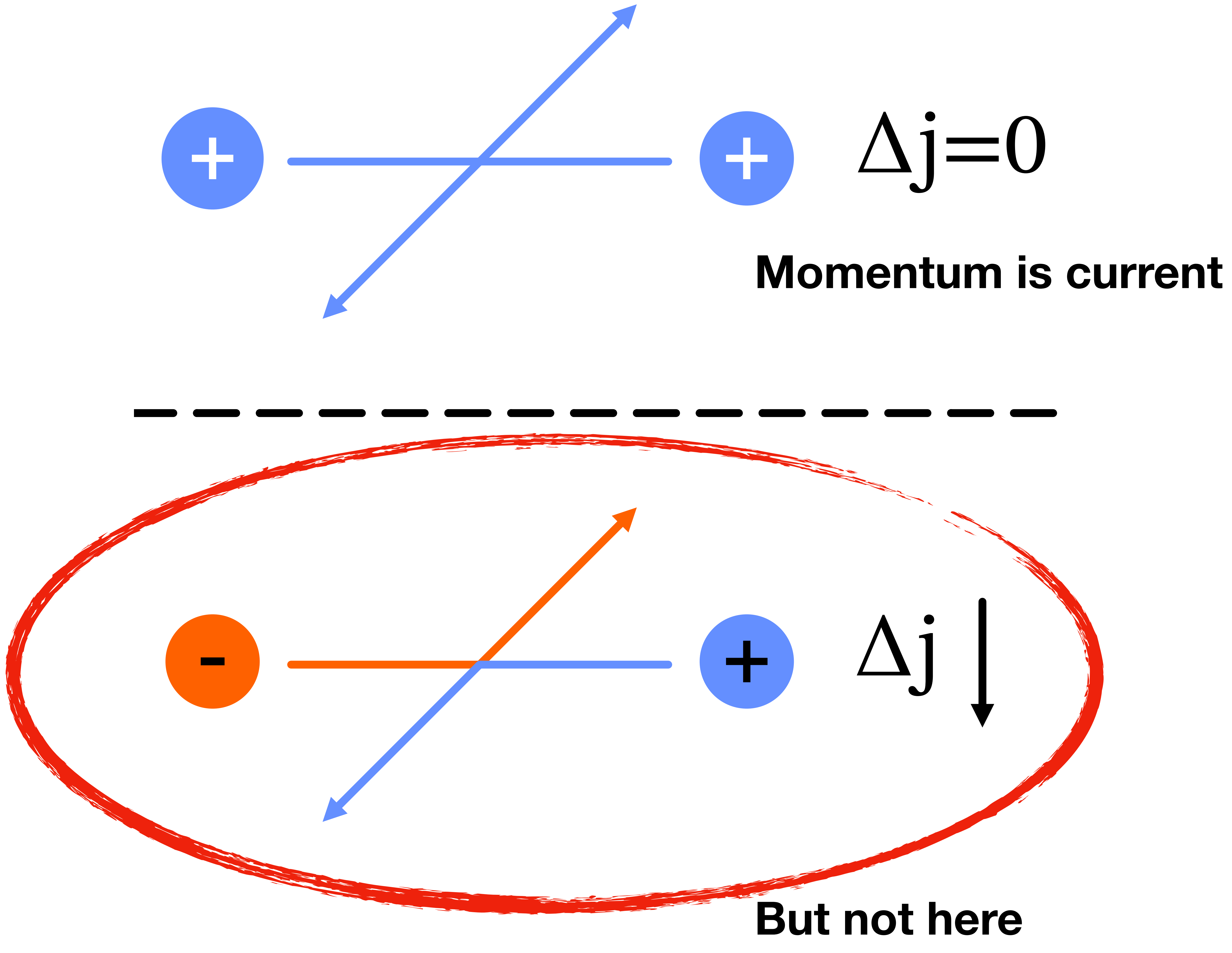
Neutral channels

How do these modes relax?





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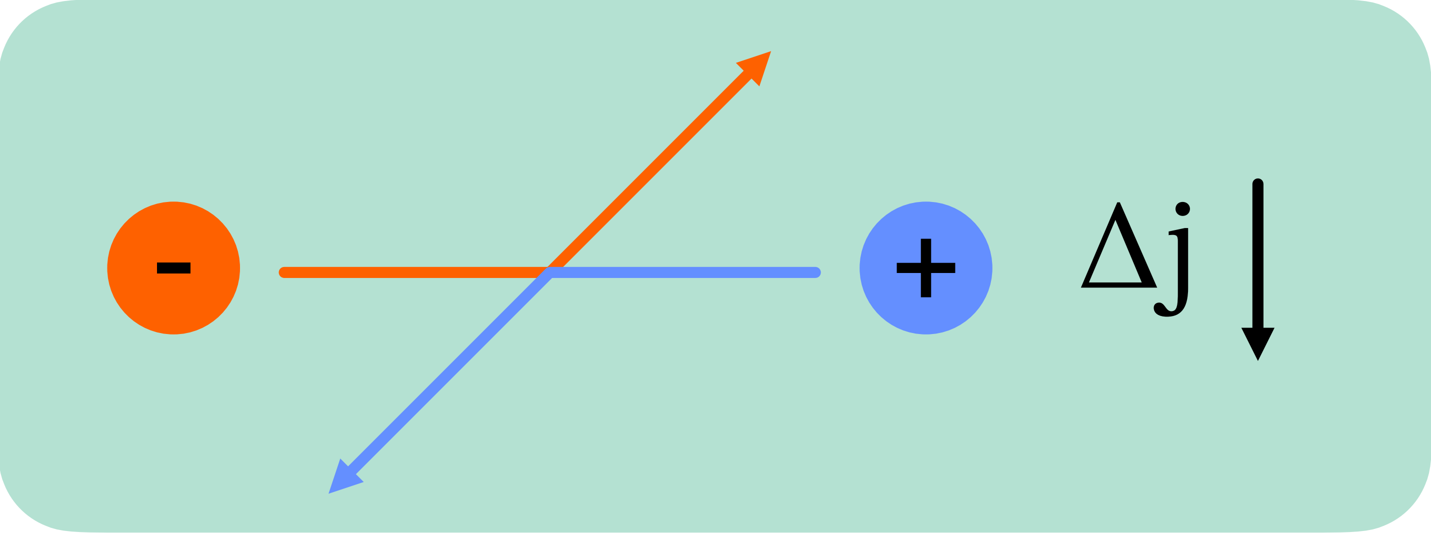
Collision integral

again by symmetry distinct channels

$$I(\mathbf{p}_i, \alpha) = -\frac{1}{T}\sum_{\beta\gamma\delta}\sum_{\mathbf{p}_j\mathbf{p}_{i'}\mathbf{p}_{j'}} (2\pi)^2\delta\Big(\sum_J{}' \mathbf{p}_J\Big)2\pi\delta\Big(\sum_J{}' \epsilon_J\Big) n_i n_j (1-n_{i'})(1-n_{j'}) W^{\alpha\beta;\gamma\delta}_{ij;i'j'}\left[\bar{\nu}_{i\alpha} + \bar{\nu}_{j\beta} - \bar{\nu}_{i'\gamma} - \bar{\nu}_{j'\delta}\right]$$

Transforming from spin-valley indices to symmetry distinct channels

Focusing on these types of scattering



$$\mathrm{tr}(\hat{X}^\mu \hat{I}) \to I^\mu_-(\mathbf{p}_i) = -\frac{1}{T}\sum_{\mathbf{p}_j\mathbf{p}_{i'}\mathbf{p}_{j'}} (2\pi)^2\delta\left(\sum_J{}' \mathbf{p}_J\right)2\pi\delta\left(\sum_J{}' \epsilon_J\right) n_i n_j (1-n_{i'})(1-n_{j'}) W^\mu_- \left(\bar{\nu}^\mu_i - \bar{\nu}^\mu_j - \bar{\nu}^\mu_{i'} + \bar{\nu}^\mu_{j'}\right)$$

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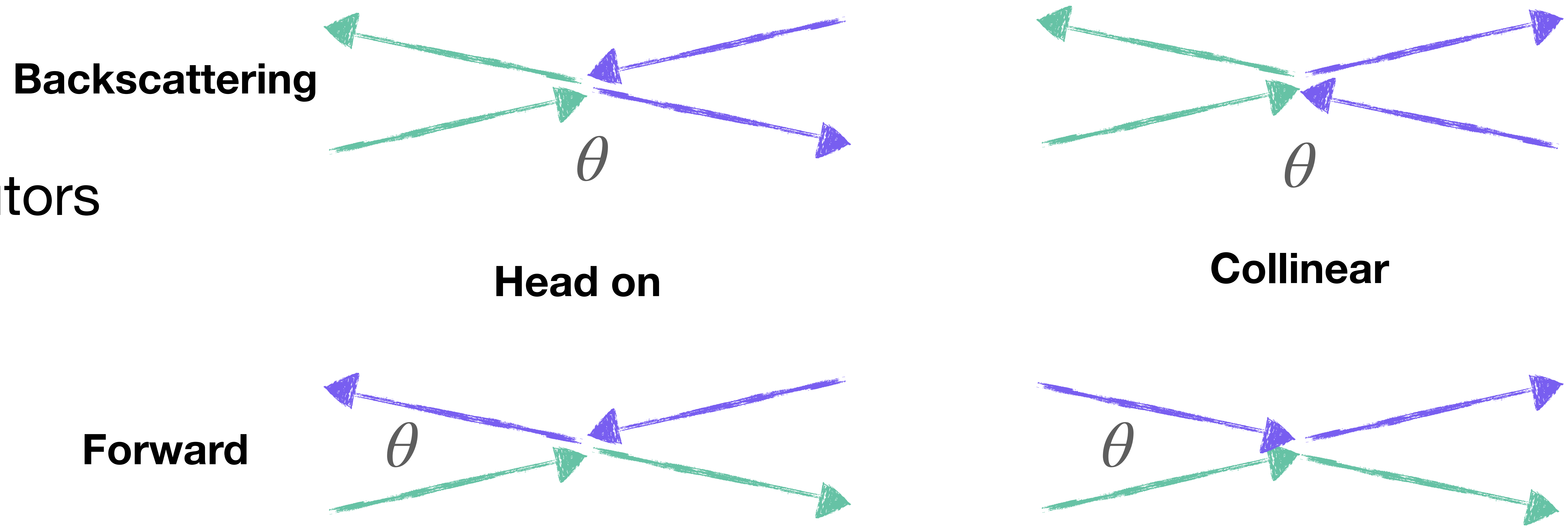
Neutral channels

Dominant contributions

- Phase space for scattering in 2D is divergent for forward and back scattering
- These are the leading contributors to transport scattering
- The two have different origins

$$\frac{1}{\tau_1^{\mu}} = \propto T^2 \int_0^{\pi-\theta_c} \frac{d\theta_{\text{sc}}}{\sin\theta_{\text{sc}}} (1-\cos\theta_{\text{sc}}) \left(W^{\mu}_{-,\text{collinear}}(\theta_{\text{sc}}) + W^{\mu}_{-,\text{head-on}}(\theta_{\text{sc}}) \right).$$



10.1103/PhysRevB.103.075422

Neutral channels

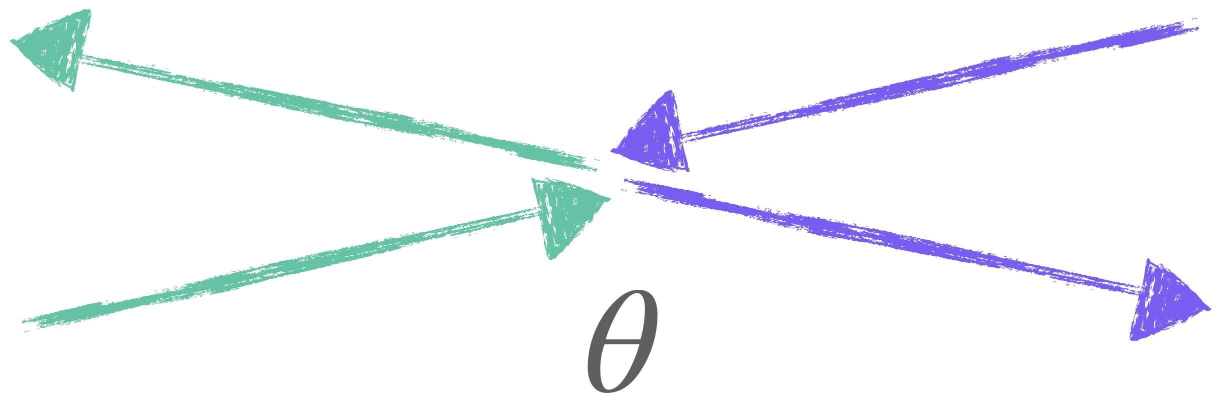
Backscattering

- Phase space for scattering in 2D is divergent for forward and back scattering

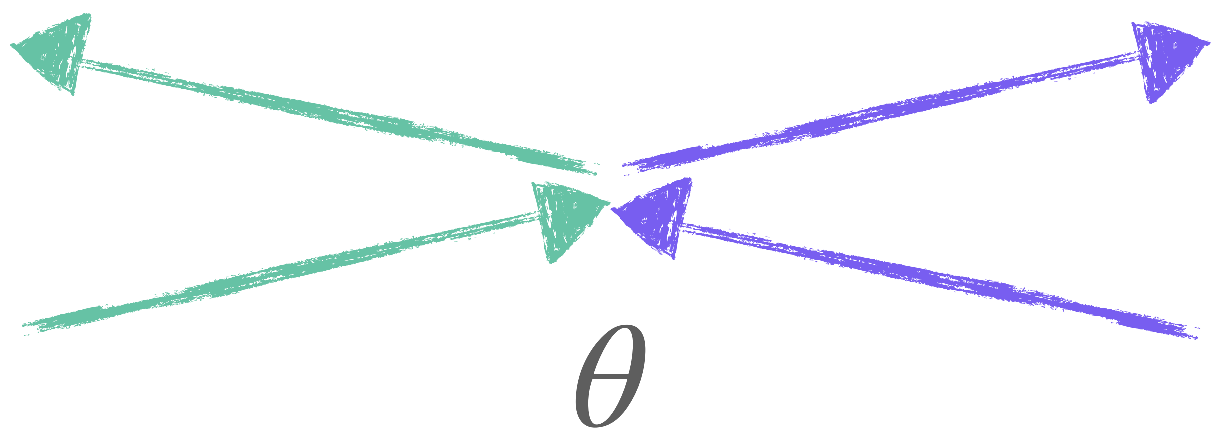
- Backscattering contribution is due to short ranged interactions

$$\lim_{\theta\to\pi} \frac{d\theta_{SC}}{\sin\theta_{SC}}(1-\cos\theta_{SC}) \to \infty$$

Head on



Collinear



Backscattering is cut off by kinematic constraints on scattering $\theta \leq \pi - \theta_c$

$$(2\pi)^2\delta\left(\sum_J{}' \mathbf{p}_J\right) 2\pi\delta\left(\sum_J{}' \epsilon_J\right)$$

$$\ln T$$

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Neutral channels

Backscattering

- Phase space for scattering in 2D is divergent for forward and back scattering
- Backscattering contribution is due to short ranged interactions
- Forward scattering due to long ranged part of Coulomb

$$\lim_{\theta\to 0} \frac{d\theta_{\text{sc}}}{\sin\theta_{\text{sc}}}(1-\cos\theta_{\text{sc}})\,|V_{\text{Coulomb}}(\theta_{\text{sc}})|^2 \to \infty$$

Head on **Collinear**

Forward scattering is cut off by the Thomas-Fermi scattering wave vector

$$\ln q_{TF}$$

10.1103/PhysRevB.103.075422

Neutral channels

Dominant contributions

- Phase space for scattering in 2D is divergent for forward and back scattering
- Leading log dependence comes from these regions

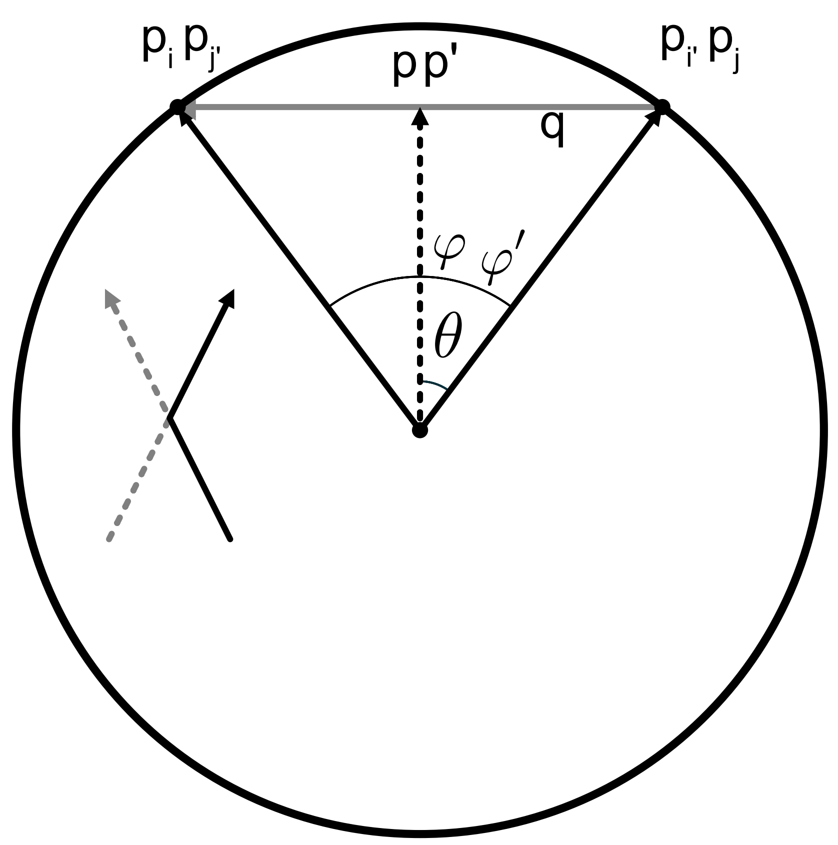
$$\frac{1}{\tau^{\mu}_{tr}} \equiv \frac{1}{\tau^{\mu}_{1}} \approx \frac{1}{\tau^{\mu}_{1,\text{Backscatter}}} + \frac{1}{\tau^{\mu}_{1,\text{Forward}}}$$

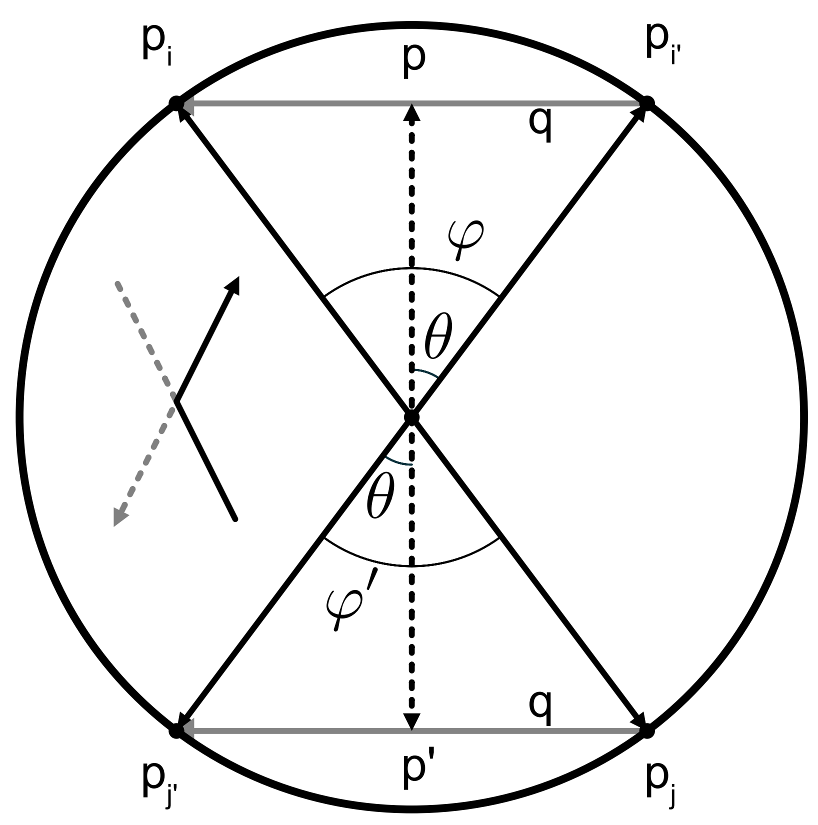
Short range

$$\frac{1}{\tau_{\text{Backscatter}}} \propto T^2 \ln \frac{\sqrt{\mu^2 - \Delta^2}}{T}$$

Long ranged (screened) Coulomb

$$\frac{1}{\tau_{\text{Forward}}} \propto T^2 \ln \frac{\sqrt{\mu^2 - \Delta^2}}{v q_{TF}}$$





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Transport rates

Two regimes

$$\Delta \sim E_F$$ **Non-relativistic**

- Generically the rates scale $T^2 \log(\cdots)$ with prefactors dependent on the regime

$$\frac{1}{\tau^{\mu}_{\mathrm{tr}}(T)} \propto \alpha^2 \frac{T^2}{E_F^2} \left(\ln(T/E_F) + \ln(E_F/vq_{TF}) \right)$$

- In the relativistic case backscattering is suppressed due to Berry curvature effects

$$\Delta \ll E_F$$ **Relativistic**

- Only short ranged interactions contribute to the backscattering amplitudes

$$\frac{1}{\tau^{\mu}_{\mathrm{tr}}(T)} \propto \frac{T^2}{E_F^2} \left\{ g^2 \ln(E_F/T) + \alpha^2 \ln(E_F/vq_{TF}) \right\}$$

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No neutral first sound

Not hydrodynamics, but diffusion

- There is no frequency regime in which neutral first sound is not overdamped
- Finite temperature transport in neutral channels is ultimately diffusive

$$\cancel{\frac{1}{\tau_1} \ll \omega \ll \frac{1}{\tau_2}}$$

$$\frac{1}{\tau_1^\mu} \propto T^2 \left[\ln(E_F/T) + \ln(E_F/vq_{TF})\right]$$

$$\gtrsim$$

$$\frac{1}{\tau_2^\mu} \propto T^2$$

First sound regime is "squeezed out"

$$\omega\tau_1 \ll 1$$

$$D^\mu \approx \frac{v_F^2}{2}\tau^\mu_{\text{tr}}(1 + F_0^\mu)$$

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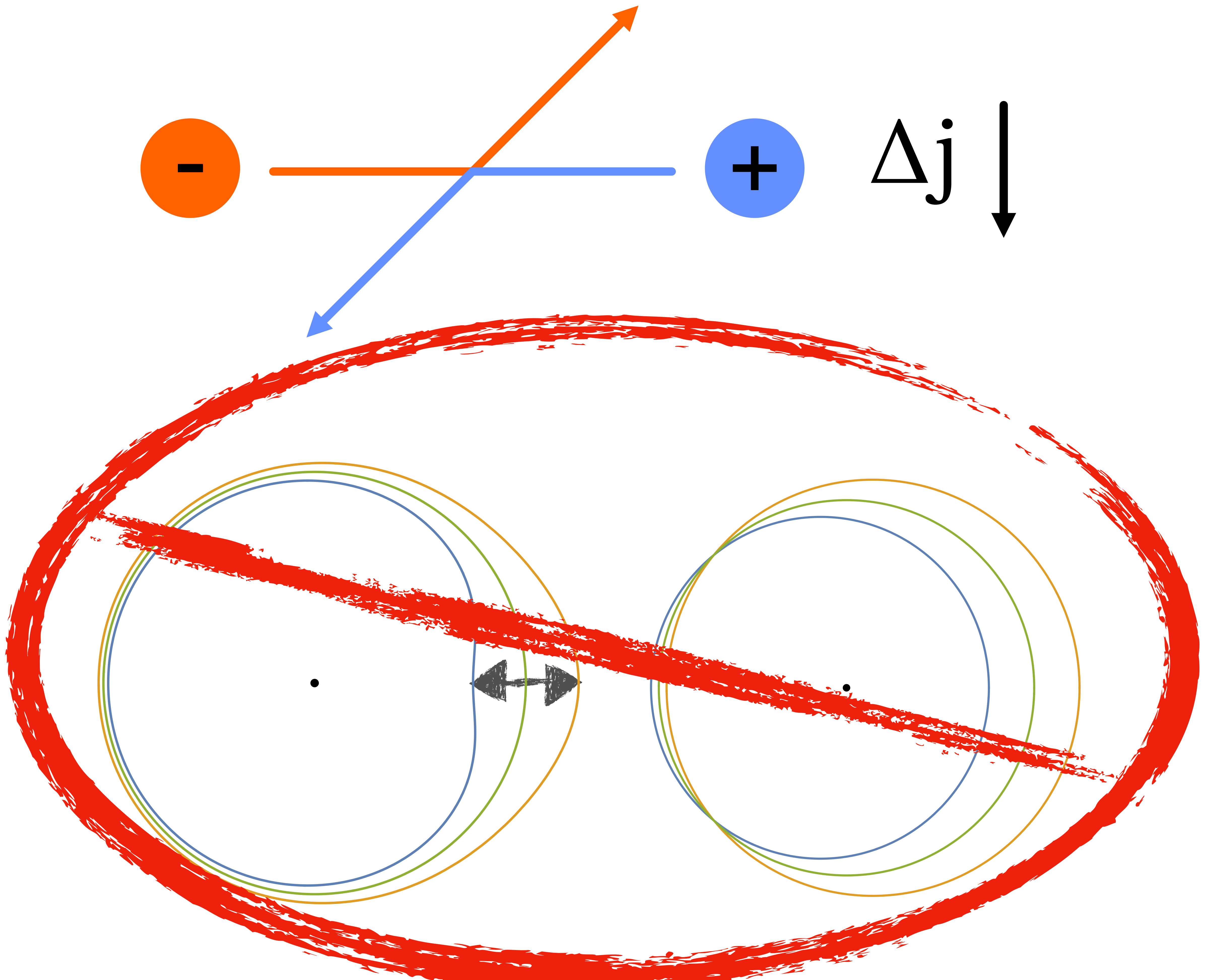
Summary

for Part I

- Symmetry dictated Fermi liquid theory of graphene
- Neutral zero sound and first sound in graphene are absent for all spin-valley channels.
- Transport of spin-valley quantum numbers is generically diffusive



ZMR, Fal'ko, Glazman, PRB, 2020

Part II - Magnetic fields and extrinsic SOC

Absence of spin zero sound and Silin modes

- Arguments of the previous section generally lead to spin zero modes being overdamped

- This is commonly true in $SU(2)$ spin invariant Fermi liquids

- In finite magnetic field there are undamped collective excitations in the spin channel

- A similar picture holds in the presence of spin orbit coupling

SOVIET PHYSICS JETP VOLUME 6 (33), NUMBER 5 May, 1958

OSCILLATIONS OF A FERMI-LIQUID IN A MAGNETIC FIELD

V. P. SILIN

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Received by JETP editor May 6, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 1227-1234 (November, 1957)

A study is made of the spin oscillations of a paramagnetic Fermi-liquid (He^3) placed in a constant magnetic field at low temperatures, where collisions can be ignored.

Spin diffusion and spin echoes in liquid ^3He at low temperature

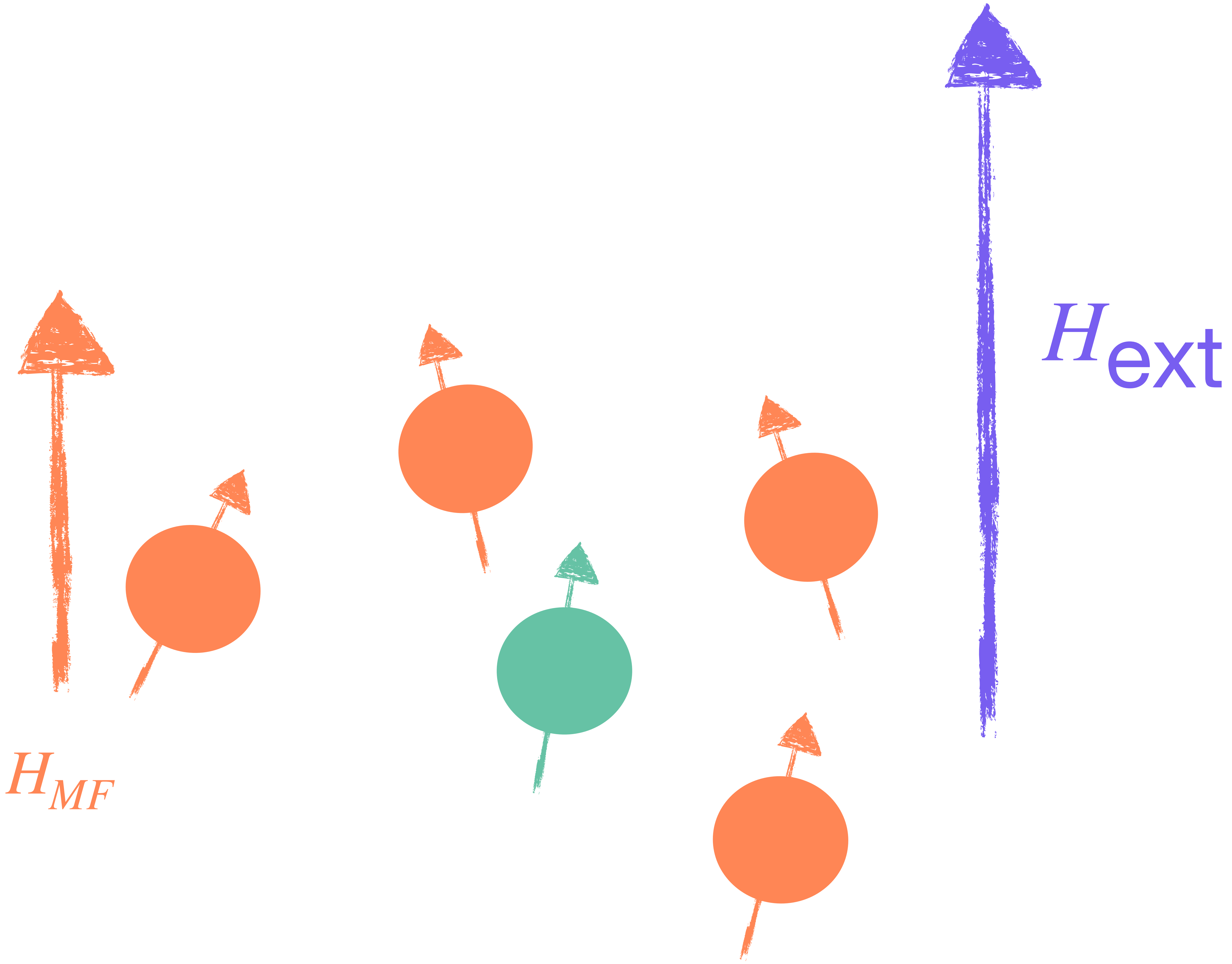
A. J. LEGGETT

School of Mathematical and Physical Sciences, University of Sussex, Falmer, Brighton

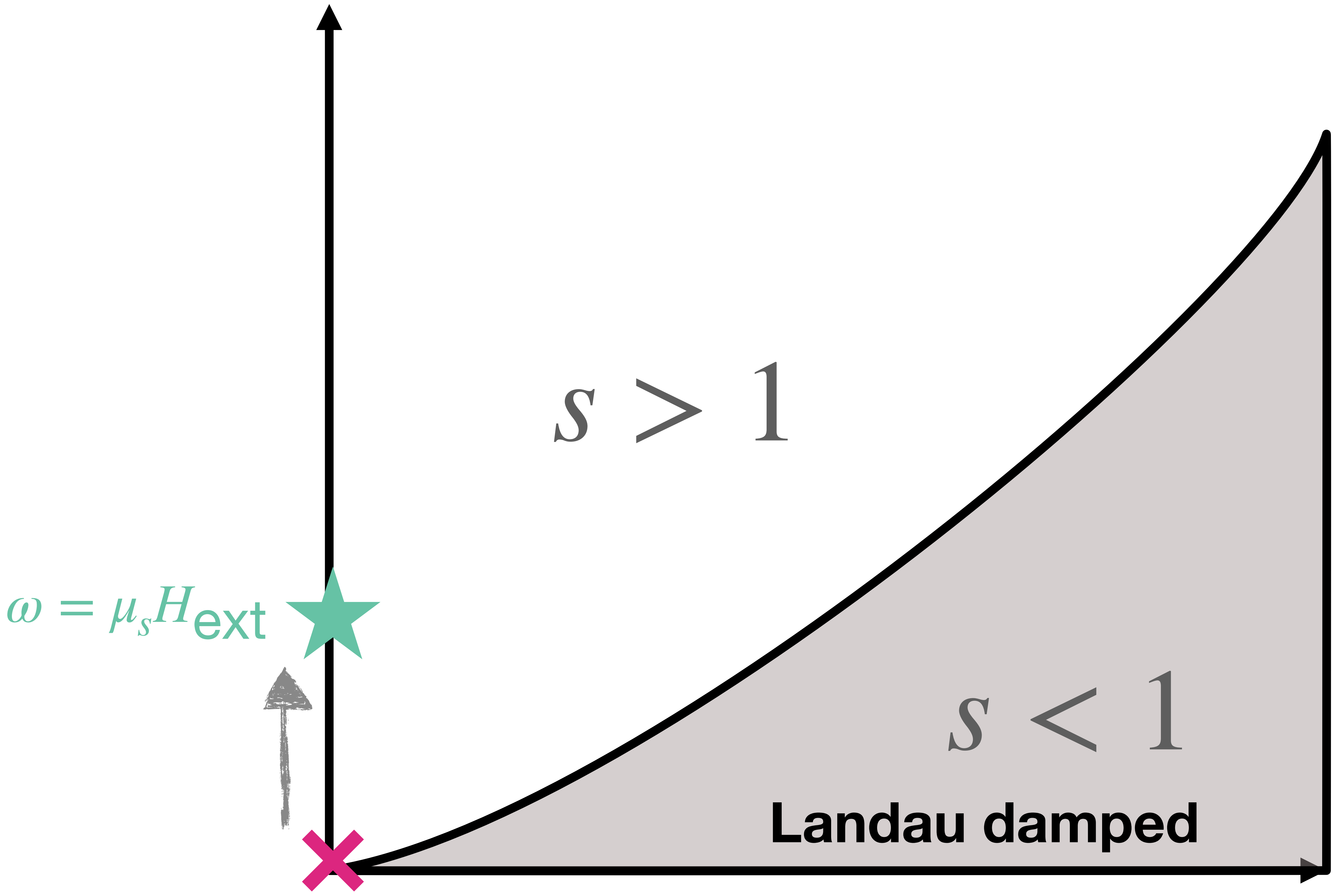
MS. received 3rd *July* 1969, *in revised form* 29*th September* 1969

Spin oscillations in magnetic field

Silin-Legget mode



OSCILLATIONS OF A FERMI-LIQUID IN A MAGNETIC FIELD

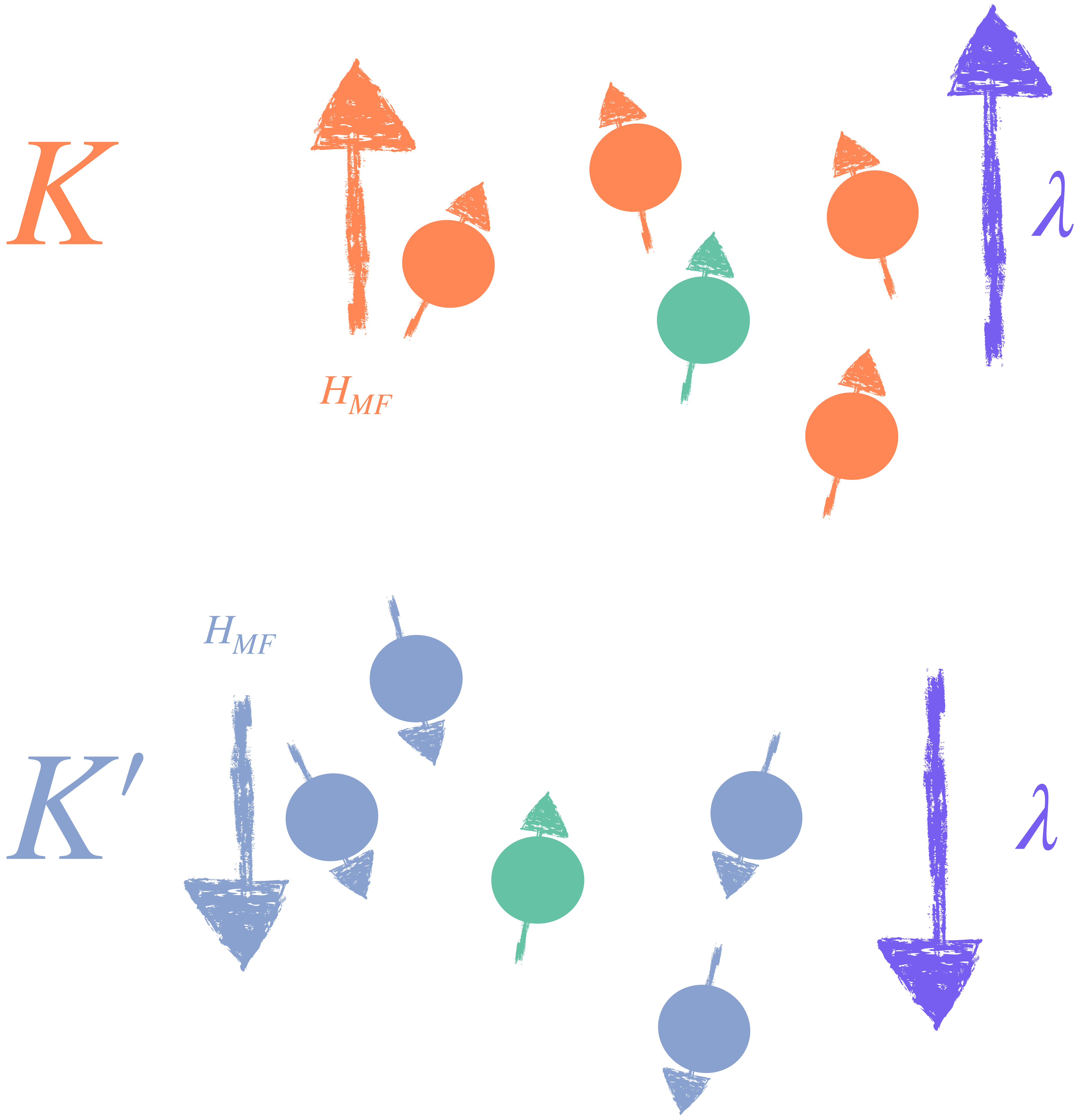


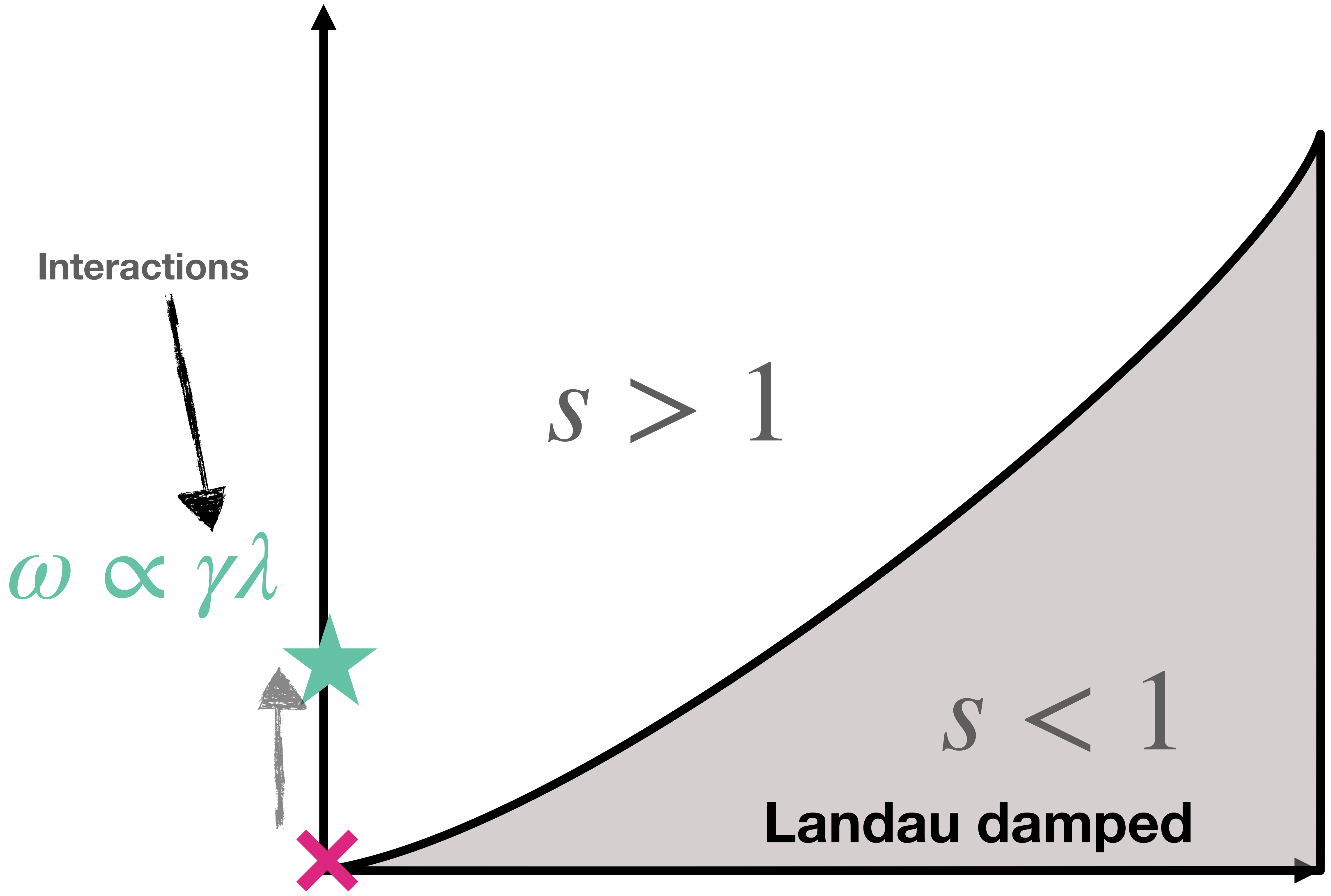
Kohn, PR 123, 1242 (1961)

Spin oscillations with SOC

Stein, v. Klitzing, Weimann, PRL 51, 130 (1983)

Dickmann, Kukushkin, PRB 71, 241310 (2005)

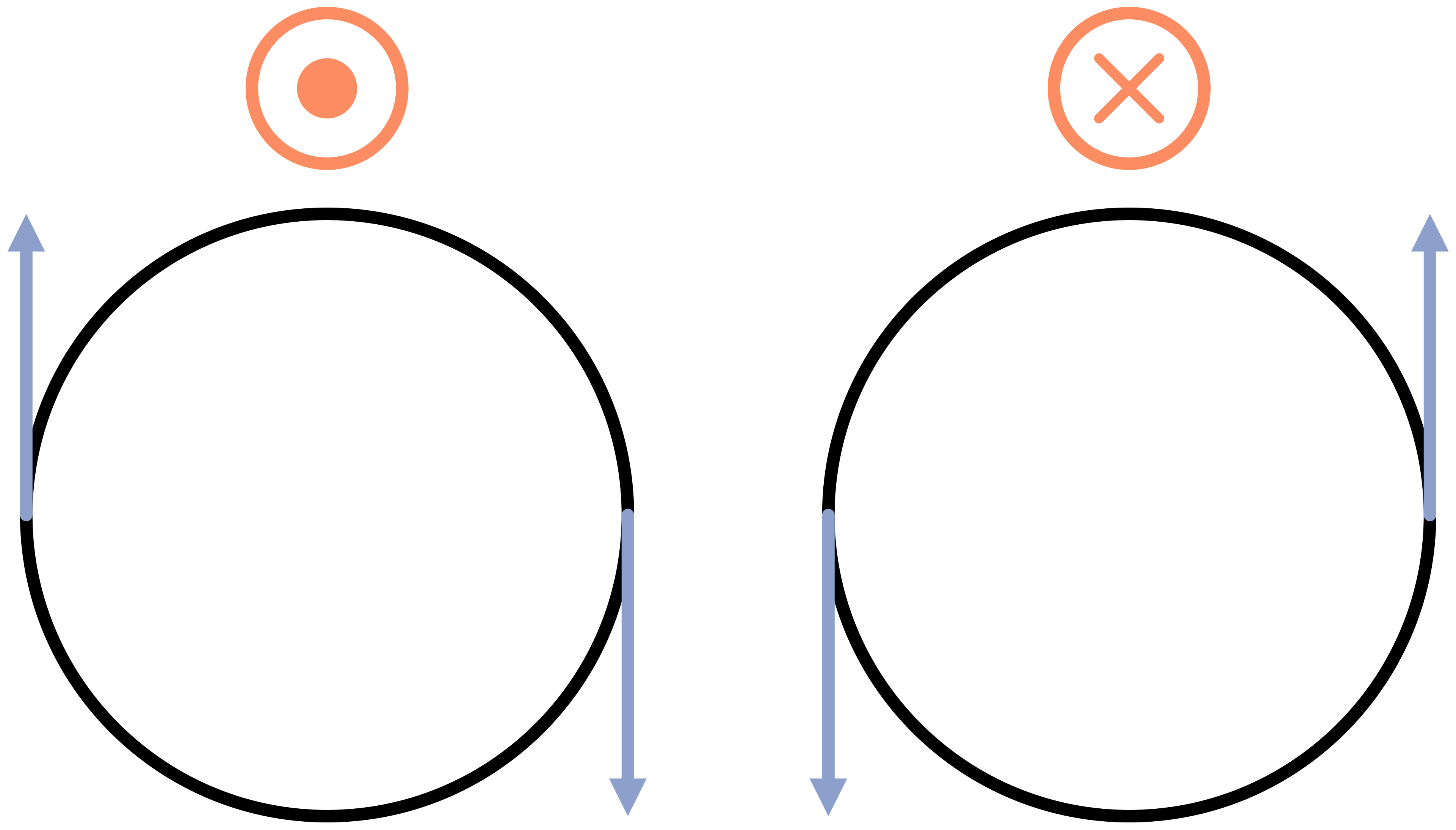




Extrinsic SOC in graphene

- Sub lattice symmetry breaking induced gap Δ
- Valley Zeeman λ
- Valley Rashba λ_R

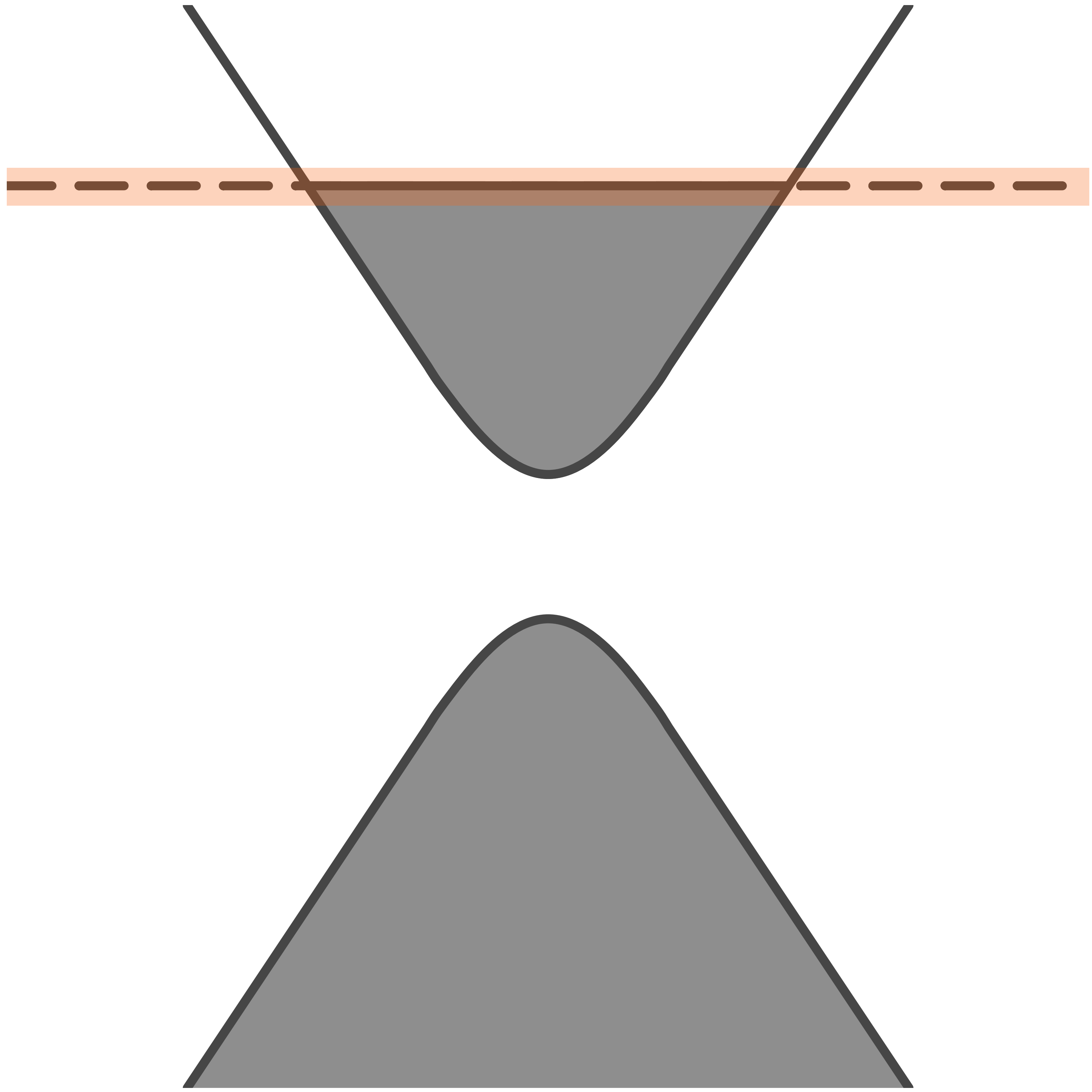
$$H_{\mathbf{p}} = v_D \mathbf{p} \cdot \boldsymbol{\Sigma} + \Delta \Sigma_z \tau_z + \lambda \tau_z \sigma_z + \lambda_R \hat{z} \cdot (\sigma \times \boldsymbol{\Sigma}).$$



e.g. Wang et al., PRX 6, 041020 (2016)

Extrinsic SOC in graphene

In the upper band



e.g. Wang et al., PRX 6, 041020 (2016)

$$H_{\mathbf{p}} = v_D \mathbf{p} \cdot \mathbf{\Sigma} + \Delta \Sigma_z \tau_z + \lambda \tau_z \sigma_z + \lambda_R \hat{z} \cdot (\sigma \times \mathbf{\Sigma}) \,.$$

Upper band projection

$$H_p^+ = \epsilon_p + a(p)\hat{z} \cdot (\sigma \times \mathbf{p}) + \lambda \hat{\sigma}_z \hat{\tau}_z$$

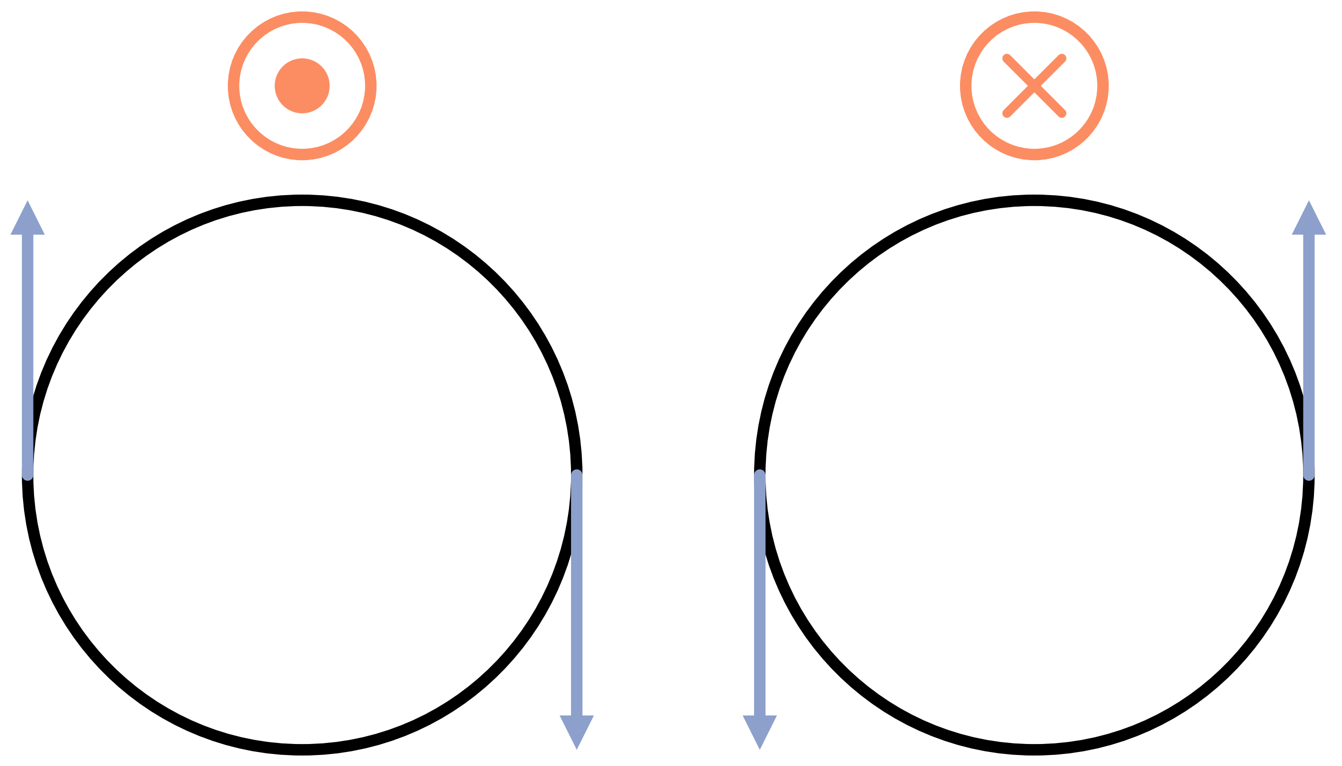
Effective Rashba Coupling $a(p) = v_D \lambda_R / \epsilon_p$

SOC Fermi liquid graphene

with magnetic field

- Previous Fermi Liquid theory plus
  - Zeeman coupling to in plane magnetic field
  - Extrinsic SOC
  - Coupling to external AC electric field $\frac{e}{c}\mathbf{A} \cdot \frac{\partial \hat{H}_{\mathbf{p}}}{\partial \mathbf{p}}$

$$U(\mathbf{p},\mathbf{p}',\mathbf{q}) = U^{d}_{\mathbf{p},\mathbf{p}',\mathbf{q}} + U^{s}_{\mathbf{p},\mathbf{p}',\mathbf{q}}\sigma\cdot\sigma + U^{v\parallel}_{\mathbf{p},\mathbf{p}',\mathbf{q}}\tau^{\parallel}\cdot\tau^{\parallel}$$
$$+U^{vz}_{\mathbf{p},\mathbf{p}',\mathbf{q}}\tau^{3}\tau^{3} + U^{m\parallel}_{\mathbf{p},\mathbf{p}',\mathbf{q}}\tau^{\parallel}\cdot\tau^{\parallel}\sigma\cdot\sigma + U^{mz}_{\mathbf{p},\mathbf{p}',\mathbf{q}}\tau^{3}\tau^{3}\sigma\cdot\sigma$$



EDSR of spin-valley modes

- In the presence of SOC, electric field couples to spin
- Here the response to the AC electric field of the probe is much stronger than to the AC magnetic field

$$H_E^+ = \frac{\mathbf{j} \cdot \mathbf{E}}{i\omega} \gg H_{\mathbf{B}}^+ = \frac{1}{2}\mu_s \mathbf{B} \cdot \sigma$$

$$\frac{\lambda_R}{\omega} \frac{c}{v_D} \frac{m_e}{m_*} \gg 1$$

Rashba, Soviet Physics Uspekhi 7, 823 (1965)

Rashba, Efros, Phys Rev Lett 91, 126405 (2003)

Maiti, Zyuzin, Maslov, PRB 91, 035106 (2015)

Equilibrium distribution

- In the presence of Zeeman and SOC, equilibrium density matrix has finite spin and valley-spin components
- At lowest order these components are proportional to the energy change from the added terms

$$\hat{\epsilon}_{\text{eq}} = \tilde{\epsilon} - \left(\frac{1}{2} \tilde{\mu}_s \mathbf{H}_0 - \tilde{a}(p)(\mathbf{p} \times \hat{z}) \right) \cdot \sigma + \tilde{\lambda} \sigma_z \tau_z$$

$$\hat{\rho}_0 = n_F + \mathbf{s}_{\text{eq}} \cdot \sigma + \mathbf{M}^z_{\text{eq}} \cdot \sigma \tau_z$$

Interactions with the equilbrium polarizations renormalize the effective couplings

$$\tilde{\mu}_s = \mu_s \frac{1}{1 + F_0^s}, \qquad \tilde{a}(p) = \frac{a(p)}{1 + F_1^s}, \qquad \tilde{\lambda} = \frac{\lambda}{1 + F_0^{mz}}$$

Collisionless transport equation

T = 0

$$\frac{\partial \delta\hat{\rho}}{\partial t} - i[\delta\hat{\epsilon}, \hat{\rho}_0] - i[\hat{\epsilon}_0, \delta\hat{\rho}] = -\frac{\partial n}{\partial \epsilon} e\mathbf{E} \cdot \nabla_{\mathbf{p}} \hat{\epsilon}_0 \,.$$

Linearized kinetic equation

Plasmon $n(\mathbf{r}, \mathbf{p}) = \frac{1}{G_s G_v} \operatorname{tr} \hat{\sigma}_0 \hat{\tau}_0 \hat{\rho}(\mathbf{r}, \mathbf{p})$

Silin mode sector

Modes separate into two sectors

Optically inactive

$$\mathbf{s}(\mathbf{r}, \mathbf{p}) = \frac{1}{G_s G_v} \operatorname{tr} \hat{\boldsymbol{\sigma}} \hat{\rho}(\mathbf{r}, \mathbf{p})$$

$$\mathbf{Y}(\mathbf{r}, \mathbf{p}) = \frac{1}{G_s G_v} \operatorname{tr} \hat{\boldsymbol{\tau}} \hat{\rho}(\mathbf{r}, \mathbf{p})$$

$j = z$

$$M_i^j(\mathbf{r}, \mathbf{p}) = \frac{1}{G_s G_v} \operatorname{tr} \hat{\tau}_i \hat{\sigma}_j \hat{\rho}(\mathbf{r}, \mathbf{p})$$

$j = \parallel$

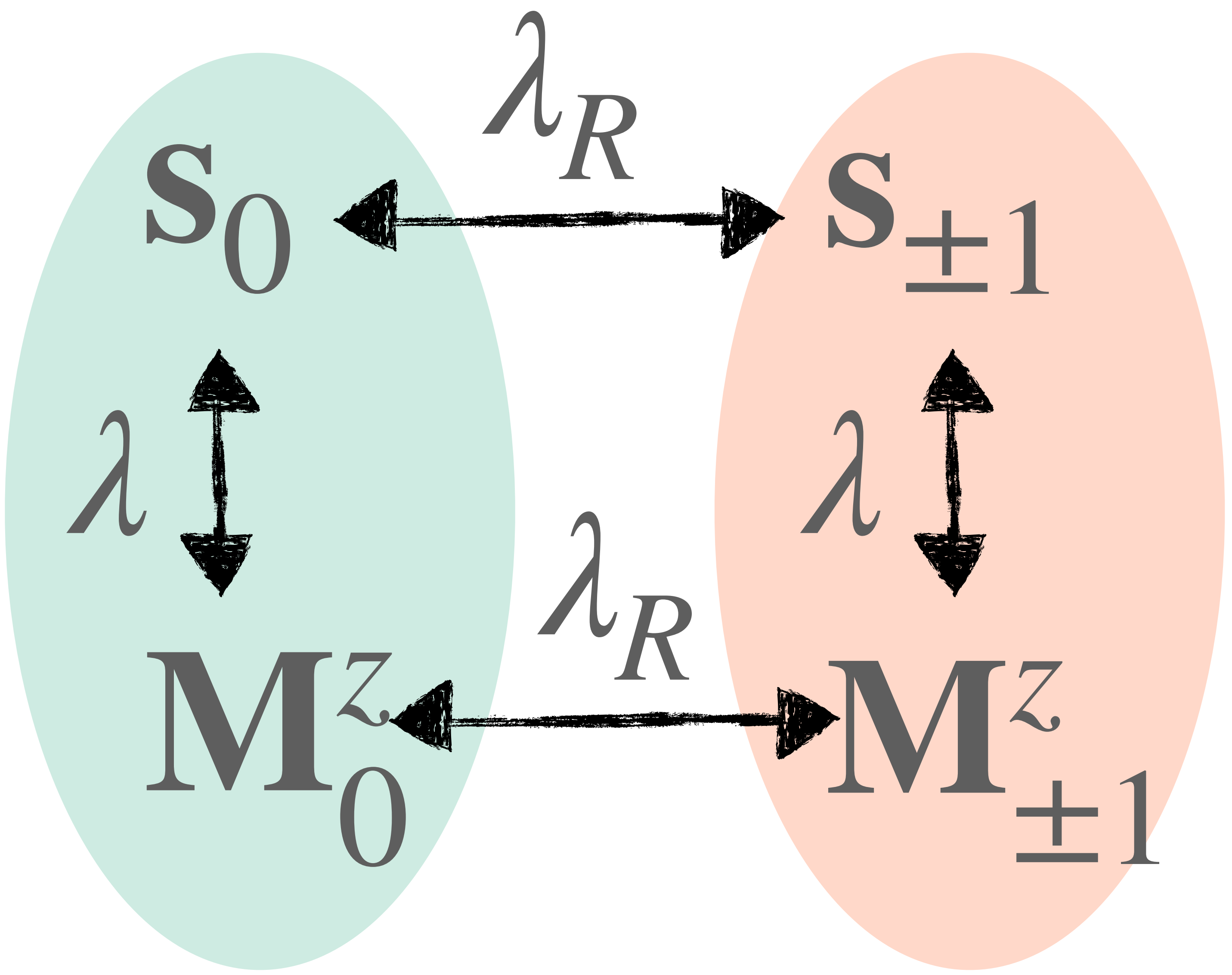
Momentum summed equations

- Here we focus on the the isotropic densities

$$\mathbf{s}_0 = \sum_{\mathbf{p}} \mathbf{s}_{\mathbf{p}}$$

$$\mathbf{M}_0^z = \sum_{\mathbf{p}} \mathbf{M}_{\mathbf{p}}^z$$

- Rashba induces coupling to other harmonics



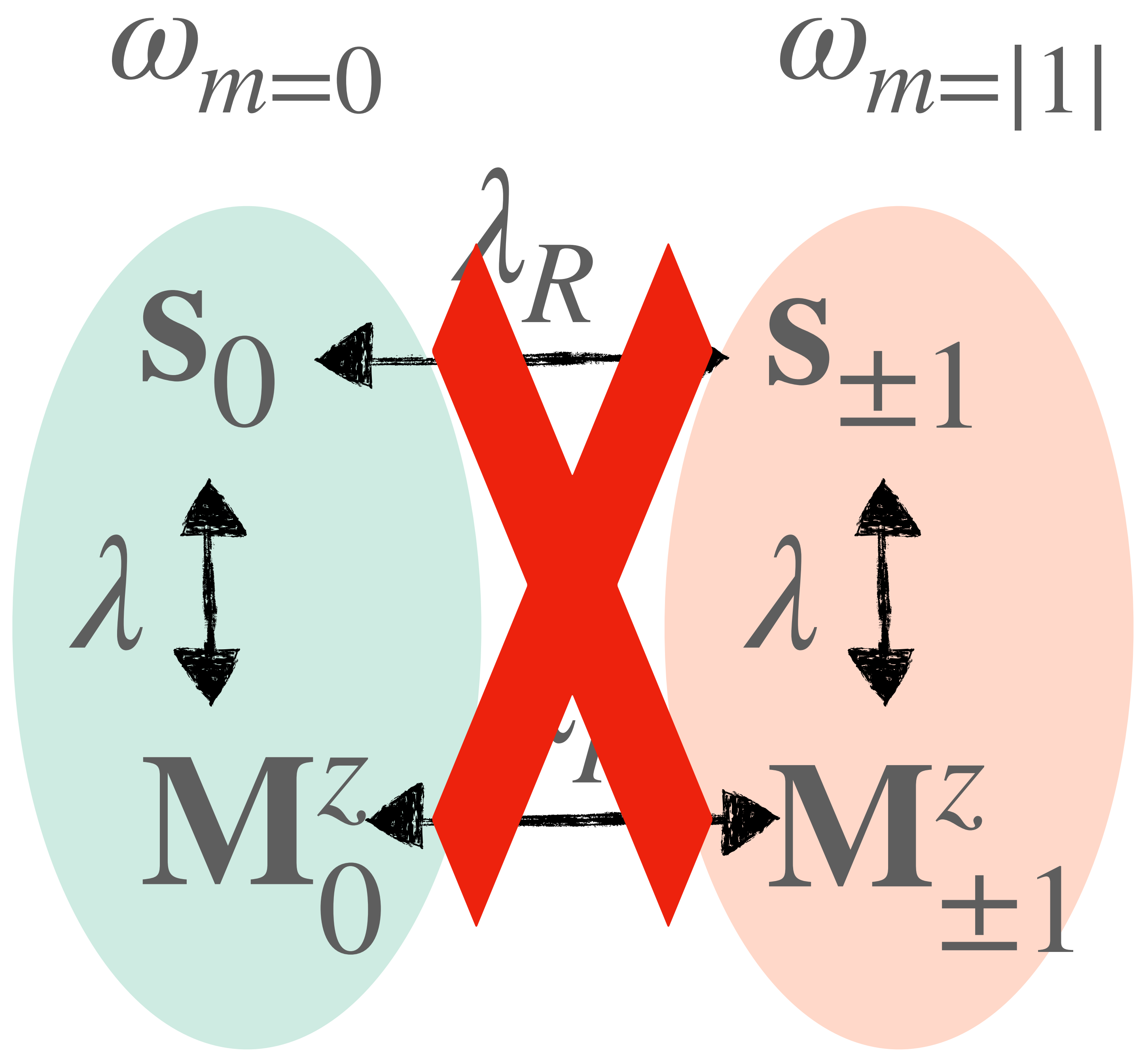
Decoupled limit

$\lambda_R \ll |\omega_{m=|1|} - \omega_{m=0}|$

But see Kumar, Maslov, PRB 95, 165140 (2017).

- For simplicity we consider the limit where angular harmonics approximately decouple

- In this limit $\mathbf{s}_0$ and $\mathbf{M}_0^z$ form a closed system of equations



Decoupled limit

$\lambda_R \ll |\omega_1 - \omega_0|$

But see Kumar, Maslov, PRB 95, 165140 (2017).

- For simplicity we consider the limit where angular harmonics approximately decouple
- In this limit $\mathbf{s}_0$ and $\mathbf{M}_0^z$ form a closed system of equations
- Spin and valley spin are mixed by the valley Zeeman coupling

$$\omega_s = \mu_s H_0, \quad \omega_m = \gamma \omega_s, \qquad \gamma \equiv \frac{1 + F_0^{mz}}{1 + F_0^s}$$

Renormalized by Interactions

$$\frac{\partial \mathbf{s}_0}{\partial t} - \omega_s \hat{x} \times \mathbf{s}_0 + 2\lambda \hat{z} \times \mathbf{M}_0^z = \mathbf{F}_s$$

$$\frac{\partial \mathbf{M}_0^z}{\partial t} - \omega_m \hat{x} \times \mathbf{M}_0^z + 2\gamma\lambda \hat{z} \times \mathbf{s}_0 = \mathbf{F}_m$$

x **axis defined by** $\mathbf{H}_0$

Re-expressing the spin/valley-spin sector

- A straightforward change of basis gives two decoupled vector equations
- There are two sectors
  - A low frequency (heavily over damped) sector $\mathbf{l}_i \parallel \mathbf{b}_i$
  - A finite frequency undamped sector containing the conventional Silin mode $\mathbf{l}_i \perp \mathbf{b}_i$

$$\dot{\mathbf{l}}_i - \mathbf{b}_i \times \mathbf{l}_i = \mathbf{f}_i,$$

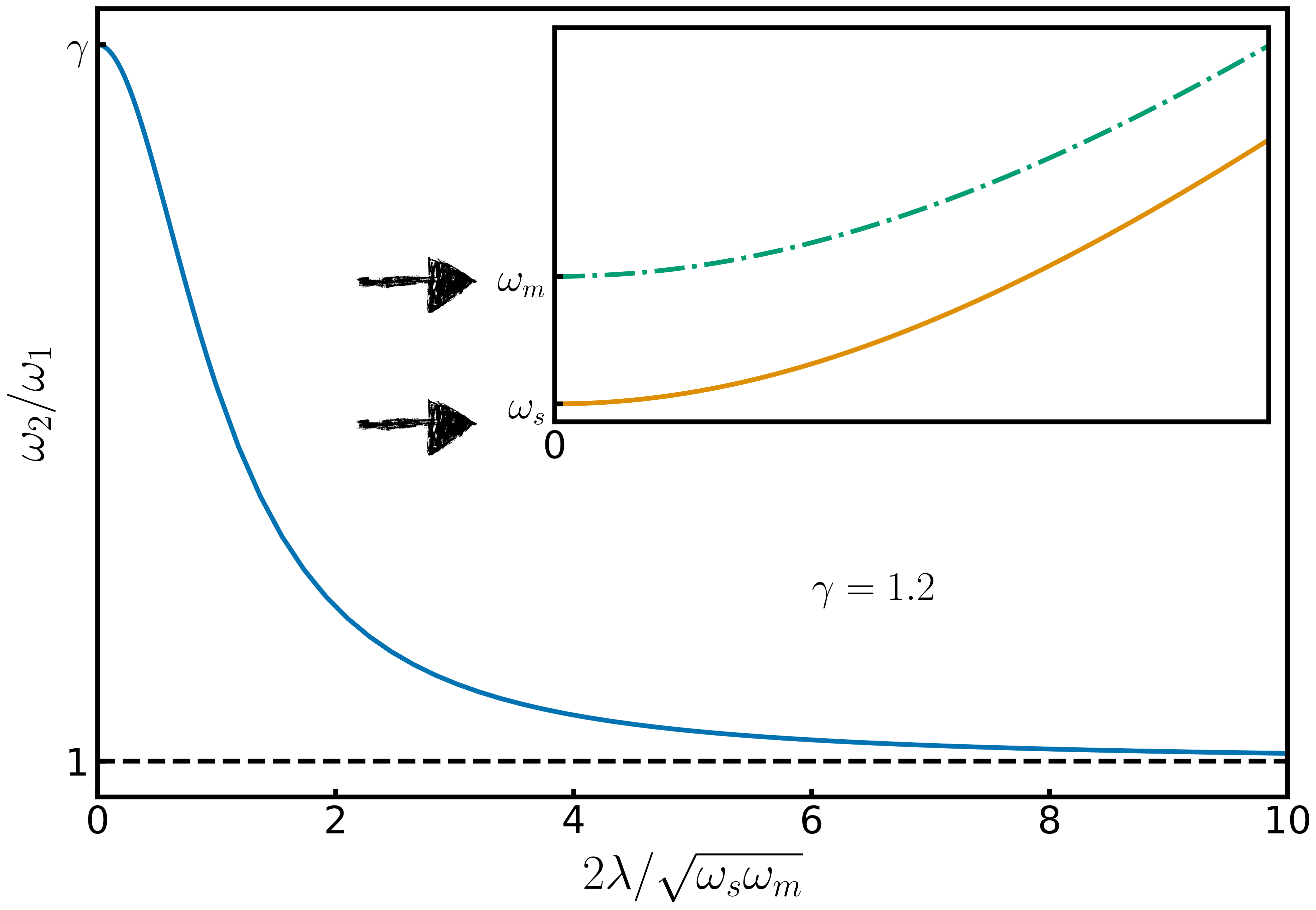
Driven precession

Valley-spin Silin modes

- There are two eigenmodes adiabatically connected to
  - the spin mode $\mathbf{s}$
  - and valley-staggered spin mode $\mathbf{M}^z$
- For $\lambda = 0$, the former must go to the Larmor frequency
- The latter mode may be renormalized away from the non-interacting frequency in general

$$\omega_1 = |\mathbf{b}_1| = \sqrt{\omega_s^2 + 4\lambda^2\gamma^{-1}}$$

$$\omega_2 = |\mathbf{b}_2| = \sqrt{\omega_m^2 + 4\lambda^2\gamma^{-1}}$$

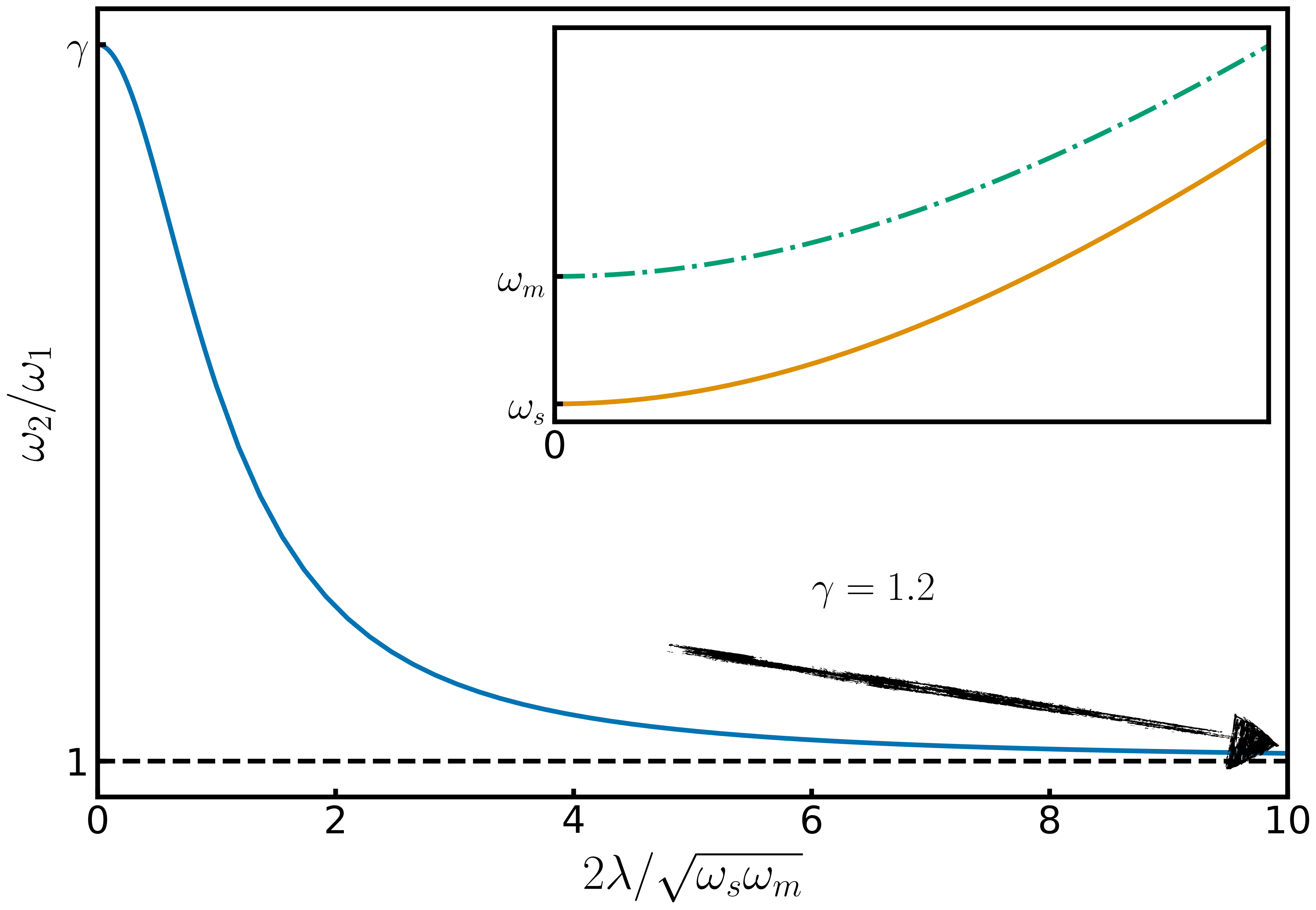


Valley-spin Silin modes

- There are two eigenmodes adiabatically connected to
  - the spin mode $\mathbf{s}$
  - and valley-staggered spin mode $\mathbf{M}^z$
- The modes do not cross, but become degenerate in $\lambda/\omega_s \to \infty$ limit

$$\omega_1 = |\mathbf{b}_1| = \sqrt{\omega_s^2 + 4\lambda^2\gamma^{-1}}$$

$$\omega_2 = |\mathbf{b}_2| = \sqrt{\omega_m^2 + 4\lambda^2\gamma^{-1}}$$



Driving via AC electric field

polarization selective driving

- Silin subspace is diagonalized by circular polarizations
$$\hat{e}_{i,\pm} = \hat{y} \pm i \frac{\mathbf{b}_i}{\omega_i} \times \hat{y}$$
- Each mode is excited by one linear polarization of electric field (with respect to $\mathbf{H}_0$)

$$\dot{\mathbf{l}}_i - \mathbf{b}_i \times \mathbf{l}_i = \mathbf{f}_i, \longrightarrow l_{i,\pm} = \frac{i f_{i\pm}}{\omega + i0 \mp \omega_i}$$

x **axis defined by** $\mathbf{H}_0$

Spin

Connected to

Valley staggered spin

$$\mathbf{l}_1 \propto E_x \longleftarrow f_{1,\pm} \propto E_x$$

$$\mathbf{l}_2 \propto E_y \longleftarrow f_{2,\pm} \propto E_y$$

Optical absorption from energy change

- Since the modes couple to E field, they contribute to the optical conductivity

- The resonant contribution can be extracted from change in the Free energy due to pumping of the spin-valley modes

- As l_i are eigenmodes they contribute two decoupled sectors to the free energy

Time averaged free energy change

$$\overline{\frac{\partial f}{\partial t}} = \int \frac{d\omega}{2\pi} \mathbf{E}(-\omega)\hat{\sigma}^R(\omega)\mathbf{E}(\omega)$$

Electric energy absorption

Fermi liquid theory

$$f = \sum_{\mathbf{k}} \mathrm{tr}[\hat{\rho}\hat{\epsilon}]$$

$$f = f_1[l_1, E_x] + f_2[l_2, E_y]$$

Contributions to conductivity

- Spin-valley modes contribute resonant peaks the real part of the conductivity

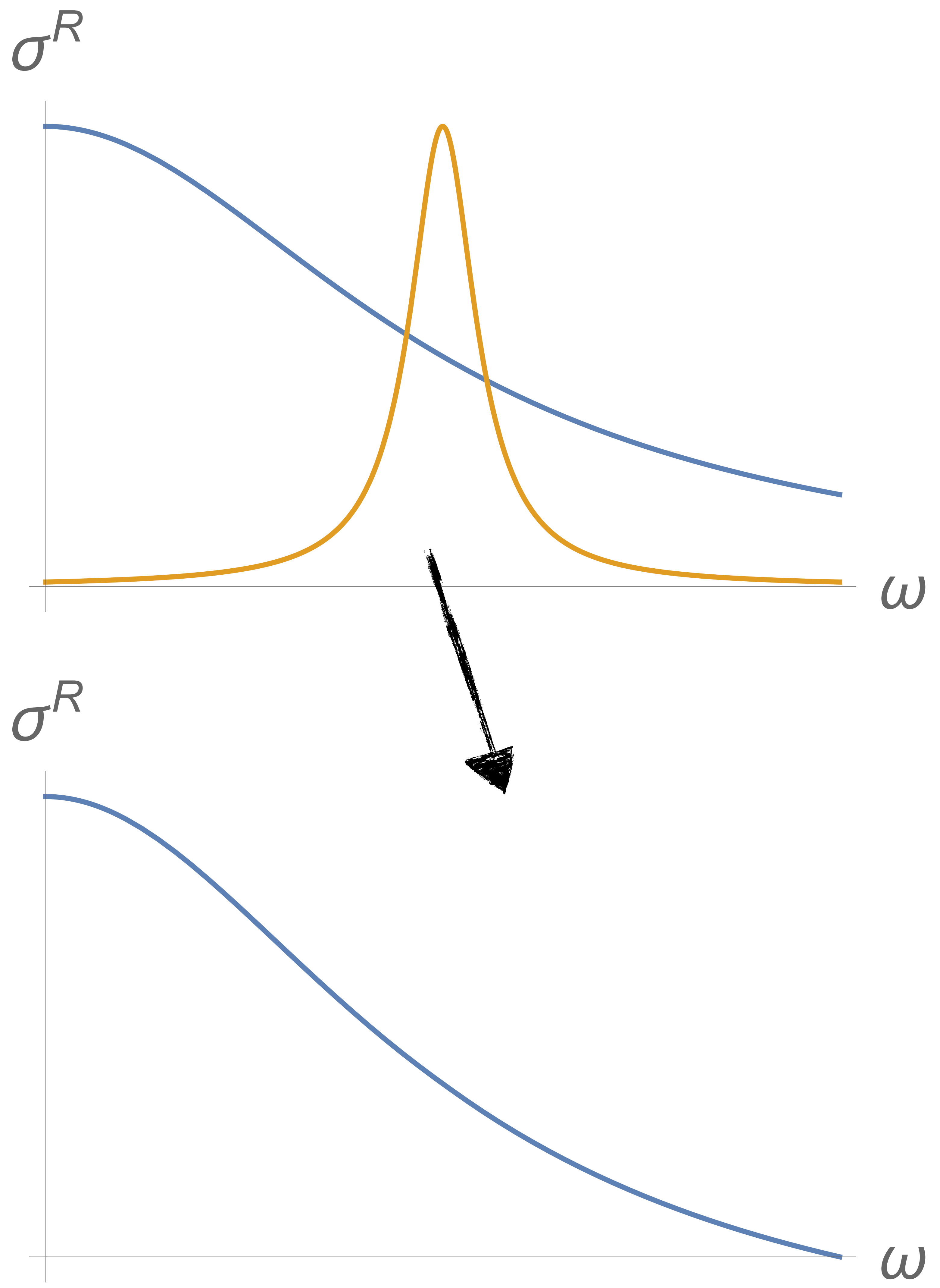
$$\Re\sigma_l^{xx}(\omega) \propto \left(F_0^s - (F_0^s - \gamma^{-1}F_0^{mz})\frac{\lambda^2}{\gamma\omega_1^2} \right) \pi\omega_1\delta(\omega^2 - \omega_1^2)$$

$$\Re\sigma_l^{yy}(\omega) \propto \frac{4\lambda^2}{\omega_2^2} \left(F_0^{mz} - (F_0^{mz} - \gamma F_0^s)\frac{\lambda^2}{\gamma\omega_2^2} \right) \pi\omega_2\delta(\omega^2 - \omega_2^2)$$

Contributions to conductivity

Damping effects

- Realistically expect D’yakonov-Perel and Elliot-Yafet spin flips
- This broadens the delta function into a Lorentzian
- Unfortunately for typical samples the resonant peaks lie in the shoulder of the Drude peak



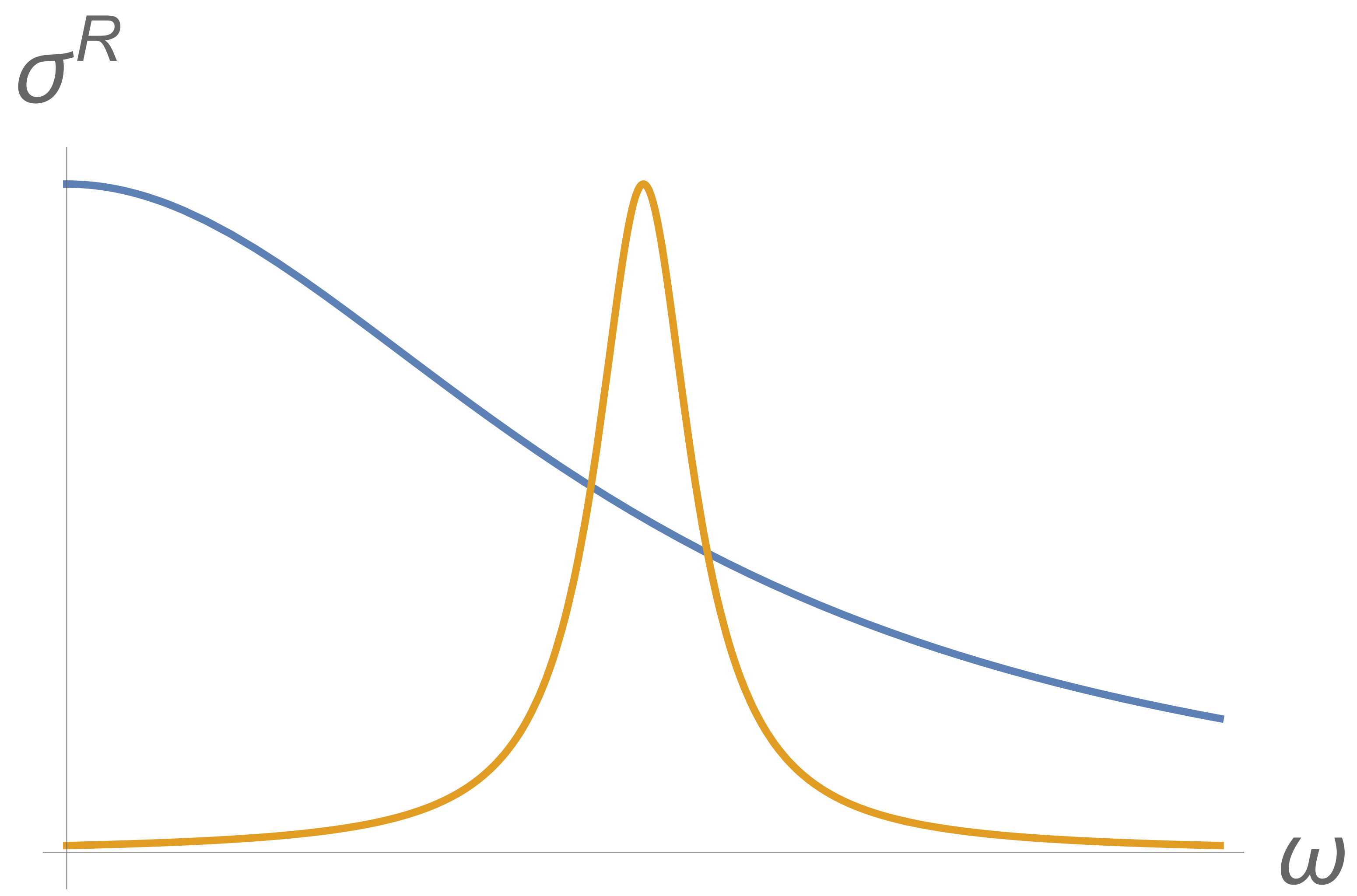
Contributions to conductivity

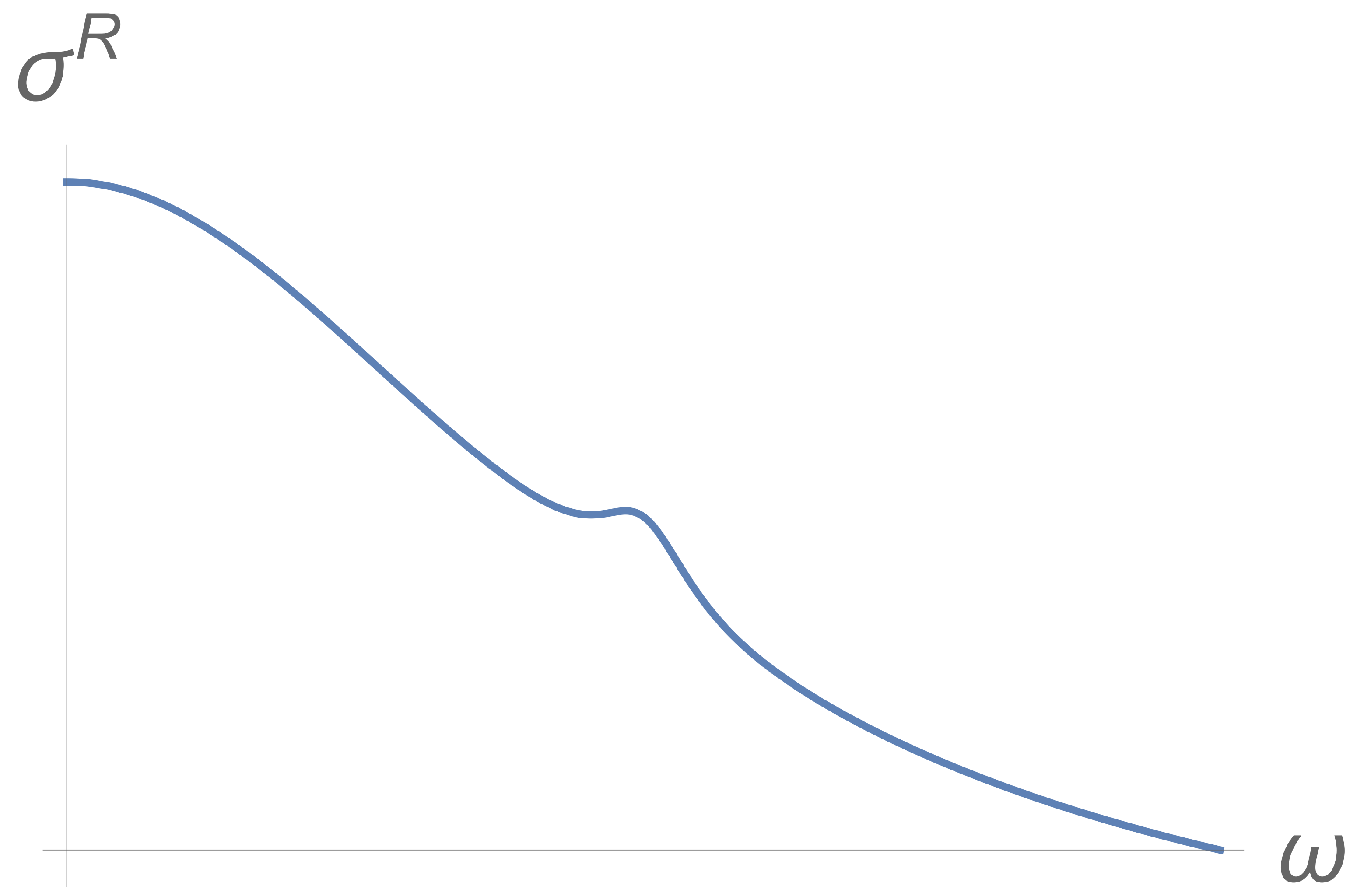
Damping effects

- Unfortunately for typical samples the resonant peaks lie in the shoulder of the Drude peak

- For larger Rashba couplings stronger signals can be obtained
  - Modes will change their angular momentum character in this case

- In samples where $\tau_s \gg \tau_{\mathrm{tr}}$ peaks should be more visible





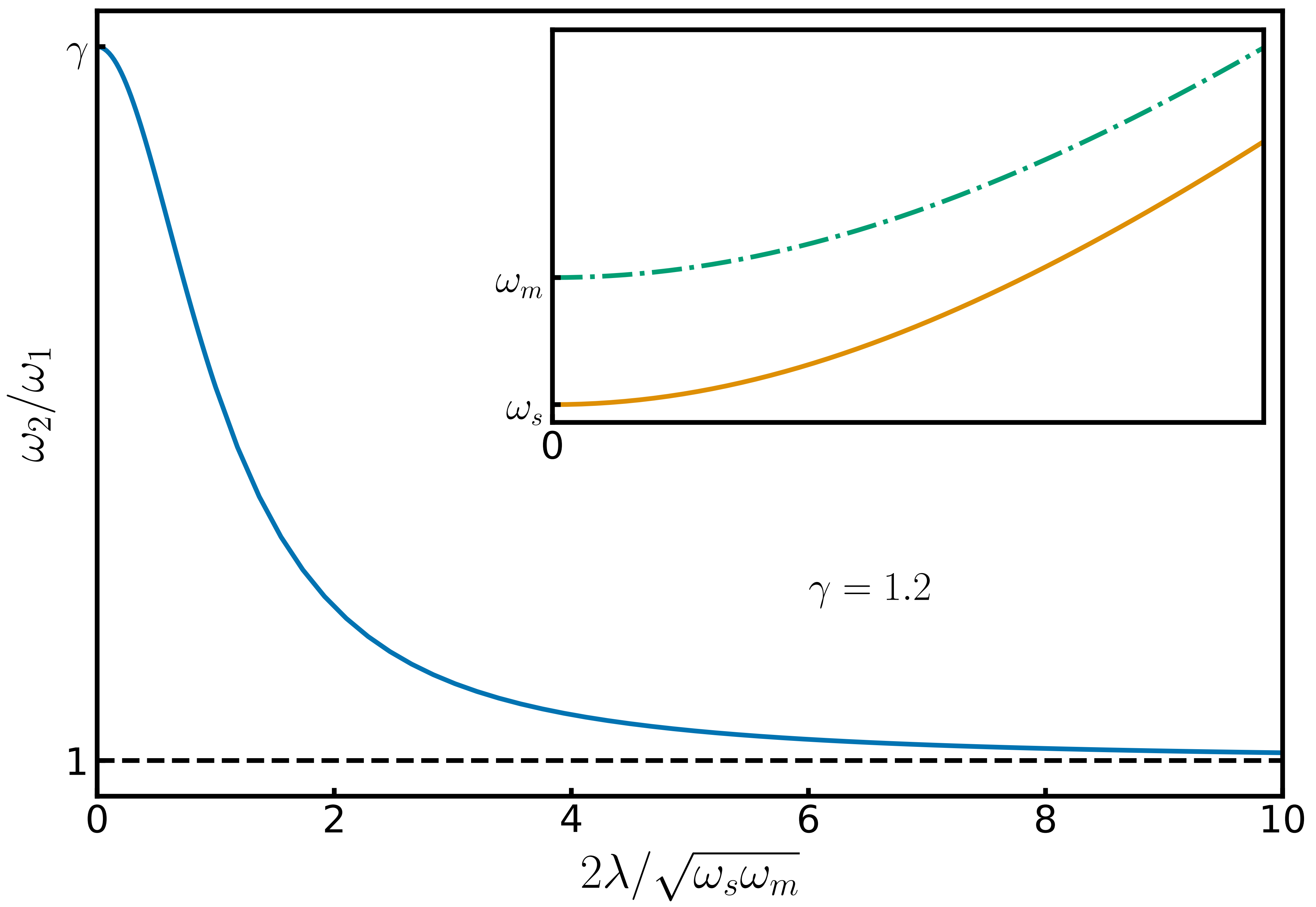
Summary

for Part II

- External Zeeman and/or extrinsic SOC promote overdamped spin-valley excitations of graphene to well-defined oscillatory modes

- Spin and valley-staggered spin modes can be excited selectively via AC electric field

- Both contribute absorption peaks to the optical conductivity



Final Summary

- Symmetry dictated Fermi liquid theory of graphene
- Neutral zero sound and first sound in graphene are overdamped for all spin-valley channels.
- Transport of spin-valley quantum numbers is generically diffusive

ZMR, Fal'ko, Glazman, PRB 103, 075422, 2020

10.1103/PhysRevB.103.075422

- External Zeeman and/or extrinsic SOC promote diffusive spin-valley excitations of graphene to well-defined oscillatory modes
- Spin and valley-staggered spin modes can be excited selectively via AC electric field
- Both contribute absorption peaks to the optical conductivity

ZMR, Maslov, Glazman

In Prep.

Thank you for your attention!

10.1103/PhysRevB.103.075422