Spin-valley collective modes of the electron liquid in graphene

Yale

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Under Review
arXiv:2107.02819


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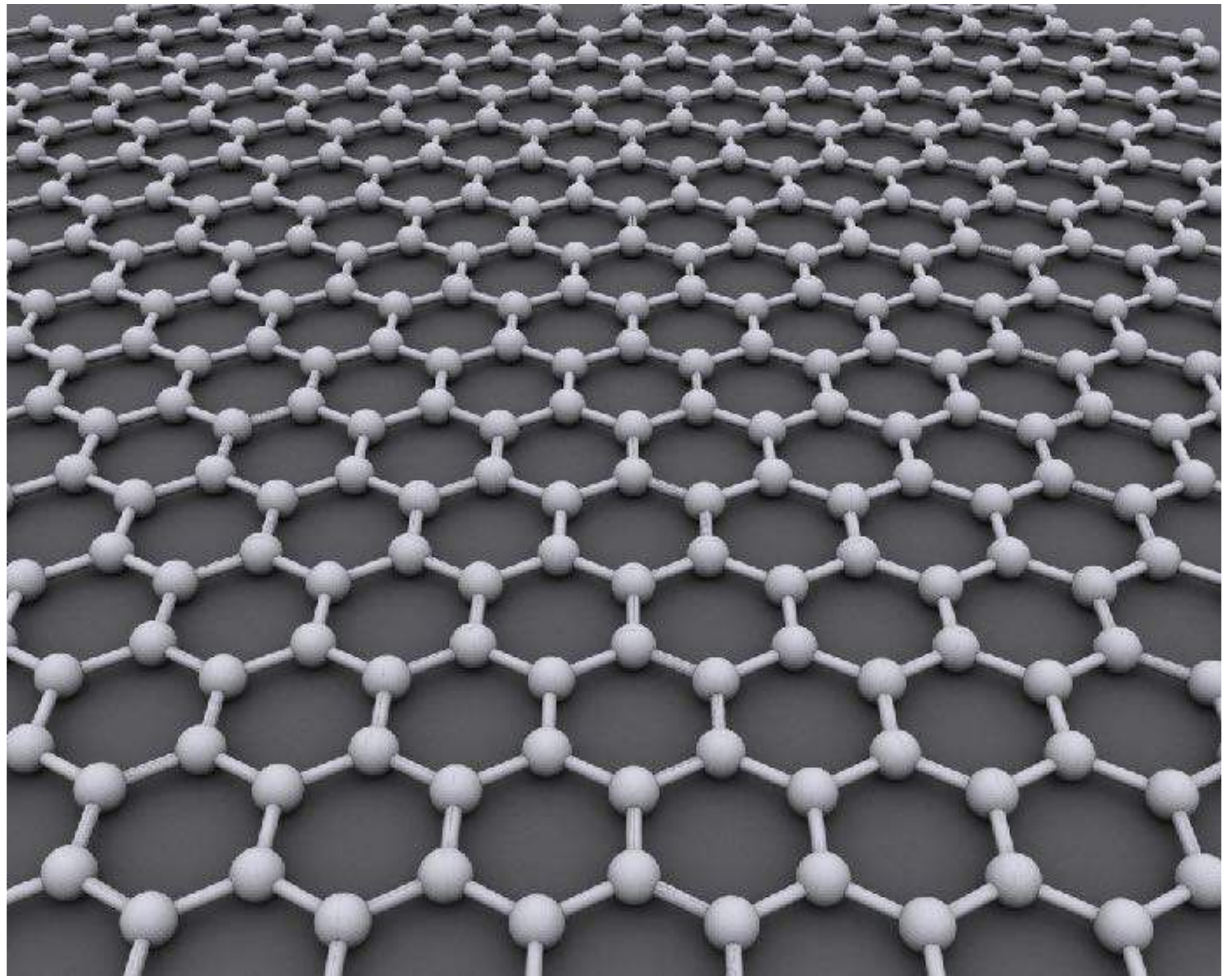
Graphene

- A material of intense interest over the past 15 years for e.g.
  - Device applications
  - Analogies with quantum electrodynamics
  - Material properties
- Valley and spin degrees of freedom are both of interest for device applications



How do excitations of the spin-valley channels spread in Fermi Liquid graphene?

Outline

Spin rotation invariant graphene

Fermi Liquid theory in doped graphene

(Absence of) neutral sound in graphene

Magnetic fields and extrinsic SOC

Spin-valley Silin modes

Geometric kinetic effects

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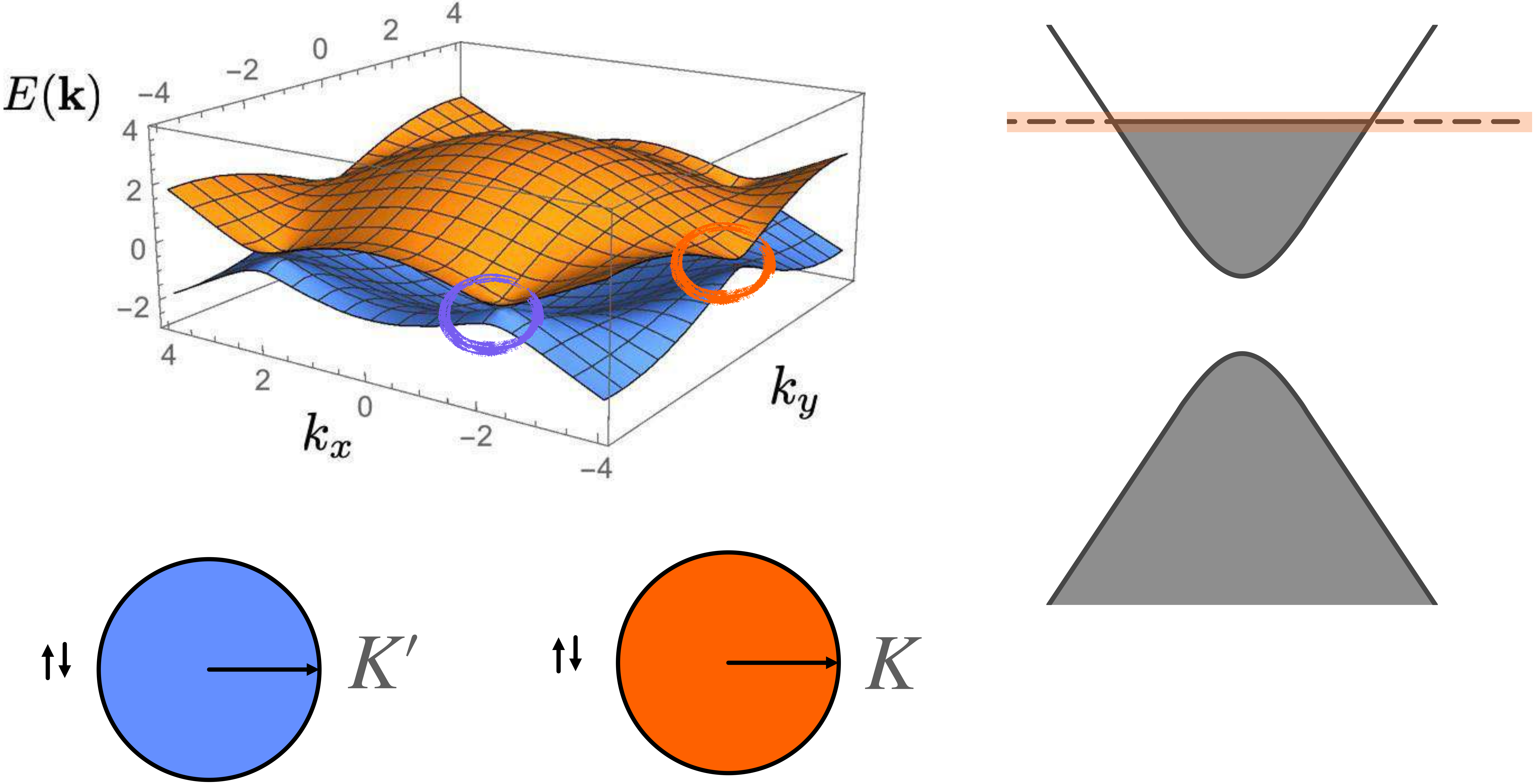
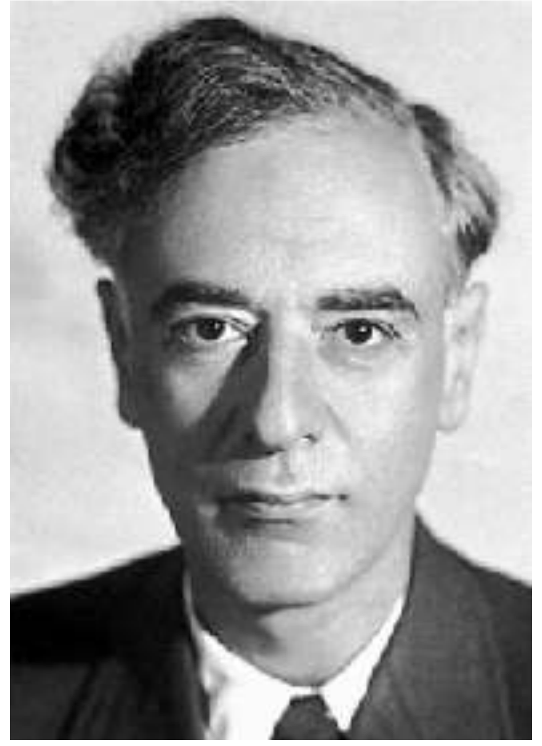


$E(\mathbf{k})$
k_x
k_y
↑↓
K'
↑↓
K

Free Energy

$$\mathscr{F} = \mathscr{F}_0 + \sum_k \xi_k \delta n_k + \frac{1}{2} \sum_{k,k'} f_{kk'} \delta n_k \delta n_{k'} + \cdots$$

What do we need to describe this system?



Bare Quasiparticle Energy

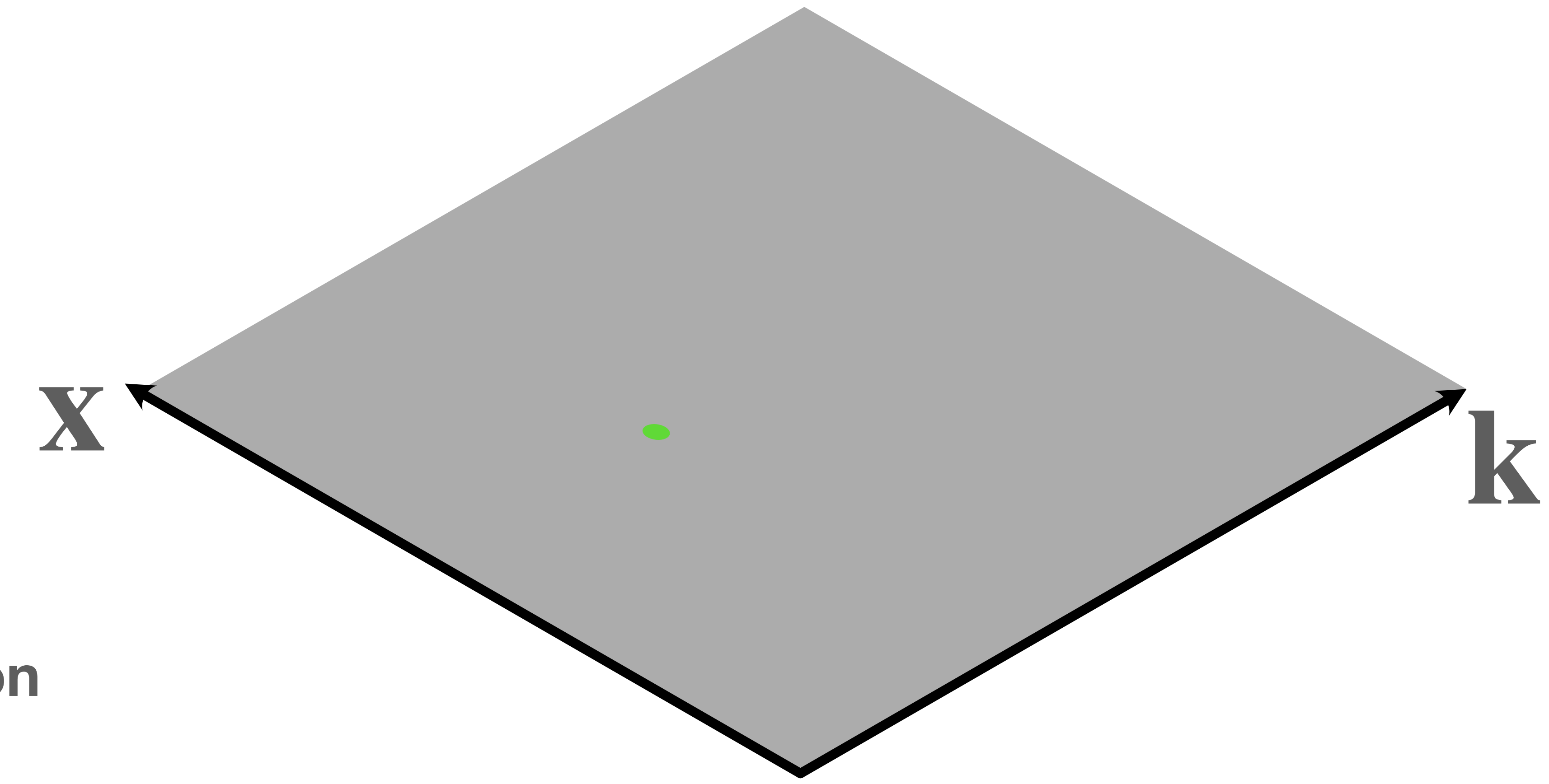
$$\xi_k$$

Occupation Function

$$\delta n_k$$

Landau Interaction Function

$$f_{kk'}$$

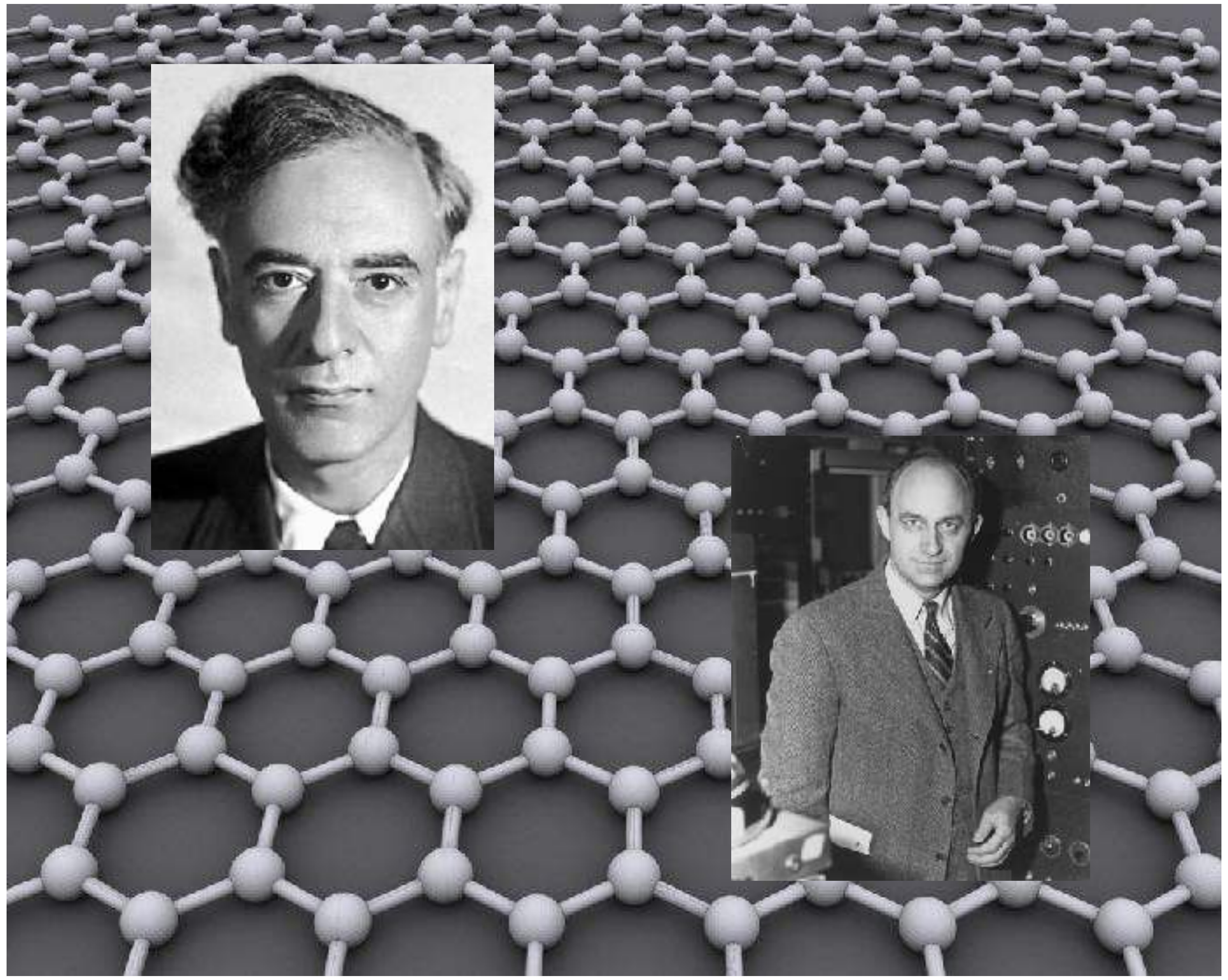


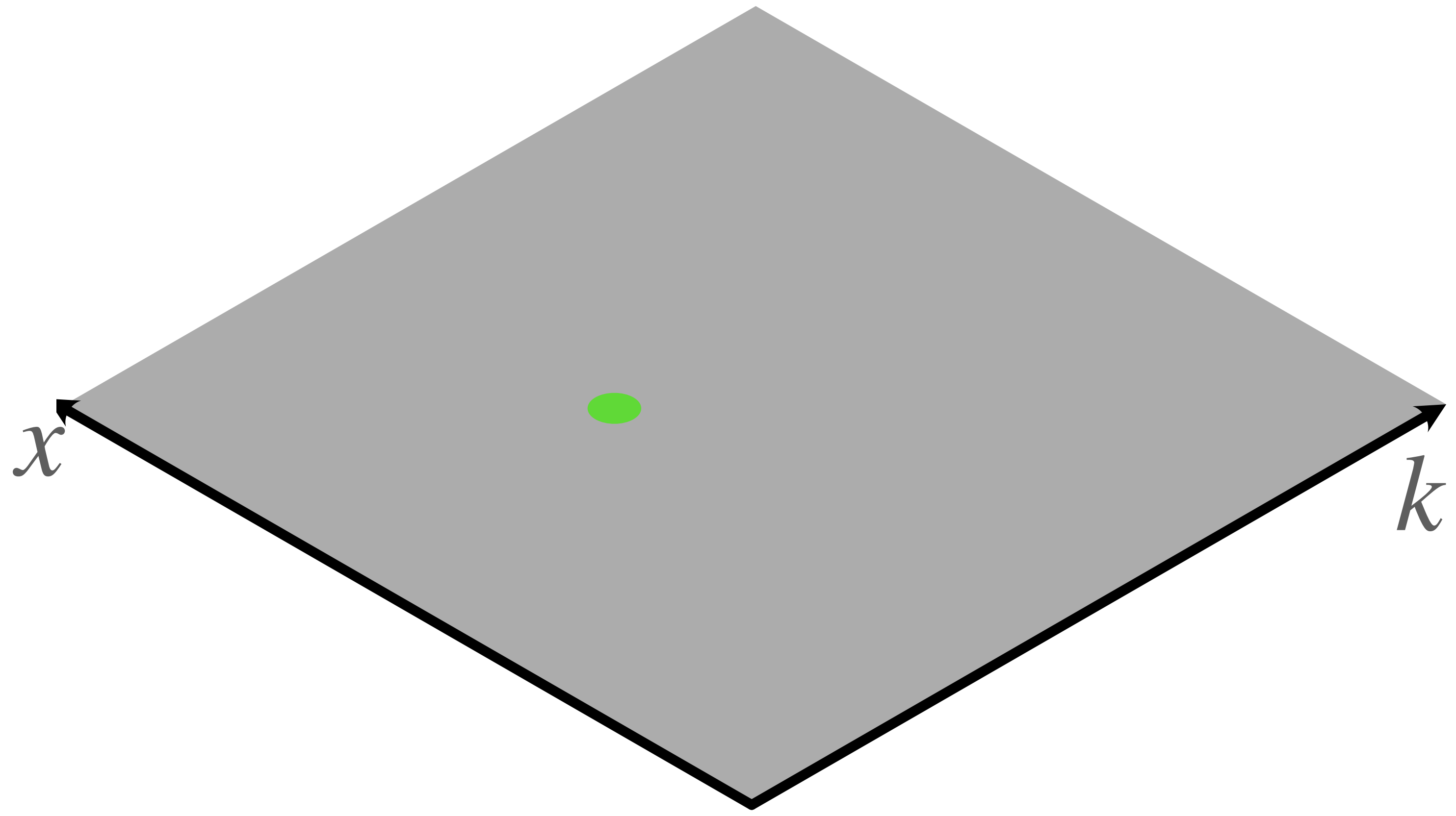
+ Evolution equation $$\frac{\partial \delta n_k}{\partial t} = \cdots$$

Free Energy

$$\mathscr{F} = \mathscr{F}_0 + \sum_k \xi_k \delta n_k + \frac{1}{2} \sum_{k,k'} f_{kk'} \delta n_k \delta n_{k'} + \cdots$$

Multicomponent Fermi Liquid theory





Free Energy

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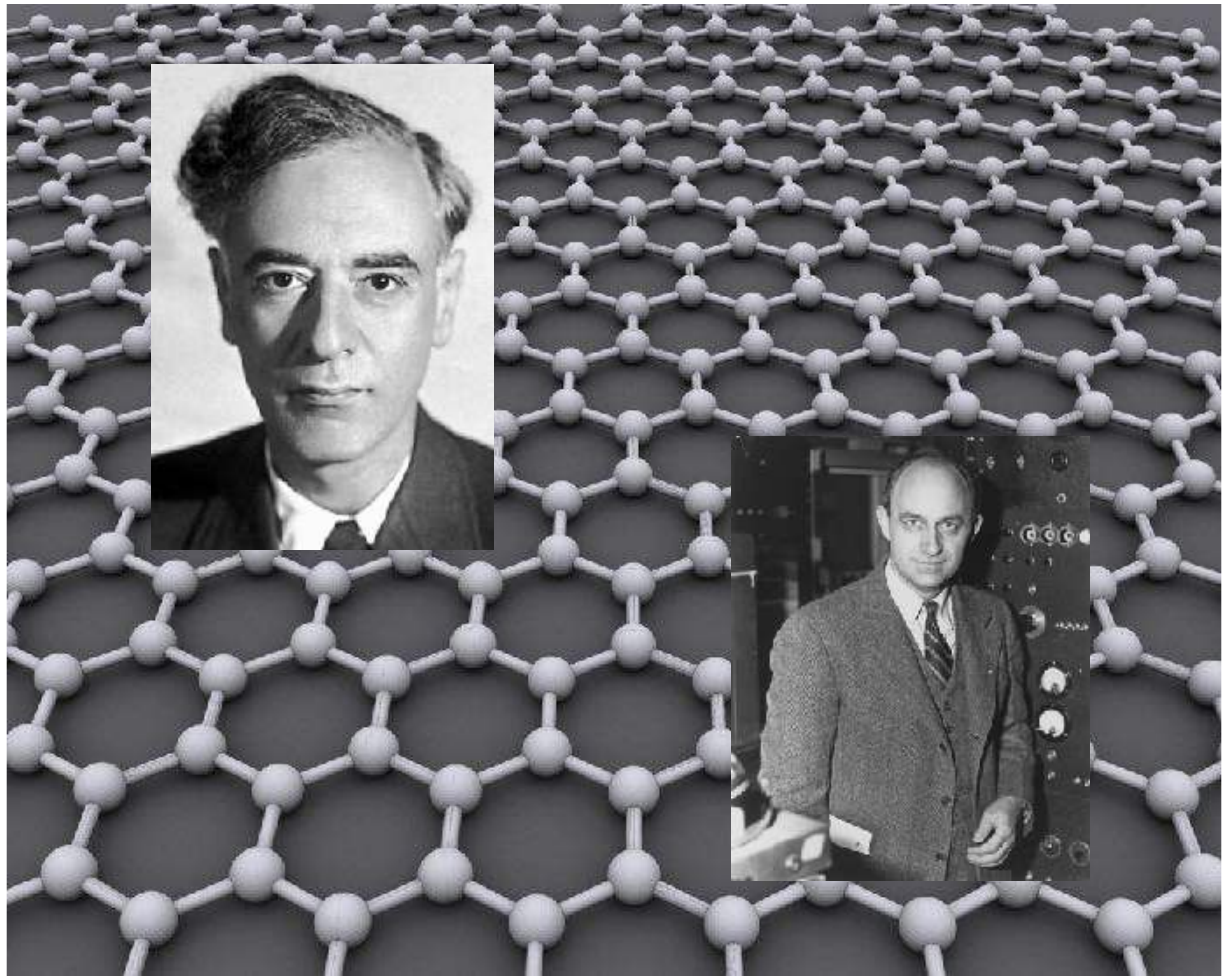
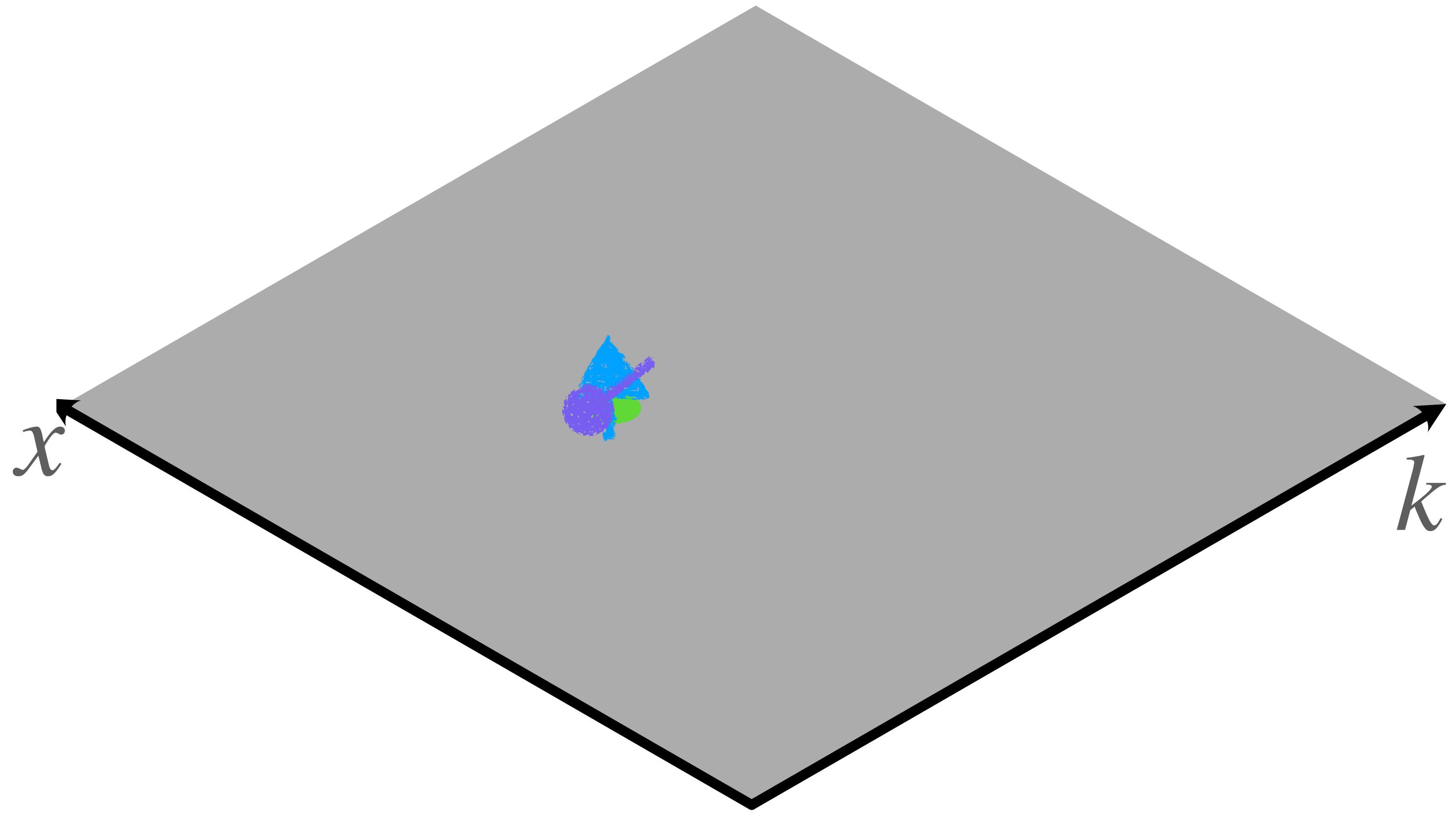
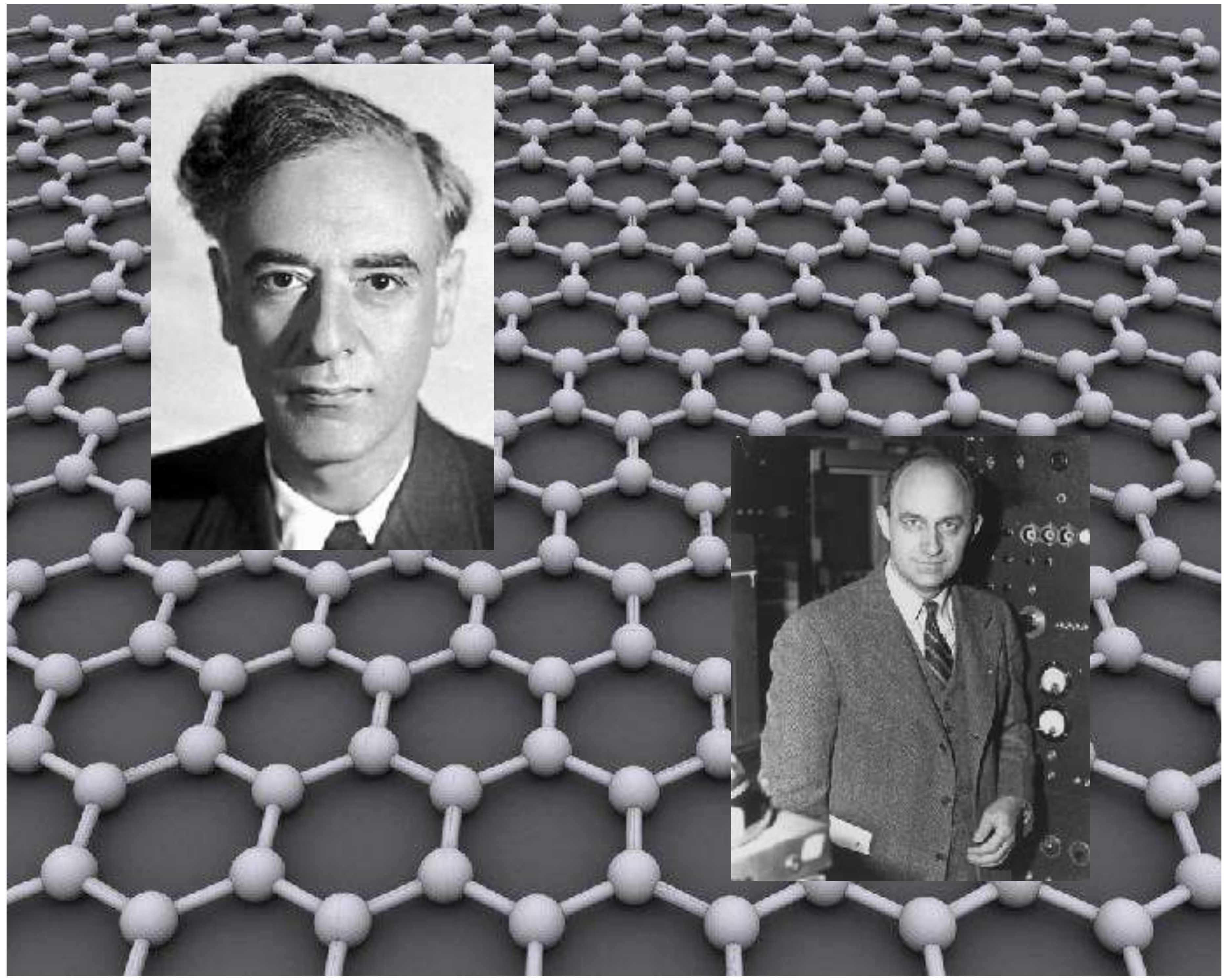




Multicomponent Fermi Liquid theory



Free Energy

$$\mathscr{F} = \mathscr{F}_0 + \sum_k \xi_k \delta n_k + \frac{1}{2} \sum_{k,k'} f_{kk'} \delta n_k \delta n_{k'} + \cdots$$

e.g

$$\sum_k \epsilon_{ij} \delta n_{ji}(k)$$

e.g

$$\sum_{k,k'} \delta n_{ij}(k) F^{ij;lm}(k,k') \delta n_{lm}(k')$$

Bare Quasiparticle Energy

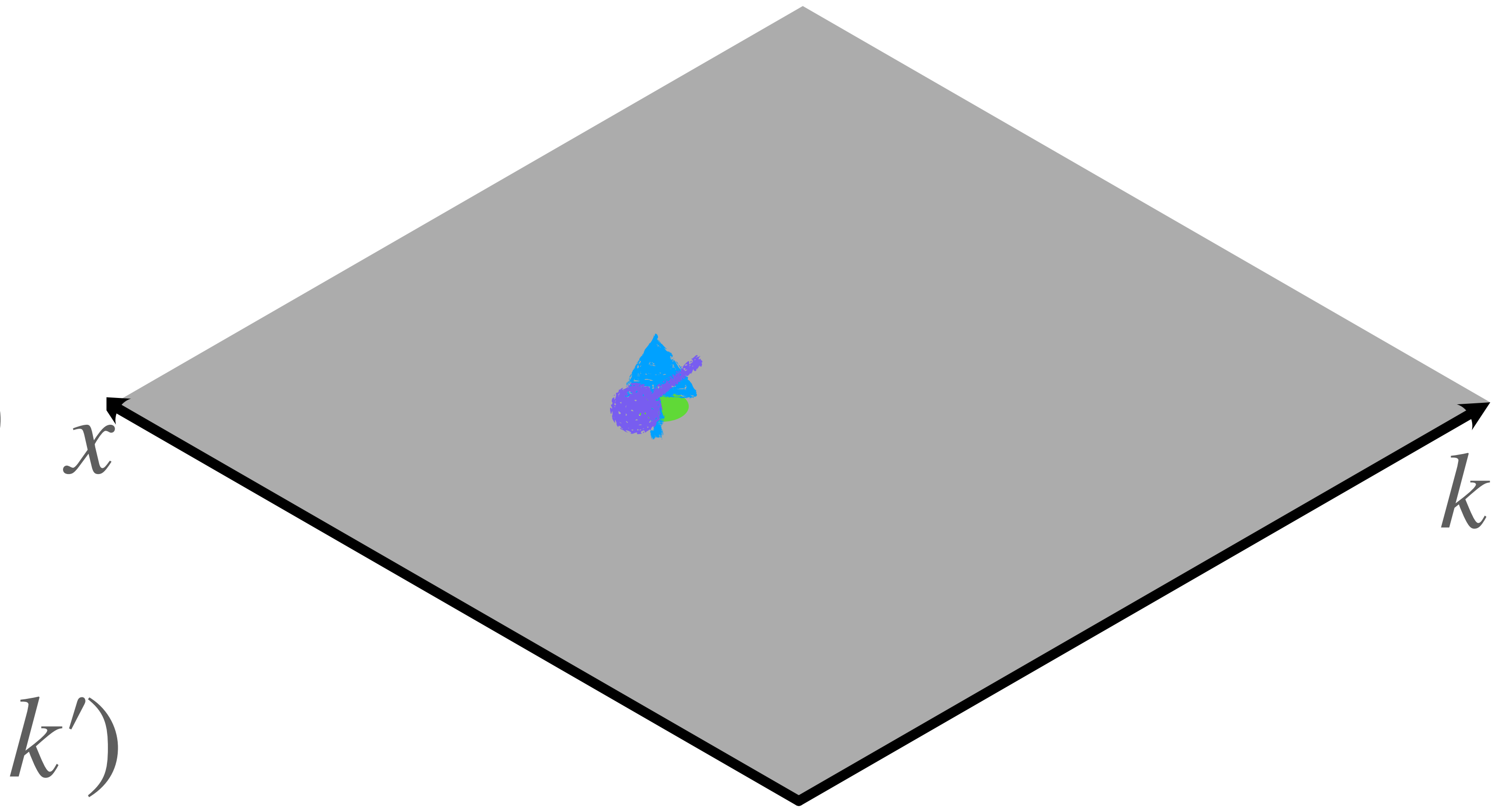
$$\xi_k \to \epsilon_{ij}(k)$$

Occupation Functions

$$\delta n_k \to \delta n_{ij}(k)$$

Landau Interaction Functions

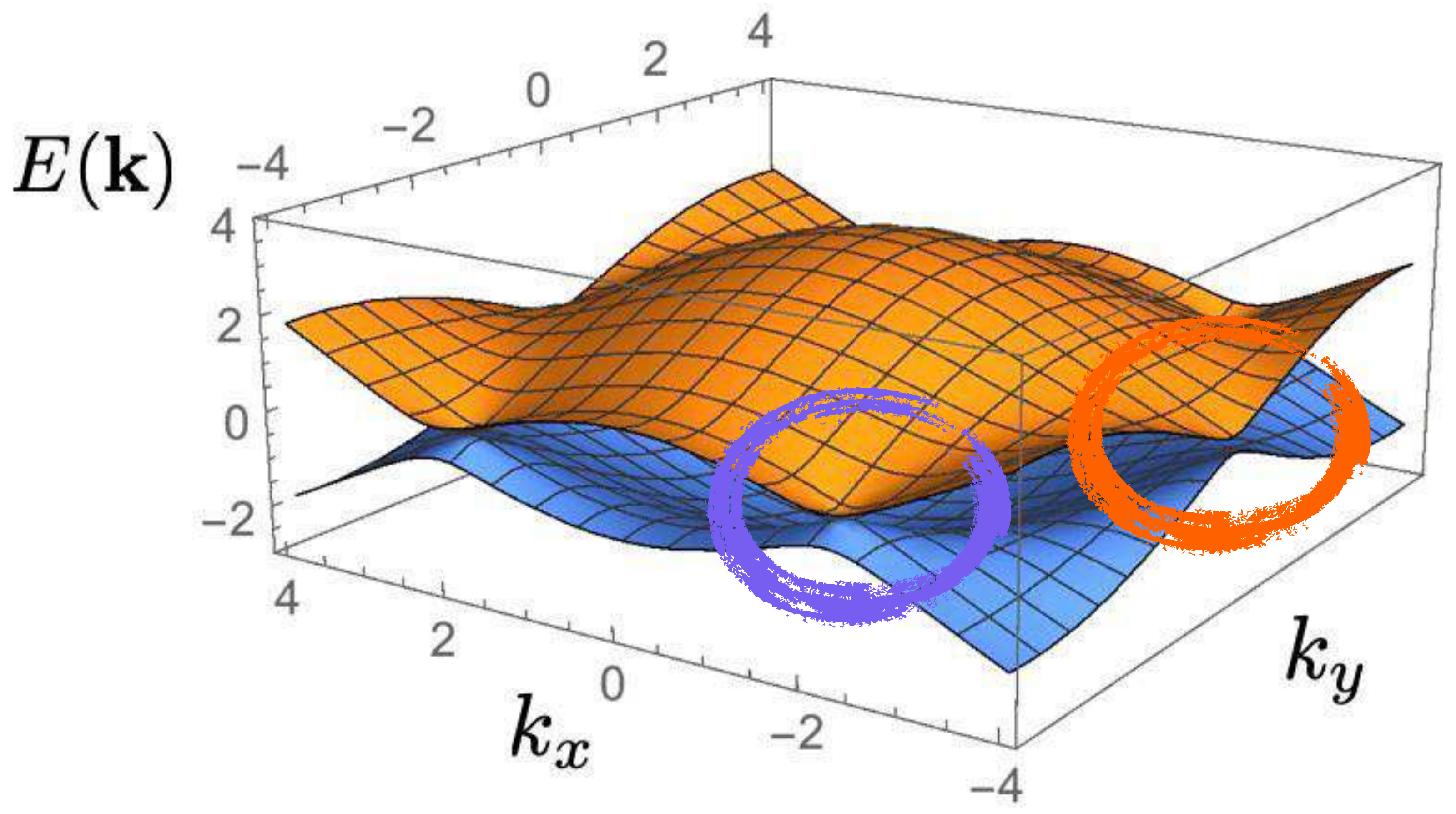
$$f_{kk'} \to f^{ij;lm}(k,k')$$



Fermi Liquid graphene

A multicomponent Fermi liquid

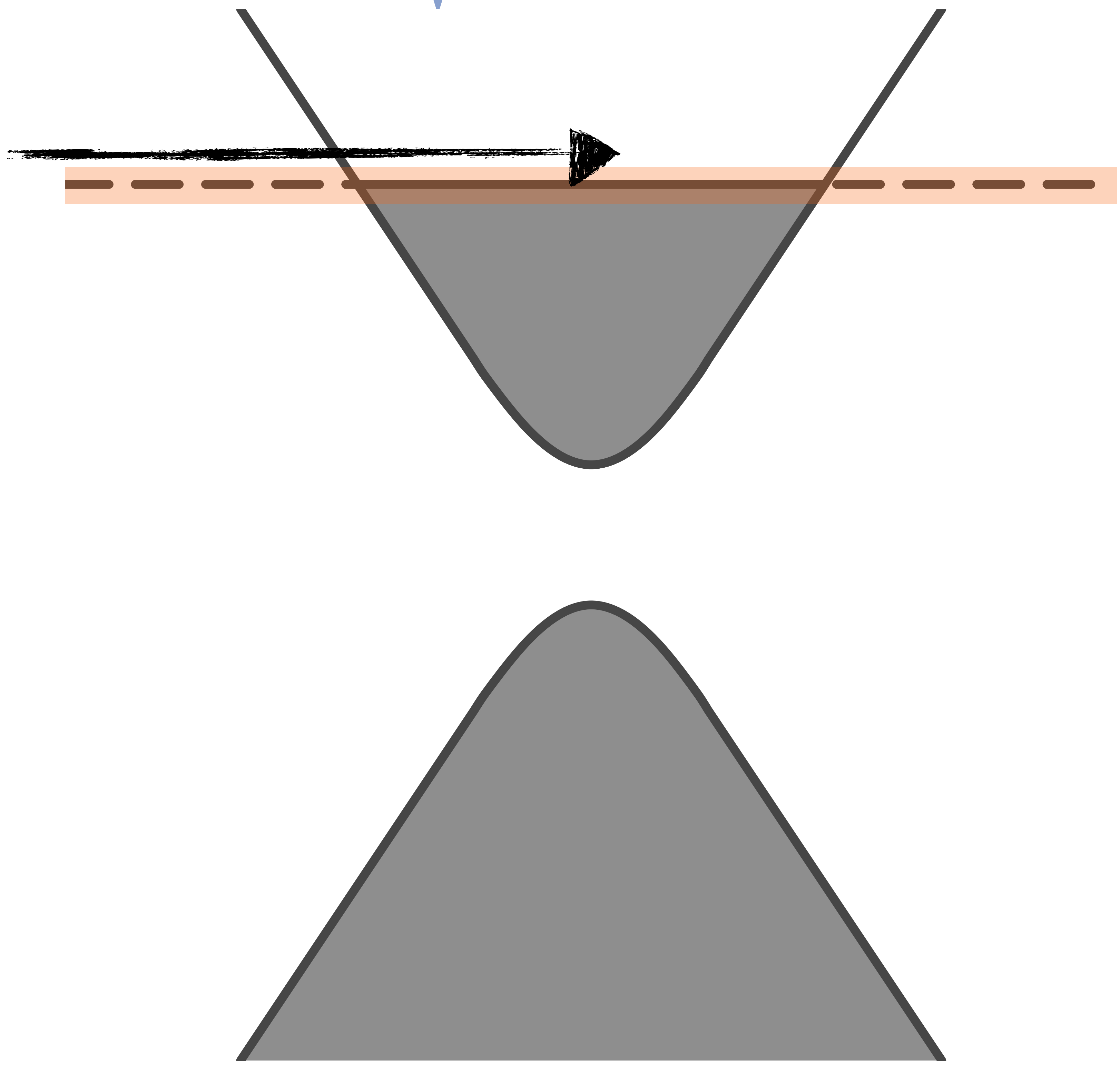
$$\epsilon_{ij}(\mathbf{p},\mathbf{r}) = \xi(\mathbf{p}) + \sum_{\mathbf{p}',lm} f_{ij;lm}(\mathbf{p}\cdot\mathbf{p}')\hat{\rho}_{lm}(\mathbf{r},\mathbf{p}'),$$

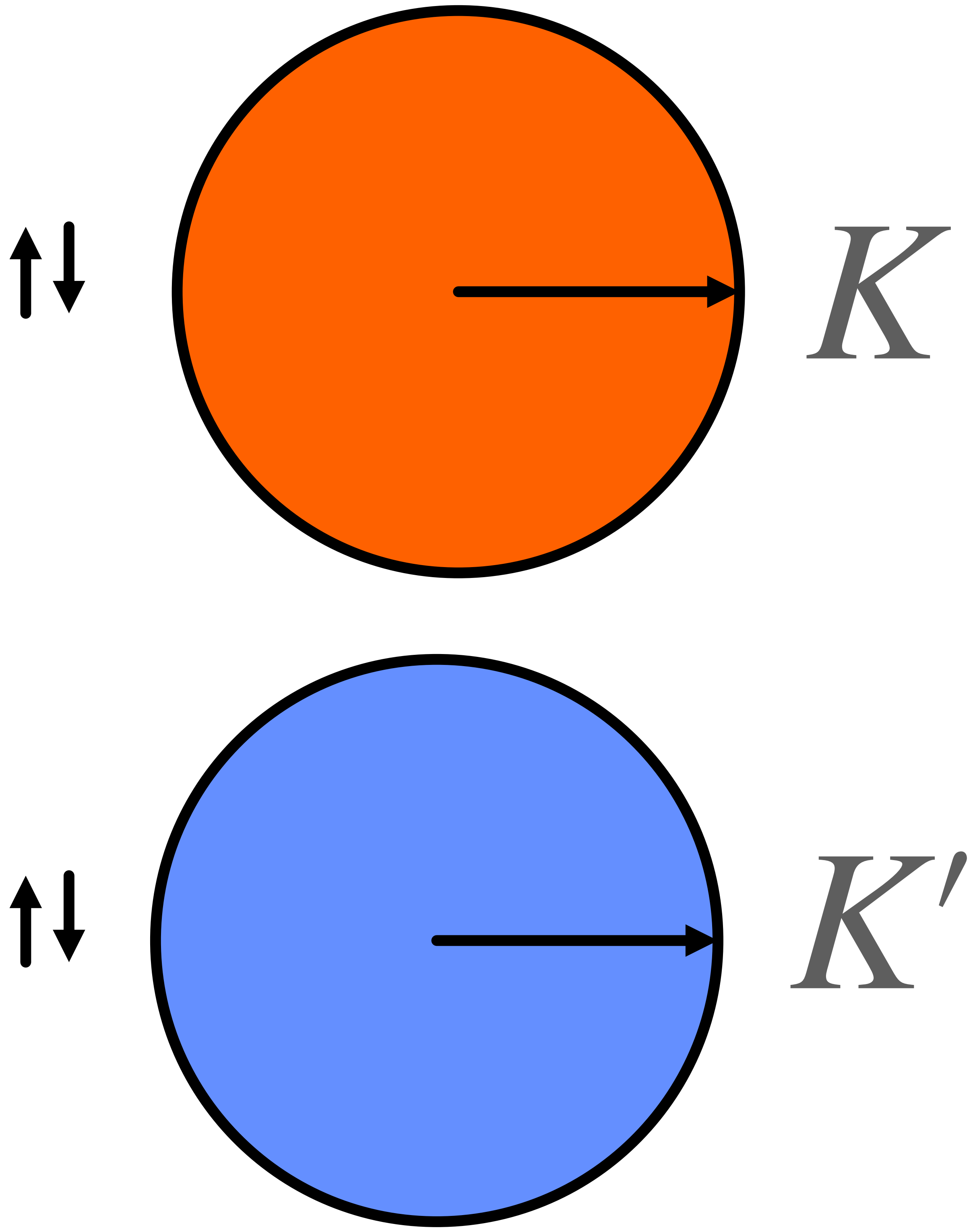


$$\xi(\mathbf{p}) = \sqrt{v^2p^2 + \Delta^2} - \mu$$

- We want to construct a Fermi liquid theory of graphene without sub lattice symmetry

- We know the non-interacting dispersion but what types of interactions can we have?



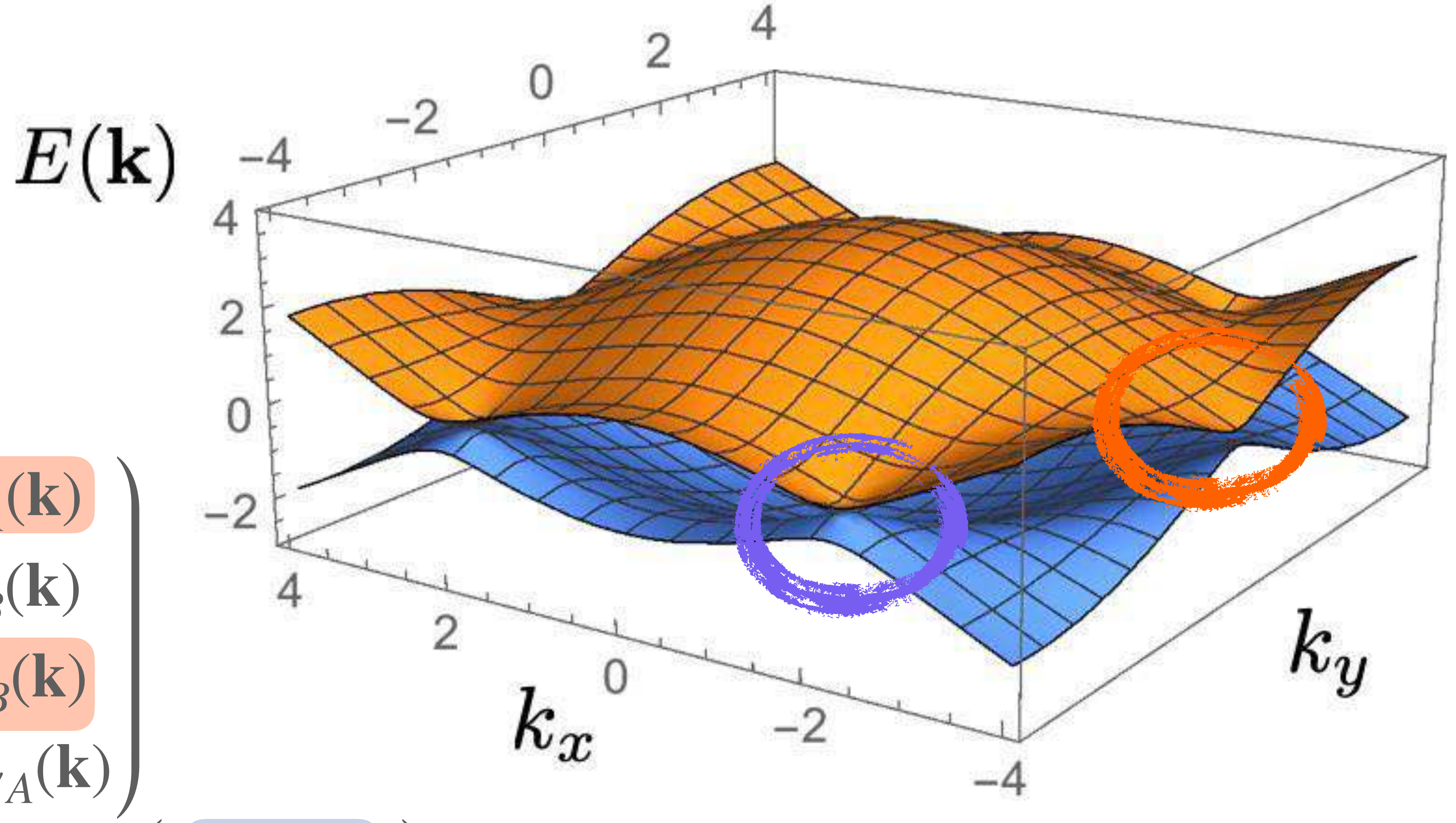


A quick note on notation

Pauli matrices

$$\Psi_{\mathbf{k}} = \begin{pmatrix} \psi_{KA}(\mathbf{k}) \\ \psi_{KB}(\mathbf{k}) \\ \psi_{K'B}(\mathbf{k}) \\ -\psi_{K'A}(\mathbf{k}) \end{pmatrix}$$

We will be largely concerned with physics of the conduction band



- $\psi_{\zeta\Sigma}$ is a spinor
- Spin matrices σ act on $\begin{pmatrix} \psi_{KA}(\mathbf{k}) \\ \psi_{KB}(\mathbf{k}) \\ \psi_{K'B}(\mathbf{k}) \\ -\psi_{K'A}(\mathbf{k}) \end{pmatrix}$
- Valley matrices τ act on $\begin{pmatrix} \psi_{KA}(\mathbf{k}) \\ \psi_{KB}(\mathbf{k}) \\ \psi_{K'B}(\mathbf{k}) \\ -\psi_{K'A}(\mathbf{k}) \end{pmatrix}$
- Sublattice matrix Σ acts on $\begin{pmatrix} \psi_{KA}(\mathbf{k}) \\ \psi_{KB}(\mathbf{k}) \\ \psi_{K'B}(\mathbf{k}) \\ -\psi_{K'A}(\mathbf{k}) \end{pmatrix}$

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Symmetry of gapped graphene

What short ranged interactions are symmetry allowed?

$$\hat{\Psi}_\sigma(\mathbf{r}) = \begin{pmatrix} u_{KA}(\mathbf{r}) & u_{KB}(\mathbf{r}) & u_{K'B}(\mathbf{r}) & -u_{K'A}(\mathbf{r}) \end{pmatrix} \cdot \hat{\vec{\psi}}_\sigma(\mathbf{r})$$

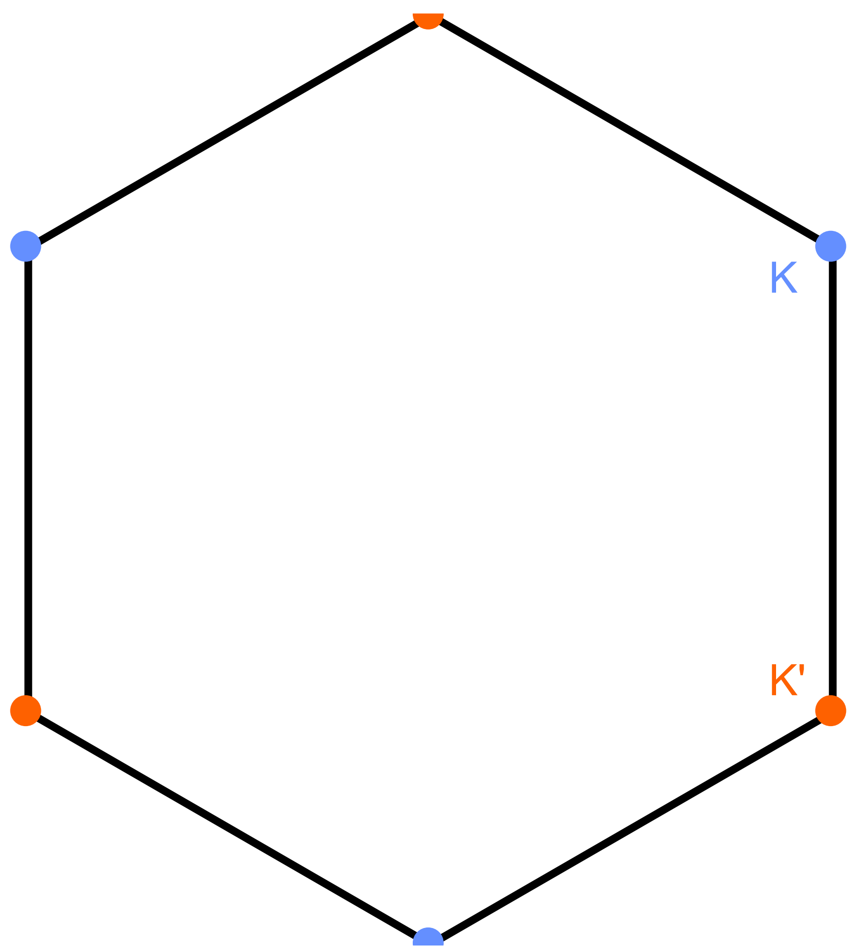
Low energy theory

- For the low energy theory we expand in terms of the Bloch wave functions at the Dirac points and slowly varying envelope functions

- These Bloch wave functions have well defined symmetry properties under lattice transformations

Expand around K and K' points

$$\hat{\vec{\psi}}_{\mathbf{k}} = \begin{pmatrix} \psi_{KA}(\mathbf{k}) \\ \psi_{KB}(\mathbf{k}) \\ \psi_{K'B}(\mathbf{k}) \\ -\psi_{K'A}(\mathbf{k}) \end{pmatrix}$$



Aleiner, Kharzeev, Tsvelik, PRB 76, 195415 (2007)

Kharitonov, PRB 85, 155439 (2012)

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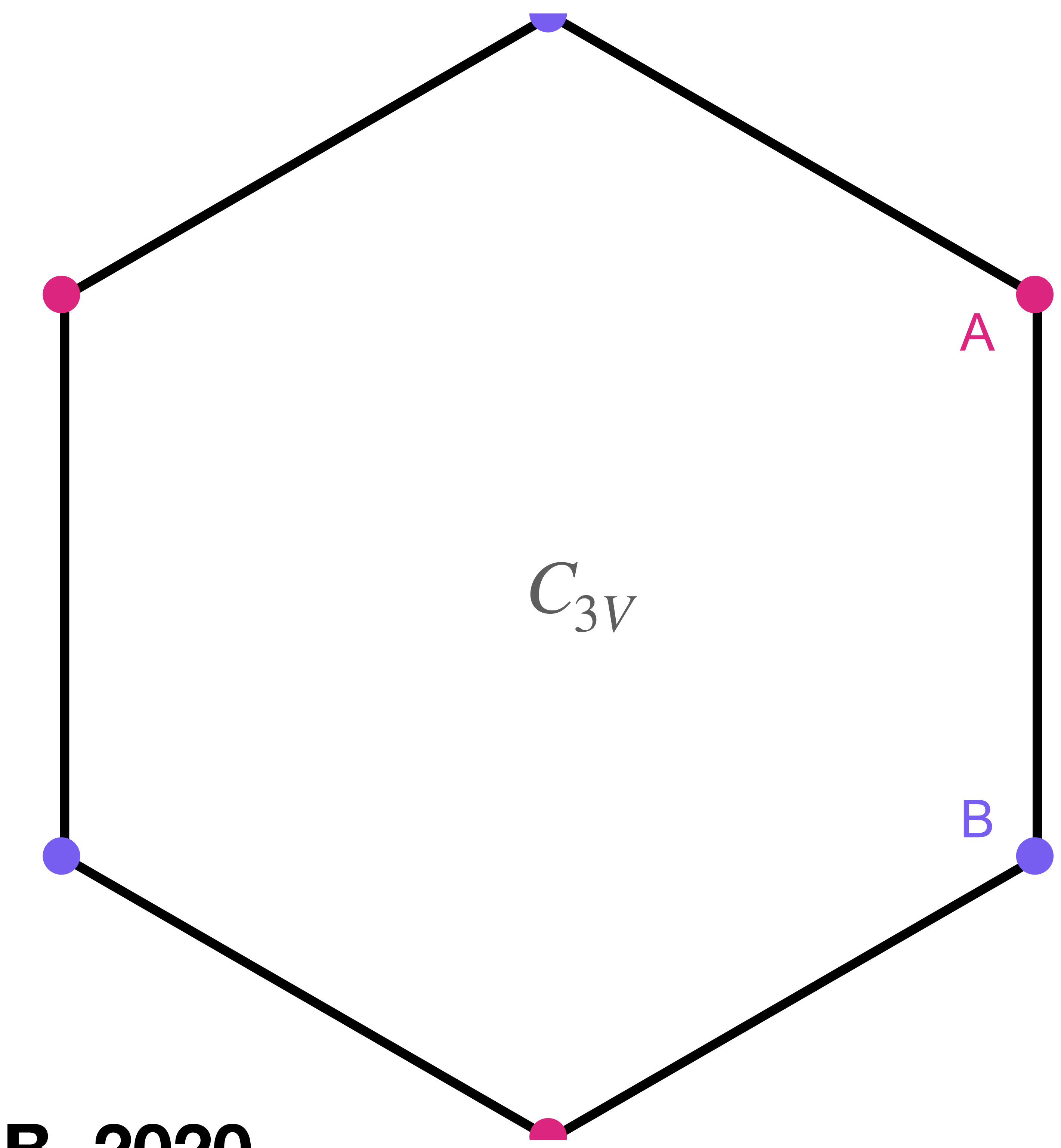
Low energy theory

Rotation

Mirror plane

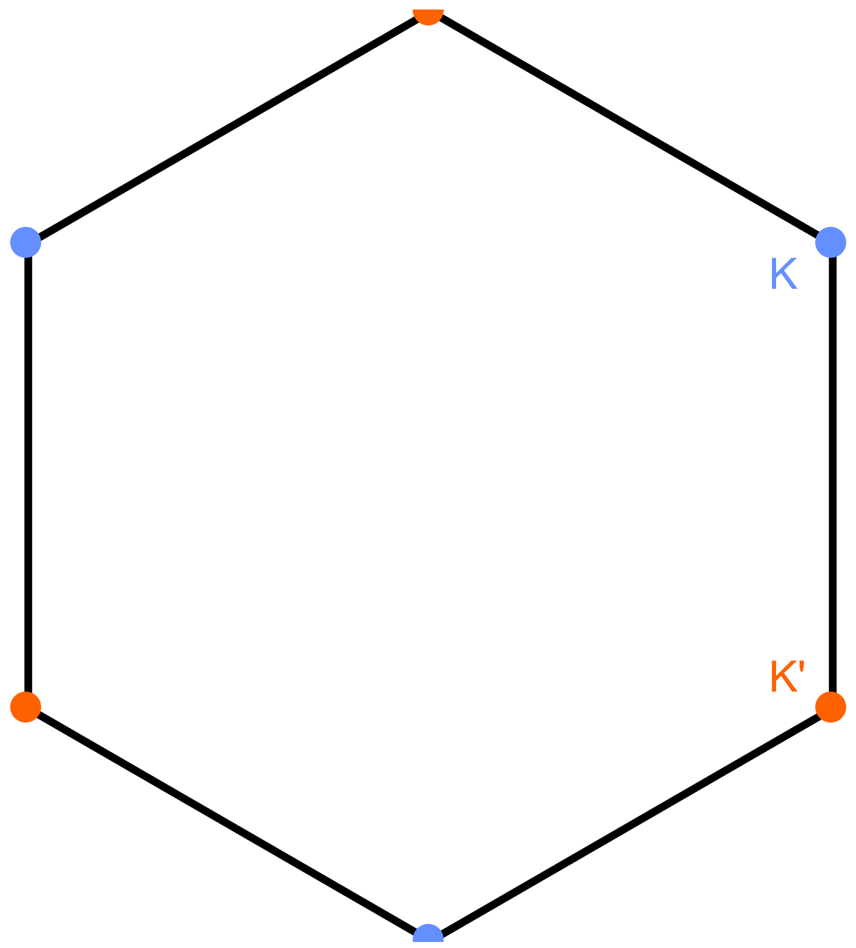
Translation

Time Reversal



Expand around K and K' points

$$\hat{\vec{\psi}}_{\mathbf{k}} = \begin{pmatrix} \psi_{KA}(\mathbf{k}) \\ \psi_{KB}(\mathbf{k}) \\ \psi_{K'B}(\mathbf{k}) \\ -\psi_{K'A}(\mathbf{k}) \end{pmatrix}$$



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Interactions from symmetry

Types of interaction

- Rewrite the Coulomb interaction Hamiltonian in terms of long wavelength operators

$$g \propto \int u^*_{\zeta\Sigma}(r)V(|r-r'|)u_{\zeta'\Sigma'}(r')$$

- Allowed interactions are
  - Long ranged Coulomb
  - Valley Pseudo-Spin Couplings
  - Density to staggered density coupling

$$\hat{H}^{\psi}_{\text{int}} = \frac{1}{2}\sum_{\mathbf{r},\mathbf{r}'} V(\mathbf{r}-\mathbf{r}') : \psi^\dagger(\mathbf{r})\psi(\mathbf{r})\psi^\dagger(\mathbf{r}')\psi(\mathbf{r}') :$$
$$+\frac{1}{2}\sum_{\mathbf{r}}\sum_{\alpha\beta}\Big[g_{\alpha\beta} : \psi^\dagger(\mathbf{r})\Sigma^\alpha\tau^\beta\psi(\mathbf{r})\psi^\dagger(\mathbf{r}')\Sigma^\alpha\tau^\beta\psi(\mathbf{r}') :$$
$$+\tilde{g}_{00} : \psi^\dagger(\mathbf{r})\Sigma^z\tau^z\psi(\mathbf{r})\psi^\dagger(\mathbf{r}')\psi(\mathbf{r}') :\Big]$$

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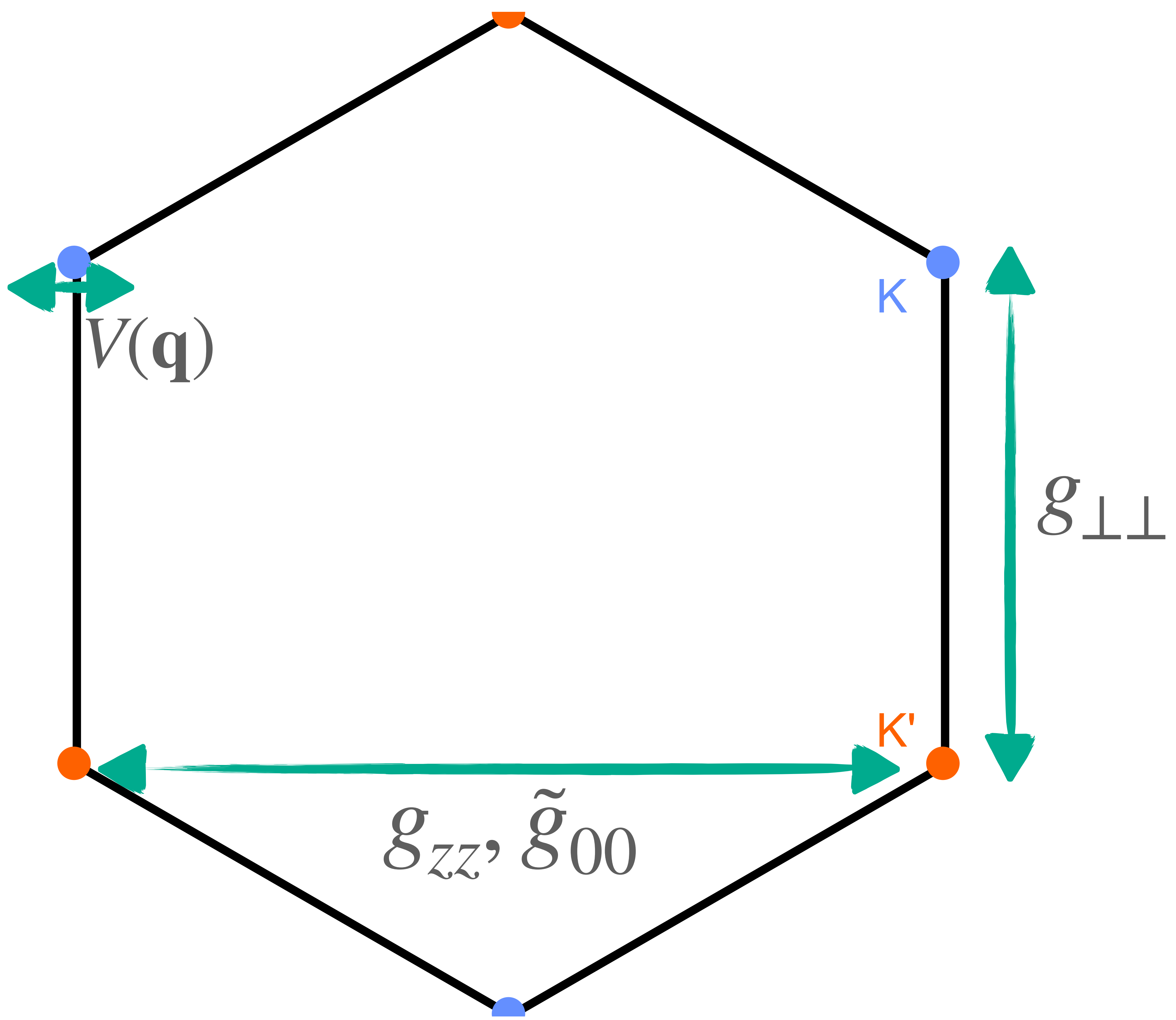
Interactions from symmetry

Energy scales



$$g \propto \int u^*_{\zeta\Sigma}(r) V(|r - r'|) u_{\zeta'\Sigma'}(r')$$

Long Range

Strongest

$V(\mathbf{q})$

$$g_{\perp\perp} \sim V(|\mathbf{K} - \mathbf{K}'|)$$

$$g_{zz} \sim V(|\mathbf{b}|)$$

$$\tilde{g}_{00} \sim \frac{g_{zz}\Delta}{v_F |\mathbf{K} - \mathbf{K}'|}$$

Weakest

Short Range

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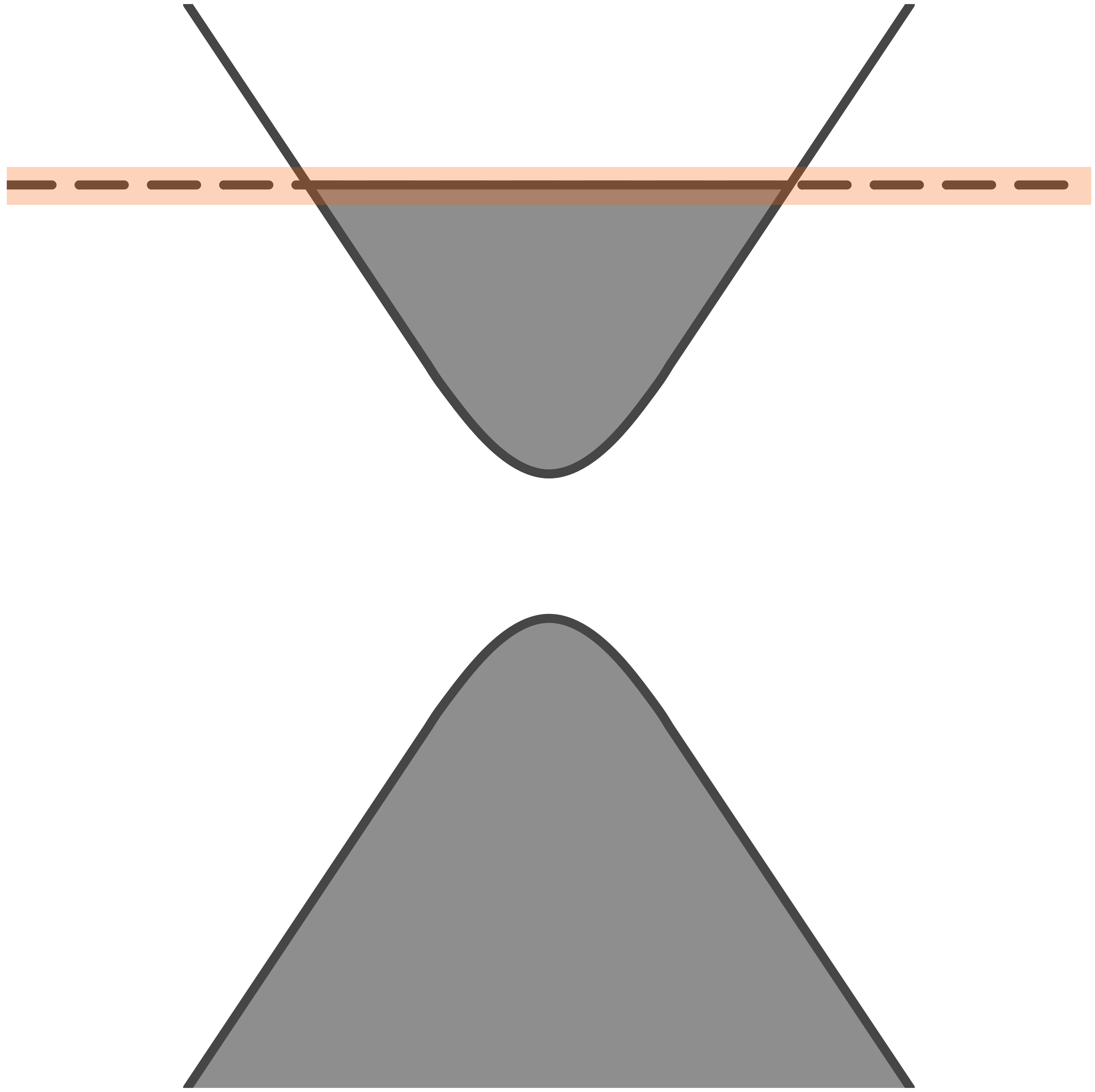
Upper band description

Interaction channels



Coulomb		Short ranged	
1		1	$\tau^3\tau^3$
		$\sigma\cdot\sigma$	$\tau^\parallel\cdot\tau^\parallel\sigma\cdot\sigma$
		$\tau^\parallel\cdot\tau^\parallel$	$\tau^3\tau^3\sigma\cdot\sigma$
$V(q)$	**Interaction functions**	$U^\mu(p,p',q)$	

Landau-Silin kinetic theory

A brief recap

$$\frac{\partial n(\mathbf{k},\mathbf{r})}{\partial t} + \frac{\partial \epsilon}{\partial \mathbf{k}} \cdot \frac{\partial}{\partial \mathbf{r}} n(\mathbf{k},\mathbf{r}) - \frac{\partial \epsilon}{\partial \mathbf{r}} \cdot \frac{\partial}{\partial \mathbf{k}} n(\mathbf{k},\mathbf{r}) = \hat{I}[n]$$

$$\mathscr{F} = \mathscr{F}_0 + \sum_k \xi_k \delta n_k + \frac{1}{2} \sum_{k,k'} f_{kk'} \delta n_k \delta n_{k'} + \cdots$$

$$\epsilon_k = \xi_k + \sum_{k'} f_{kk'} \delta n_{k'}$$

Landau-Silin kinetic theory

A brief recap

$$\frac{\partial n(\mathbf{k},\mathbf{r})}{\partial t} + \frac{\partial \epsilon}{\partial \mathbf{k}} \cdot \frac{\partial}{\partial \mathbf{r}} n(\mathbf{k},\mathbf{r}) - \frac{\partial \epsilon}{\partial \mathbf{r}} \cdot \frac{\partial}{\partial \mathbf{k}} n(\mathbf{k},\mathbf{r}) = \hat{I}[n]$$

$$\updownarrow$$

$$\frac{\partial n(\mathbf{k},\mathbf{r})}{\partial t} + \dot{\mathbf{r}} \cdot \frac{\partial}{\partial \mathbf{r}} n(\mathbf{k},\mathbf{r}) + \dot{\mathbf{p}} \cdot \frac{\partial}{\partial \mathbf{k}} n(\mathbf{k},\mathbf{r}) = \hat{I}[n]$$

$$\mathcal{F} = \mathcal{F}_0 + \sum_k \xi_k \delta n_k + \frac{1}{2} \sum_{k,k'} f_{kk'} \delta n_k \delta n_{k'} + \cdots$$

$$\epsilon_k = \xi_k + \sum_{k'} f_{kk'} \delta n_{k'}$$

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$$\updownarrow$$

$$\frac{\partial n(\mathbf{k},\mathbf{r})}{\partial t} + \dot{\mathbf{r}} \cdot \frac{\partial}{\partial \mathbf{r}} n(\mathbf{k},\mathbf{r}) + \dot{\mathbf{p}} \cdot \frac{\partial}{\partial \mathbf{k}} n(\mathbf{k},\mathbf{r}) = \hat{I}[n]$$

$$\mathcal{F} = \mathcal{F}_0 + \sum_k \xi_k \delta n_k + \frac{1}{2} \sum_{k,k'} f_{kk'} \delta n_k \delta n_{k'} + \cdots$$

$$\epsilon_k = \xi_k + \sum_{k'} f_{kk'} \delta n_{k'}$$

Now linearize

$$n = n_F(\bar{\epsilon}) + \delta n$$

Landau-Silin kinetic theory

A brief recap

$$\frac{\partial \delta n(\mathbf{k},\mathbf{r})}{\partial t} + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}} \overline{\delta n}(\mathbf{k},\mathbf{r}) + \frac{\partial n_F}{\partial \epsilon}\Big|_{\bar{\epsilon}} \mathbf{v} \cdot \mathbf{F} = \hat{I}[n]$$

$$\mathscr{F} = \mathscr{F}_0 + \sum_k \xi_k \delta n_k + \frac{1}{2} \sum_{k,k'} f_{kk'} \delta n_k \delta n_{k'} + \cdots$$

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Landau-Silin kinetic theory

A brief recap

$$\frac{\partial \delta n(\mathbf{k},\mathbf{r})}{\partial t}+\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\overline{\delta n}(\mathbf{k},\mathbf{r})+\frac{\partial n_F}{\partial \epsilon}\Big|_{\bar{\epsilon}}\mathbf{v}\cdot\mathbf{F}=\hat{I}[n]$$

Velocity

$$\mathcal{F}=\mathcal{F}_0+\sum_k \xi_k \delta n_k+\frac{1}{2}\sum_{k,k'} f_{kk'}\delta n_k \delta n_{k'}+\cdots$$

$$\epsilon_k=\xi_k+\sum_{k'} f_{kk'}\delta n_{k'}$$

Now linearize

$$n=n_F(\bar{\epsilon})+\delta n$$

Landau-Silin kinetic theory

A brief recap

$$\frac{\partial \delta n(\mathbf{k},\mathbf{r})}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\overline{\delta n}(\mathbf{k},\mathbf{r}) + \left.\frac{\partial n_F}{\partial \epsilon}\right|_{\bar{\epsilon}} \mathbf{v}\cdot\mathbf{F} = \hat{I}[n]$$

Velocity

Force

$$\mathscr{F} = \mathscr{F}_0 + \sum_k \xi_k \delta n_k + \frac{1}{2}\sum_{k,k'} f_{kk'}\delta n_k \delta n_{k'} + \cdots$$

$$\epsilon_k = \xi_k + \sum_{k'} f_{kk'}\delta n_{k'}$$

Now linearize

$$n = n_F(\bar{\epsilon}) + \delta n$$

Landau-Silin kinetic theory

A brief recap

$$\frac{\partial \delta n(\mathbf{k},\mathbf{r})}{\partial t}+\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\overline{\delta n}(\mathbf{k},\mathbf{r})+\left.\frac{\partial n_F}{\partial \epsilon}\right|_{\bar{\epsilon}}\mathbf{v}\cdot\mathbf{F}=\hat{I}[n]$$

Velocity

Deviation from local equilibrium

Force

$$\overline{\delta n}=\delta n-\frac{\partial n}{\partial \epsilon}\delta\epsilon$$

Interactions hide here

$$\mathscr{F}=\mathscr{F}_0+\sum_k \xi_k \delta n_k+\frac{1}{2}\sum_{k,k'} f_{kk'}\delta n_k \delta n_{k'}+\cdots$$

$$\epsilon_k=\xi_k+\sum_{k'} f_{kk'}\delta n_{k'}$$

Now linearize

$$n=n_F(\bar{\epsilon})+\delta n$$

Landau-Silin kinetic theory

multicomponent

$$\frac{\partial \delta n(\mathbf{k},\mathbf{r})}{\partial t}+\mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\overline{\delta n}(\mathbf{k},\mathbf{r})+\left.\frac{\partial n_F}{\partial \epsilon}\right|_{\bar{\epsilon}}\mathbf{v}\cdot\mathbf{F}=\hat{I}[n]$$

$$\mathscr{F}=\mathscr{F}_0+\sum_k \xi_k\delta n_k+\frac{1}{2}\sum_{k,k'} f_{kk'}\delta n_k\delta n_{k'}+\cdots$$

e.g

$$\sum_k \epsilon_{ij}\delta n_{ji}(k)$$

e.g

$$\sum_{k,k'} \delta n_{ij}(k)f^{ij;lm}(k,k')\delta n_{lm}(k')$$

$$\hat{\rho}=n_F(\bar{\epsilon})+\delta\hat{\rho}$$

Landau-Silin kinetic theory

multicomponent

$$\frac{\partial \delta n(\mathbf{k},\mathbf{r})}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\overline{\delta n}(\mathbf{k},\mathbf{r}) + \frac{\partial n_F}{\partial \epsilon}\bigg|_{\bar\epsilon} \mathbf{v}\cdot\mathbf{F} = \hat{I}[n]$$

$$\mathscr{F} = \mathscr{F}_0 + \sum_k \xi_k \delta n_k + \frac{1}{2}\sum_{k,k'} f_{kk'}\delta n_k \delta n_{k'} + \cdots$$

e.g

$$\sum_k \epsilon_{ij}\delta n_{ji}(k)$$

e.g

$$\sum_{k,k'} \delta n_{ij}(k) f^{ij;lm}(k,k')\delta n_{lm}(k')$$

Now with matrices

$$\frac{\partial \delta\hat\rho(\mathbf{k},\mathbf{r})}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\delta\hat{\bar\rho}(\mathbf{k},\mathbf{r}) + \frac{\partial n}{\partial \epsilon}\bigg|_{\bar\epsilon} \mathbf{v}\cdot\hat{\mathbf{F}} = \hat{I}[\delta\hat\rho]$$

$$\hat\rho = n_F(\bar\epsilon) + \delta\hat\rho$$

Landau-Silin kinetic theory

multicomponent

$$\frac{\partial \delta\hat{\rho}(\mathbf{k},\mathbf{r})}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\delta\hat{\bar{\rho}}(\mathbf{k},\mathbf{r}) + \left.\frac{\partial n}{\partial \epsilon}\right|_{\bar{\epsilon}} \mathbf{v}\cdot\hat{\mathbf{F}} = \hat{I}[\delta\hat{\rho}]$$

$$\hat{\rho} = n_F(\bar{\epsilon}) + \delta\hat{\rho}$$

Landau-Silin kinetic theory

multicomponent

$$\frac{\partial \delta\hat{\rho}(\mathbf{k},\mathbf{r})}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\delta\hat{\bar{\rho}}(\mathbf{k},\mathbf{r}) + \frac{\partial n}{\partial \epsilon}\bigg|_{\bar{\epsilon}} \mathbf{v}\cdot\hat{\mathbf{F}} = \hat{I}[\delta\hat{\rho}]$$

$$\hat{\rho} = n_F(\bar{\epsilon}) + \delta\hat{\rho}$$

Recall

Short ranged

$$1 \qquad \tau^3\tau^3$$

$$\sigma\cdot\sigma \qquad \tau^{\parallel}\cdot\tau^{\parallel}\sigma\cdot\sigma$$

$$\tau^{\parallel}\cdot\tau^{\parallel} \qquad \tau^3\tau^3\sigma\cdot\sigma$$

Landau-Silin kinetic theory

multicomponent

$$\frac{\partial \delta\hat{\rho}(\mathbf{k},\mathbf{r})}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\delta\hat{\bar{\rho}}(\mathbf{k},\mathbf{r}) + \frac{\partial n}{\partial \epsilon}\Big|_{\bar{\epsilon}}\mathbf{v}\cdot\hat{\mathbf{F}} = \hat{I}[\delta\hat{\rho}]$$

$$\hat{\rho} = n_F(\bar{\epsilon}) + \delta\hat{\rho}$$

$$\hat{\rho} = n_F + \sum_{\mu} \hat{X}^{\mu}\delta\rho^{\mu}$$

Natural variables

Recall

Short ranged

1

$\sigma\cdot\sigma$

$\tau^{\|}\cdot\tau^{\|}$

$\tau^3\tau^3$

$\tau^{\|}\cdot\tau^{\|}\sigma\cdot\sigma$

$\tau^3\tau^3\sigma\cdot\sigma$

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Landau-Silin kinetic theory

Dynamics

$$\hat{\rho} = n_F + \sum_{\mu} \hat{X}^{\mu} \delta\rho^{\mu}$$

$$n(\mathbf{r}, \mathbf{p}) = \frac{1}{G_s G_v} \operatorname{tr} \hat{\sigma}_0 \hat{\tau}_0 \hat{\rho}(\mathbf{r}, \mathbf{p})$$

$$\mathbf{s}(\mathbf{r}, \mathbf{p}) = \frac{1}{G_s G_v} \operatorname{tr} \hat{\boldsymbol{\sigma}} \hat{\rho}(\mathbf{r}, \mathbf{p})$$

$$\mathbf{Y}(\mathbf{r}, \mathbf{p}) = \frac{1}{G_s G_v} \operatorname{tr} \hat{\boldsymbol{\tau}} \hat{\rho}(\mathbf{r}, \mathbf{p})$$

$$M_i^j(\mathbf{r}, \mathbf{p}) = \frac{1}{G_s G_v} \operatorname{tr} \hat{\tau}_i \hat{\sigma}_j \hat{\rho}(\mathbf{r}, \mathbf{p})$$

$\parallel$, $\perp$

Spin and valley degeneracy

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Landau-Silin kinetic theory

Dynamics

- Collective modes associated with each channel

$$\hat{\rho} = n_F + \sum_{\mu} \hat{X}^{\mu} \delta\rho^{\mu}$$

$$n(\mathbf{r},\mathbf{p}) = \frac{1}{G_s G_v} \operatorname{tr} \hat{\sigma}_0 \hat{\tau}_0 \hat{\rho}(\mathbf{r},\mathbf{p})$$

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$$M_i^j(\mathbf{r},\mathbf{p}) = \frac{1}{G_s G_v} \operatorname{tr} \hat{\tau}_i \hat{\sigma}_j \hat{\rho}(\mathbf{r},\mathbf{p})$$

$\parallel, \perp$

Spin and valley degeneracy

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Landau-Silin kinetic theory

Dynamics

$$\hat{\rho} = n_F + \sum_{\mu} \hat{X}^{\mu} \delta\rho^{\mu}$$

- Collective modes associated with each channel
- Equations decouple

$$\frac{\partial \delta\rho^{\mu}(\mathbf{k},\mathbf{r})}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\delta\bar{\rho}^{\mu}(\mathbf{k},\mathbf{r}) + \frac{\partial n}{\partial \epsilon}\bigg|_{\bar{\epsilon}} \mathbf{v}\cdot\mathbf{F}^{\mu} = \frac{1}{G_s G_v}\,\mathrm{tr}\,\hat{X}^{\mu}\hat{I}[\delta\hat{\rho}]$$

$$n(\mathbf{r},\mathbf{p}) = \frac{1}{G_s G_v}\,\mathrm{tr}\,\hat{\sigma}_0\hat{\tau}_0\hat{\rho}(\mathbf{r},\mathbf{p})$$

$$\mathbf{s}(\mathbf{r},\mathbf{p}) = \frac{1}{G_s G_v}\,\mathrm{tr}\,\hat{\sigma}\hat{\rho}(\mathbf{r},\mathbf{p})$$

$$\mathbf{Y}(\mathbf{r},\mathbf{p}) = \frac{1}{G_s G_v}\,\mathrm{tr}\,\hat{\tau}\hat{\rho}(\mathbf{r},\mathbf{p})$$

$$M_i^j(\mathbf{r},\mathbf{p}) = \frac{1}{G_s G_v}\,\mathrm{tr}\,\hat{\tau}_i\hat{\sigma}_j\hat{\rho}(\mathbf{r},\mathbf{p})$$

$\parallel$, $\perp$

Spin and valley degeneracy

10.1103/PhysRevB.103.075422

Conventional Sound $\omega \propto v_F q$

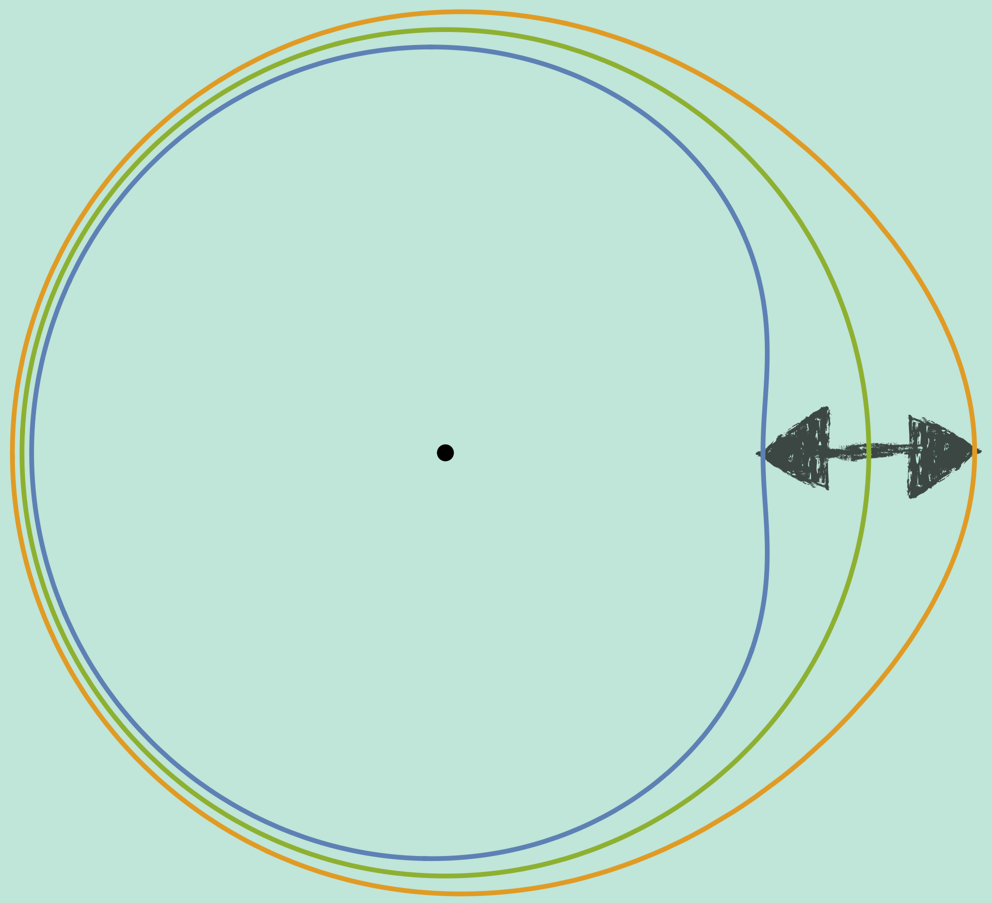
(uncharged) 2D FL

$$\frac{\partial n(\mathbf{k},\mathbf{r})}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\overline{\delta n}(\mathbf{k},\mathbf{r}) + \left.\frac{\partial n_F}{\partial \epsilon}\right|_{\bar{\epsilon}} \mathbf{v}\cdot\mathbf{F} = \hat{I}[n]$$

Zero Sound

$$\omega \gg \frac{1}{\tau}$$

Collisionless



- For charged liquids the Coulomb potential turns both into the plasmon

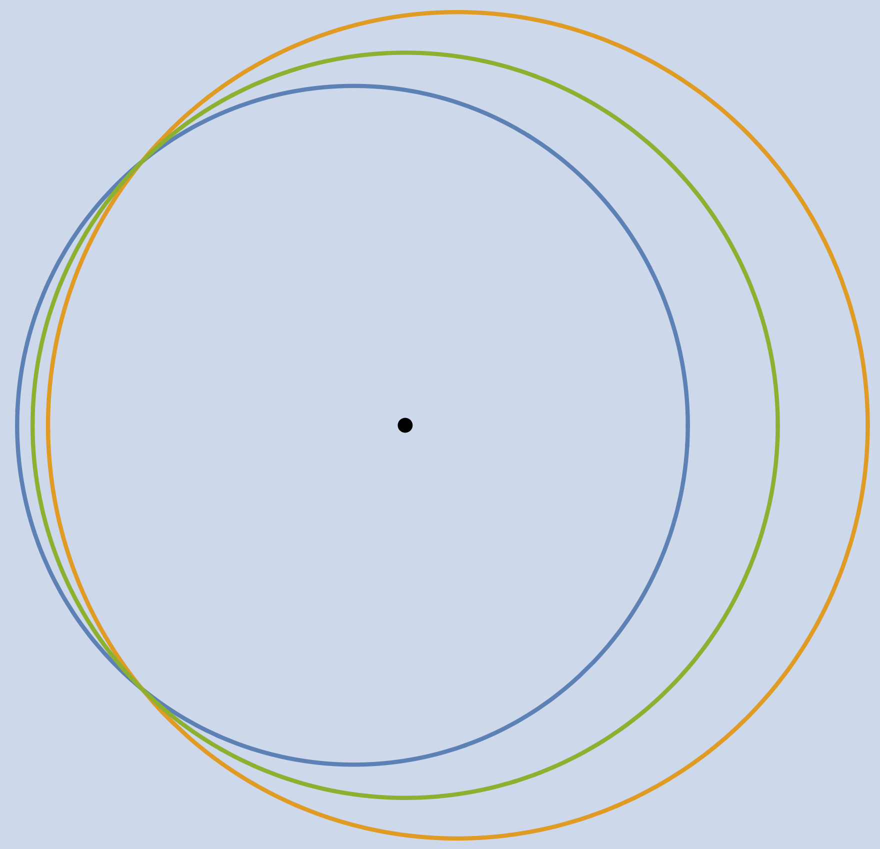
$$\omega \approx \sqrt{\frac{4\pi e^2 n}{m_*}q}$$

Plasmon

First Sound

$$\frac{1}{\tau_1} \ll \omega \ll \frac{1}{\tau_2}$$

Collisional



10.1103/PhysRevB.103.075422

Other spin-valley channels

Well studied and generic to 2D FL

Plasmon $n(\mathbf{r}, \mathbf{p}) = \frac{1}{G_s G_v} \mathrm{tr}\, \hat{\sigma}_0 \hat{\tau}_0 \hat{\rho}(\mathbf{r}, \mathbf{p})$

What analogues of first and zero sound exist here?

$$\mathbf{s}(\mathbf{r}, \mathbf{p}) = \frac{1}{G_s G_v} \mathrm{tr}\, \hat{\boldsymbol{\sigma}} \hat{\rho}(\mathbf{r}, \mathbf{p})$$

Multi valley materials

$$\mathbf{Y}(\mathbf{r}, \mathbf{p}) = \frac{1}{G_s G_v} \mathrm{tr}\, \hat{\boldsymbol{\tau}} \hat{\rho}(\mathbf{r}, \mathbf{p})$$

$$M_i^j(\mathbf{r}, \mathbf{p}) = \frac{1}{G_s G_v} \mathrm{tr}\, \hat{\tau}_i \hat{\sigma}_j \hat{\rho}(\mathbf{r}, \mathbf{p})$$

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Neutral sound modes

What kills first and zero sound

$$\frac{\partial \delta\rho^{\mu}(\mathbf{k},\mathbf{r})}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\delta\bar{\rho}^{\mu}(\mathbf{k},\mathbf{r}) + \frac{\partial n}{\partial \epsilon}\bigg|_{\bar{\epsilon}} \mathbf{v}\cdot\mathbf{F}^{\mu} = \frac{1}{G_s G_v}\,\mathrm{tr}\,\hat{X}^{\mu}\hat{I}[\delta\hat{\rho}]$$

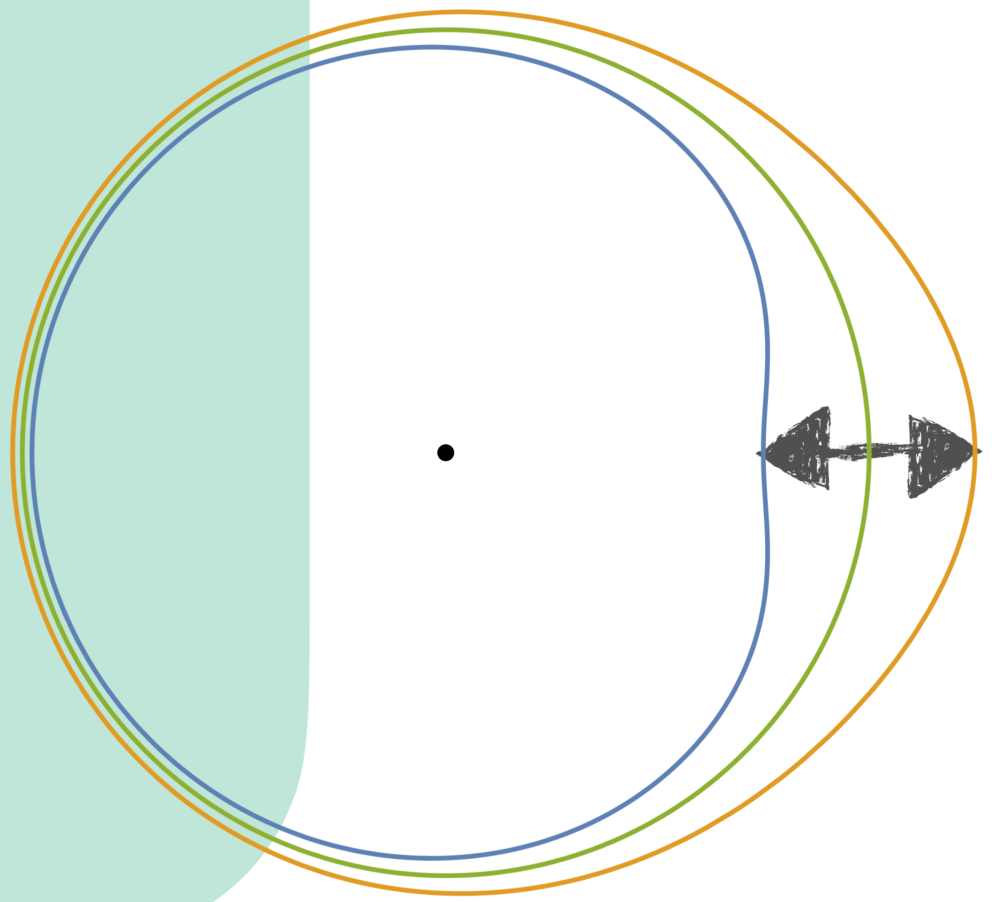
Zero Sound

$$\omega \gg \frac{1}{\tau}$$

Regime generically exists at low enough temperature

Collisionless

Can be killed by Landau damping

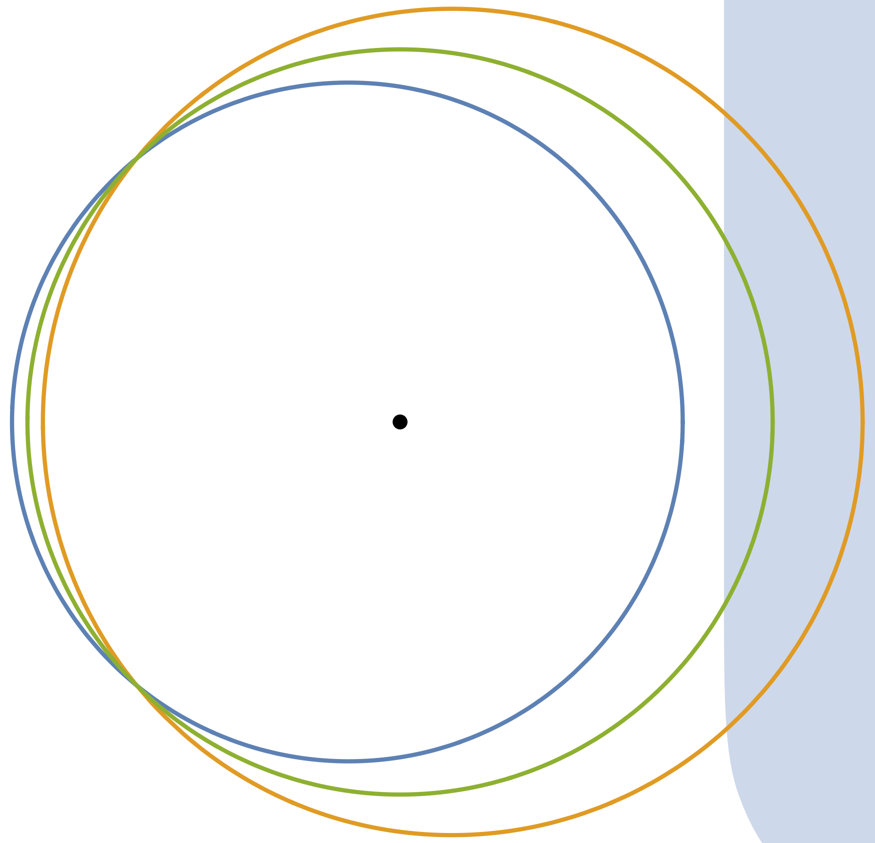


First Sound

$$\frac{1}{\tau_1} \ll \omega \ll \frac{1}{\tau_2}$$

Not guaranteed to exist

Collisional



Can be killed by collisional damping

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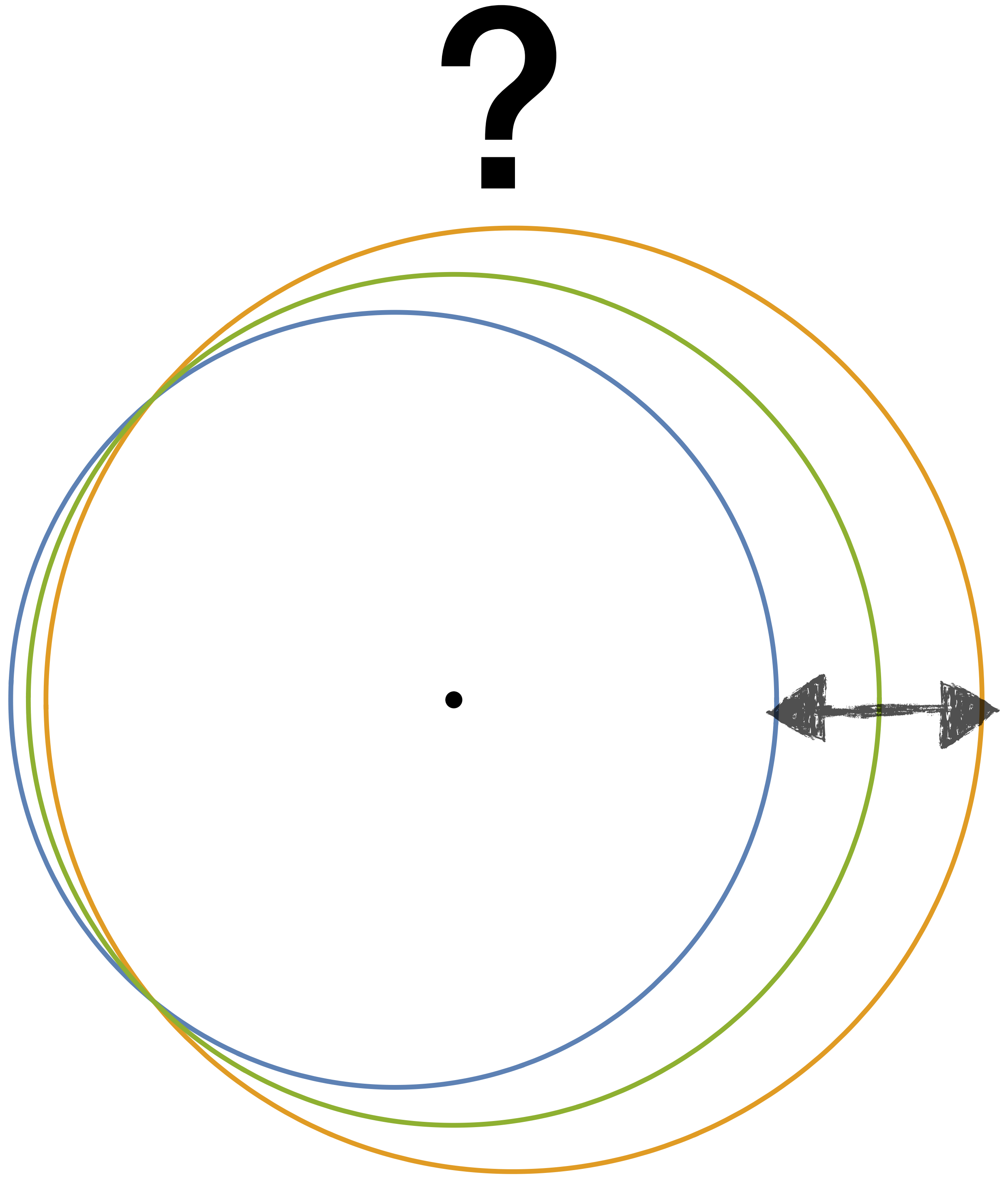
First sound

collisional kinetics

$$\frac{\partial \delta\rho^{\mu}(\mathbf{k},\mathbf{r})}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\delta\bar{\rho}^{\mu}(\mathbf{k},\mathbf{r}) = \frac{1}{G_s G_v}\,\mathrm{tr}\,\hat{X}^{\mu}\hat{I}[\delta\hat{\rho}]$$



10.1103/PhysRevB.103.075422

First sound

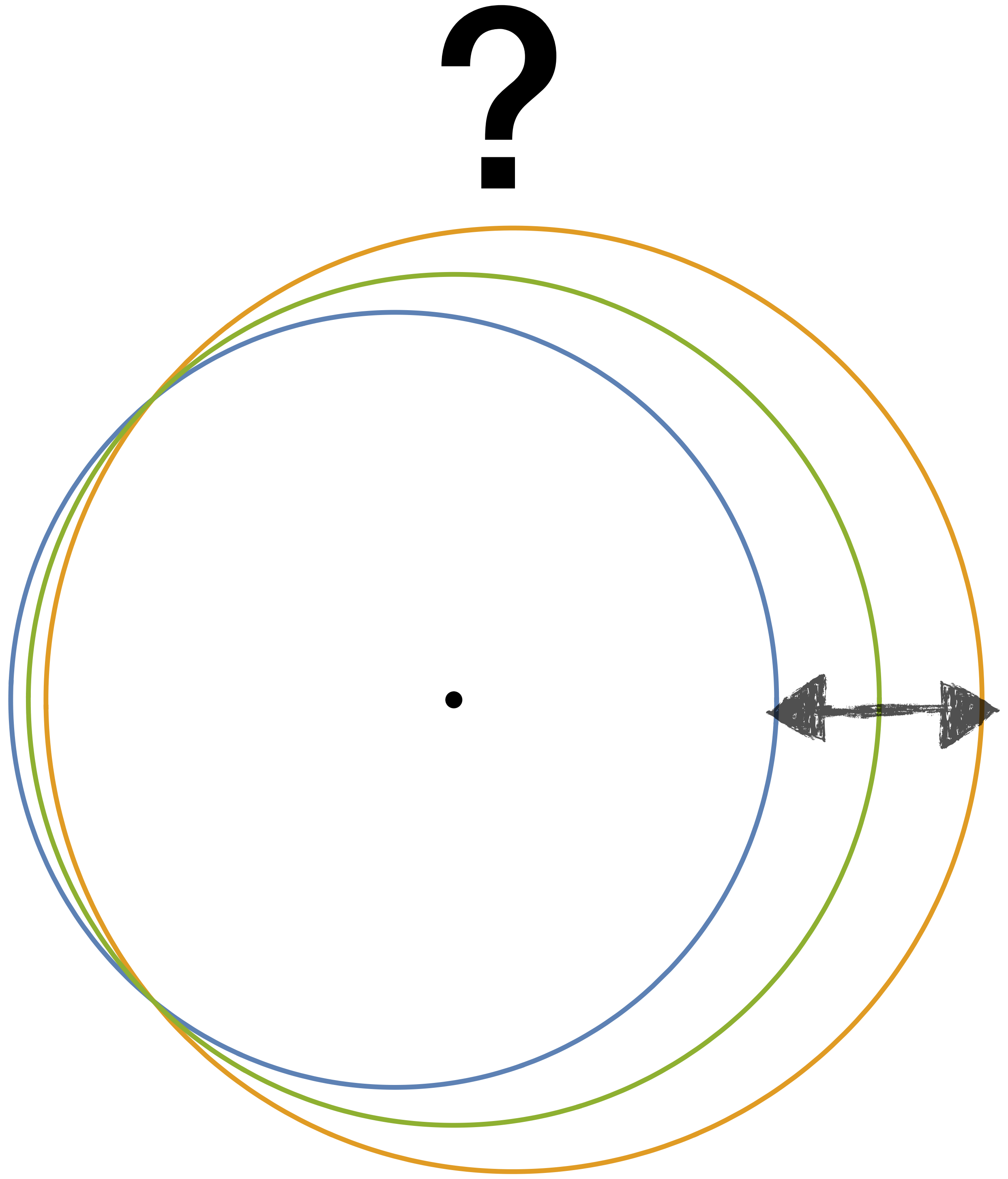
collisional kinetics

$$\frac{\partial \delta\rho^{\mu}(\mathbf{k},\mathbf{r})}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\delta\bar{\rho}^{\mu}(\mathbf{k},\mathbf{r}) = \frac{1}{G_s G_v}\,\mathrm{tr}\,\hat{X}^{\mu}\hat{I}[\delta\hat{\rho}]$$

$$\delta\rho^{\mu}(\mathbf{k},\mathbf{r}) = -\frac{\partial n}{\partial \epsilon}\sum_m e^{im\phi_{\mathbf{k}_F}}\nu_m^{\mu}(\mathbf{r})$$

$$\frac{\partial n}{\partial \epsilon}\sum_m \frac{\nu_m^{\mu}}{\tau_m^{\mu}}$$

Expand in angular harmonics



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First sound

collisional kinetics

$$\frac{\partial \delta\rho^{\mu}(\mathbf{k},\mathbf{r})}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\delta\bar{\rho}^{\mu}(\mathbf{k},\mathbf{r}) = \frac{1}{G_s G_v}\,\mathrm{tr}\,\hat{X}^{\mu}\hat{I}[\delta\hat{\rho}]$$

$$\delta\rho^{\mu}(\mathbf{k},\mathbf{r}) = -\frac{\partial n}{\partial \epsilon}\sum_m e^{im\phi_{\mathbf{k}_F}}\nu_m^{\mu}(\mathbf{r})$$

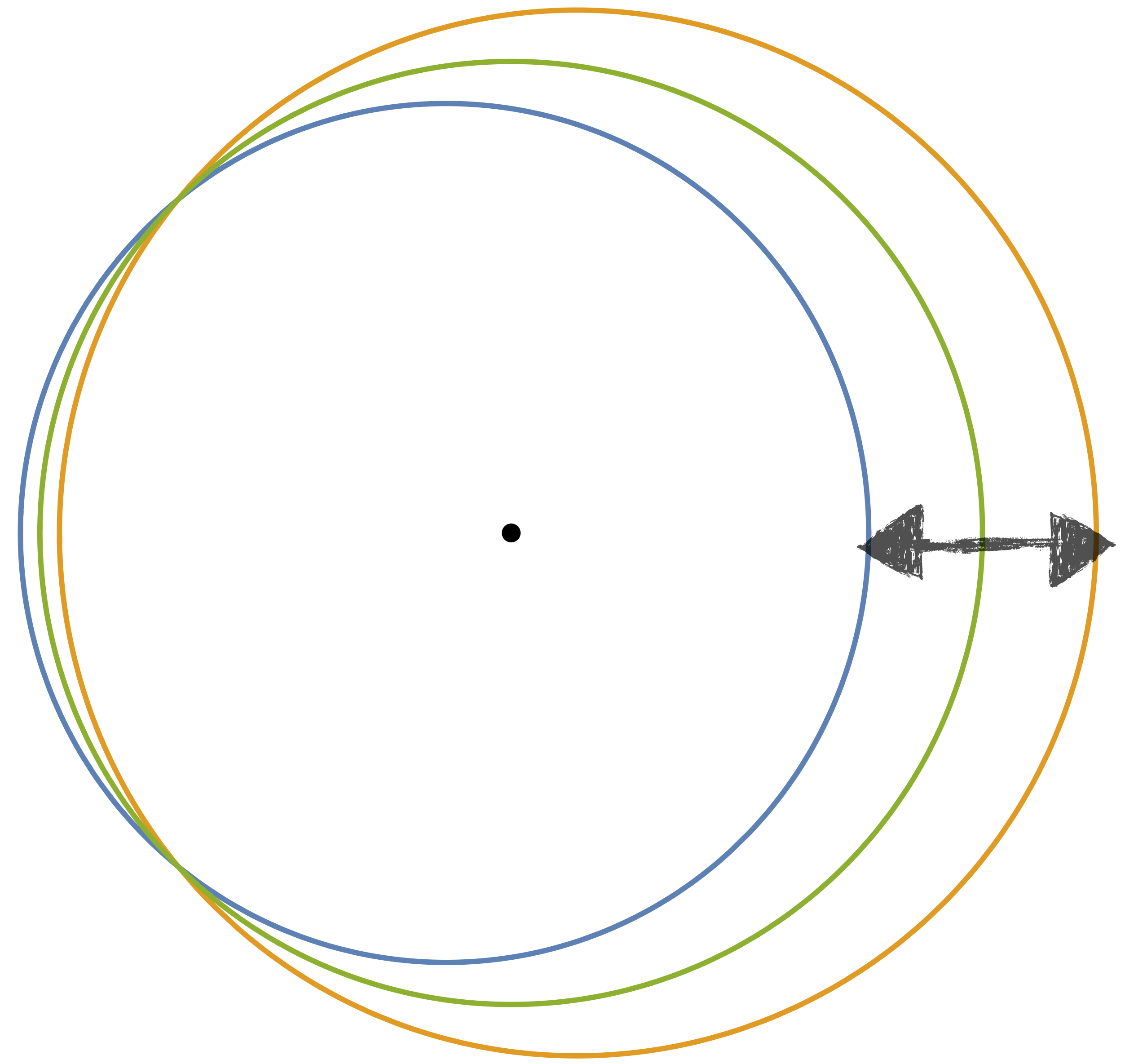
$$\frac{\partial n}{\partial \epsilon}\sum_m \frac{\nu_m^{\mu}}{\tau_m^{\mu}}$$

Expand in angular harmonics

occurs when

$$\frac{1}{\tau_1} \ll \omega \ll \frac{1}{\tau_2}$$

?



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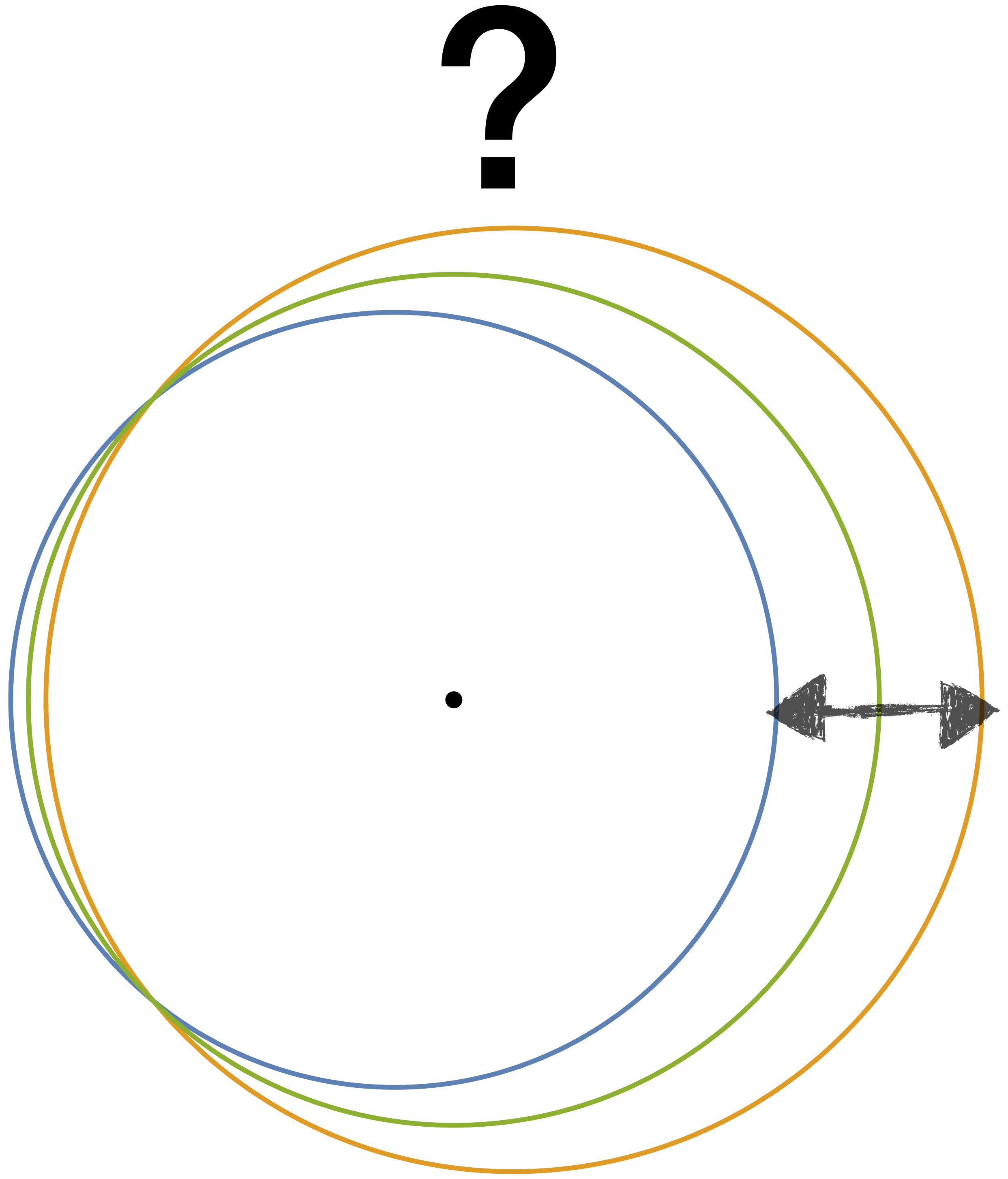
First sound

is there a hydrodynamic regime in neutral channels

$m = 0$	$\sum_p \delta\rho_p^\mu$	**Density**	**Conserved**
$m = \lvert 1 \rvert$	$\sum_p \cos\phi_{p_F}\delta\rho_p^\mu$	**Current**	
$m = \lvert 2 \rvert$	$\sum_p \cos^2\phi_{p_F}\delta\rho_p^\mu$		**Not**
	$\vdots$		

$m = 1$ and $m = 2$ are key



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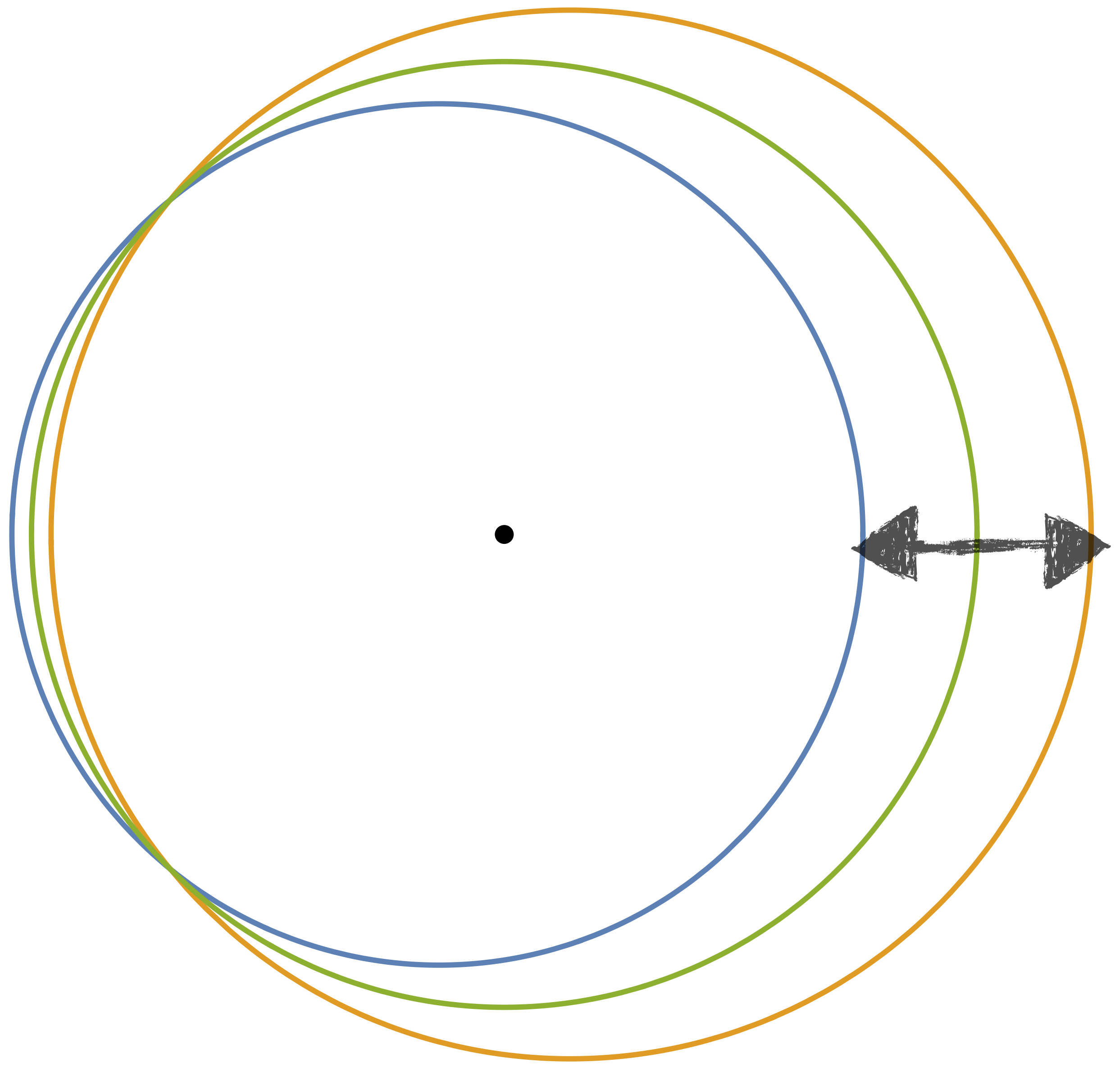
$m = 0$	$\sum_p \delta\rho_p^\mu$	**Density**	**Conserved**
$m = \|1\|$	$\sum_p \cos\phi_{p_F}\delta\rho_p^\mu$	**Current**	
$m = \|2\|$	$\sum_p \cos^2\phi_{p_F}\delta\rho_p^\mu$		**Not**
	$\vdots$		

$m = 1$ and $m = 2$ are key

occurs when

$$\frac{1}{\tau_1} \ll \omega \ll \frac{1}{\tau_2}$$

?



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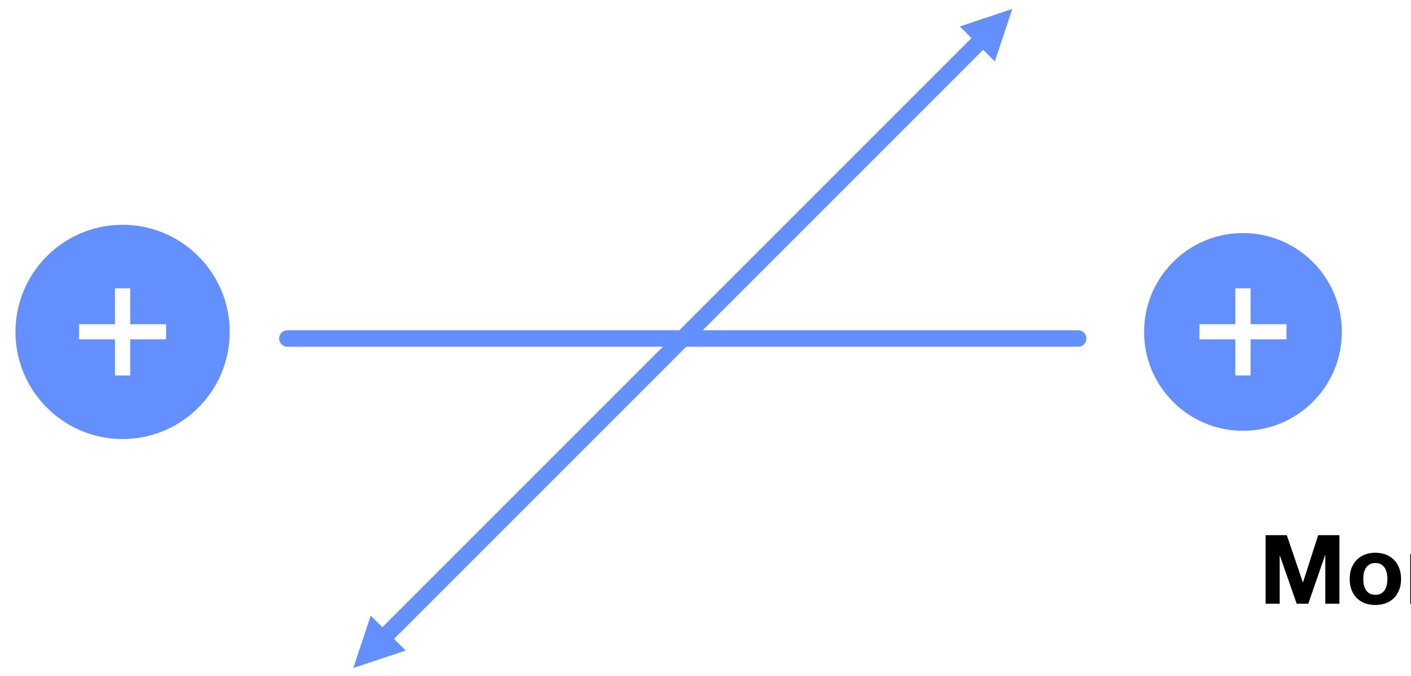
Neutral channels

How do these modes relax?



$\Delta j=0$

Momentum is current

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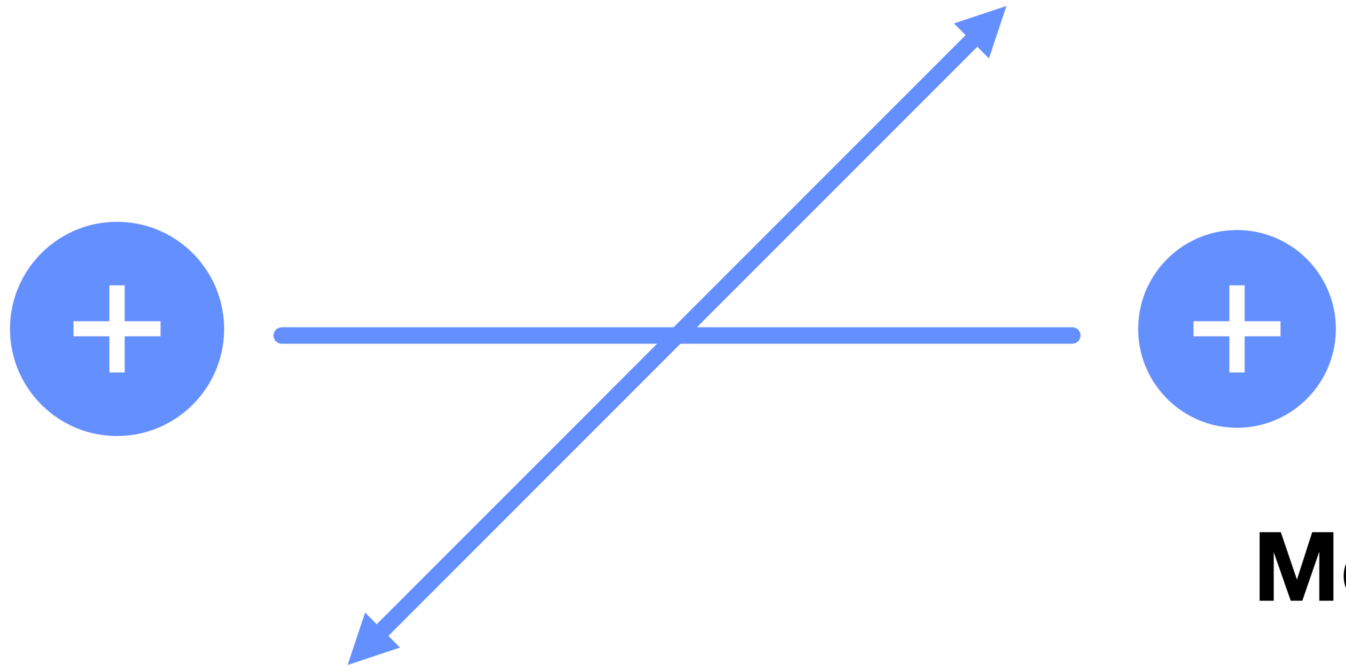
Neutral channels

How do these modes relax?



$\Delta j = 0$

Momentum is current

$\tau_1^{\mu} \to \infty$

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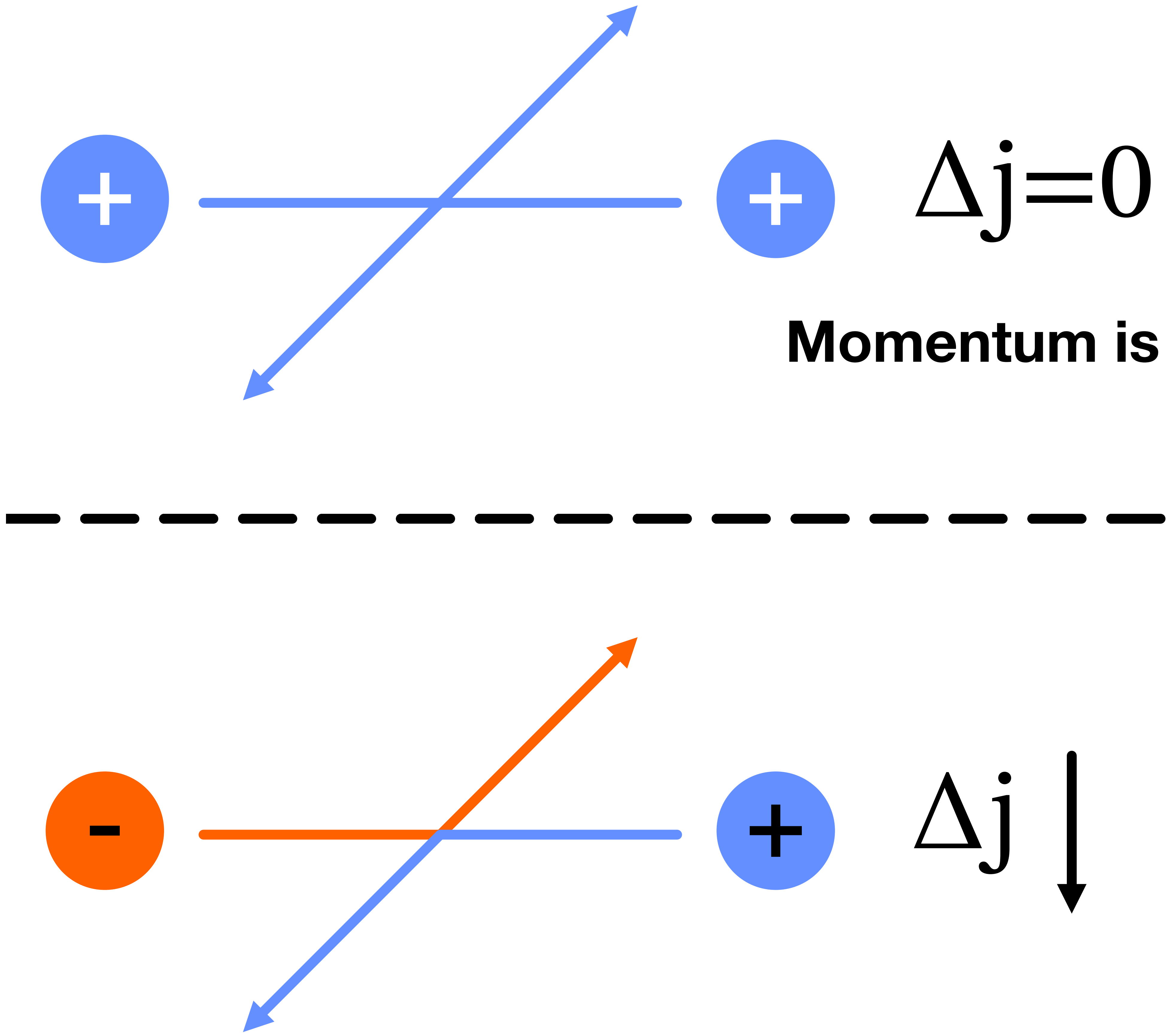
Neutral channels

How do these modes relax?



$\Delta j = 0$ $\tau_1^\mu \to \infty$

Momentum is current

$\Delta j \downarrow$

But not here

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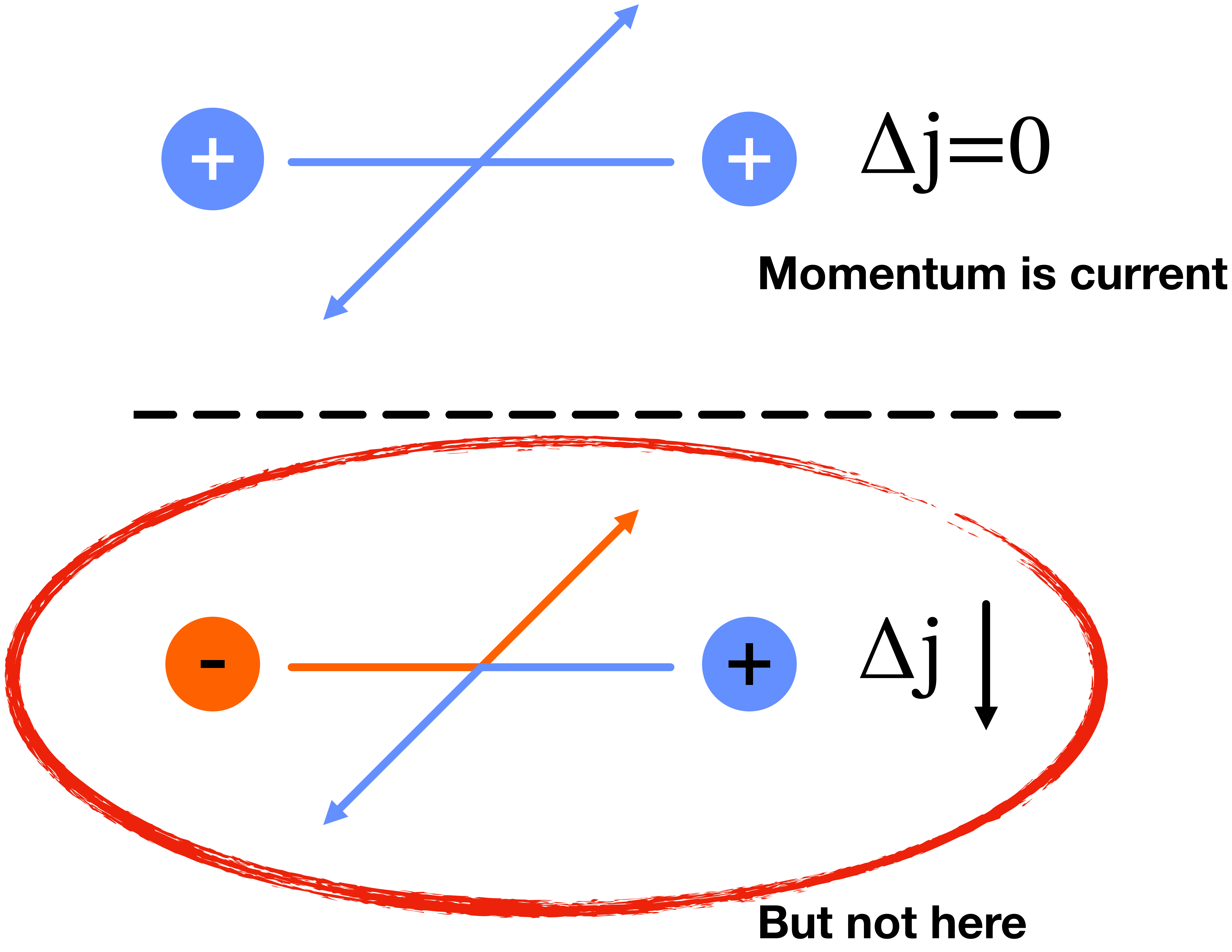
Neutral channels

How do these modes relax?



$\tau_1^\mu \to \infty$

τ_1^μ finite

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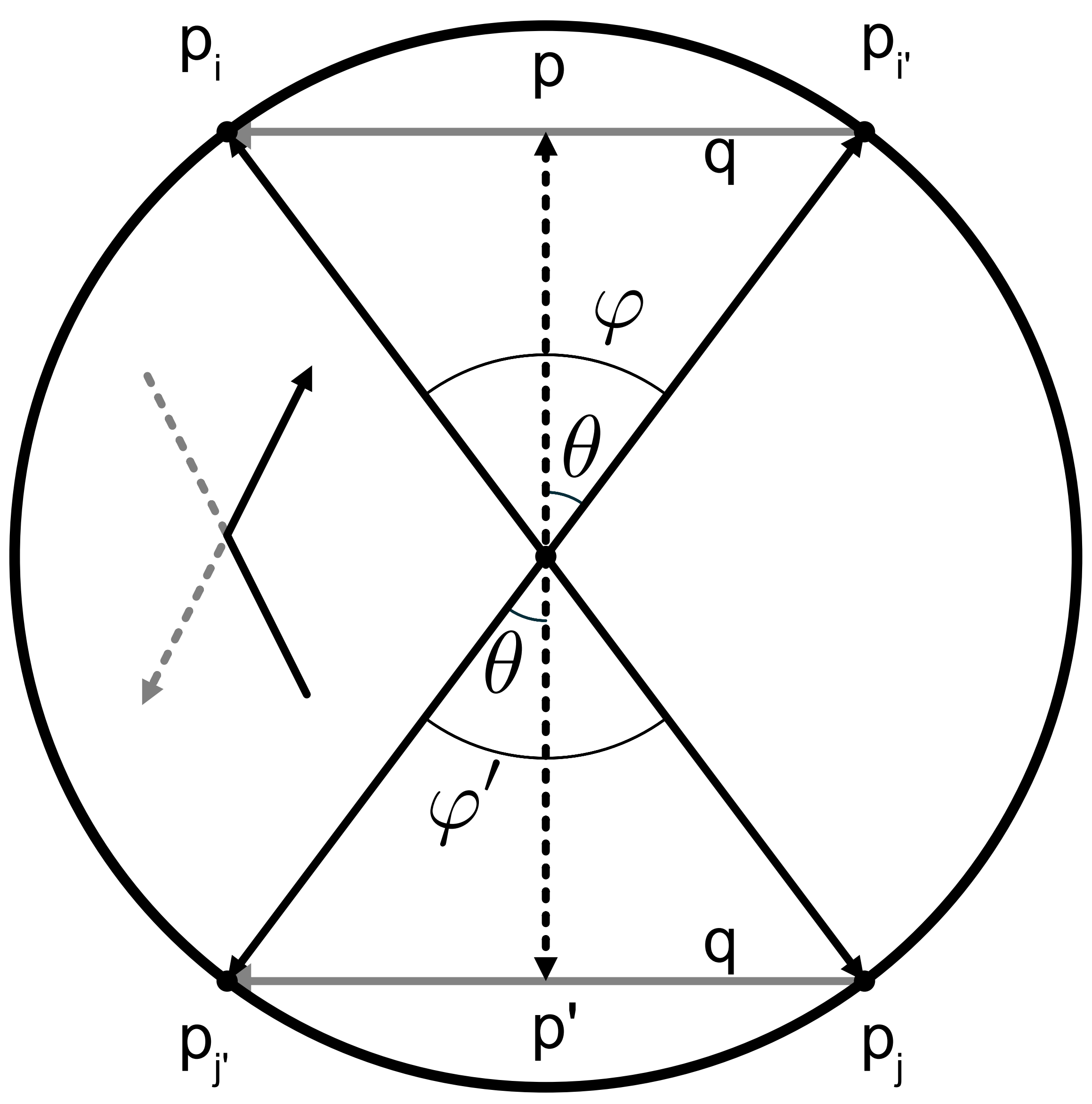
Collision integral in 2D

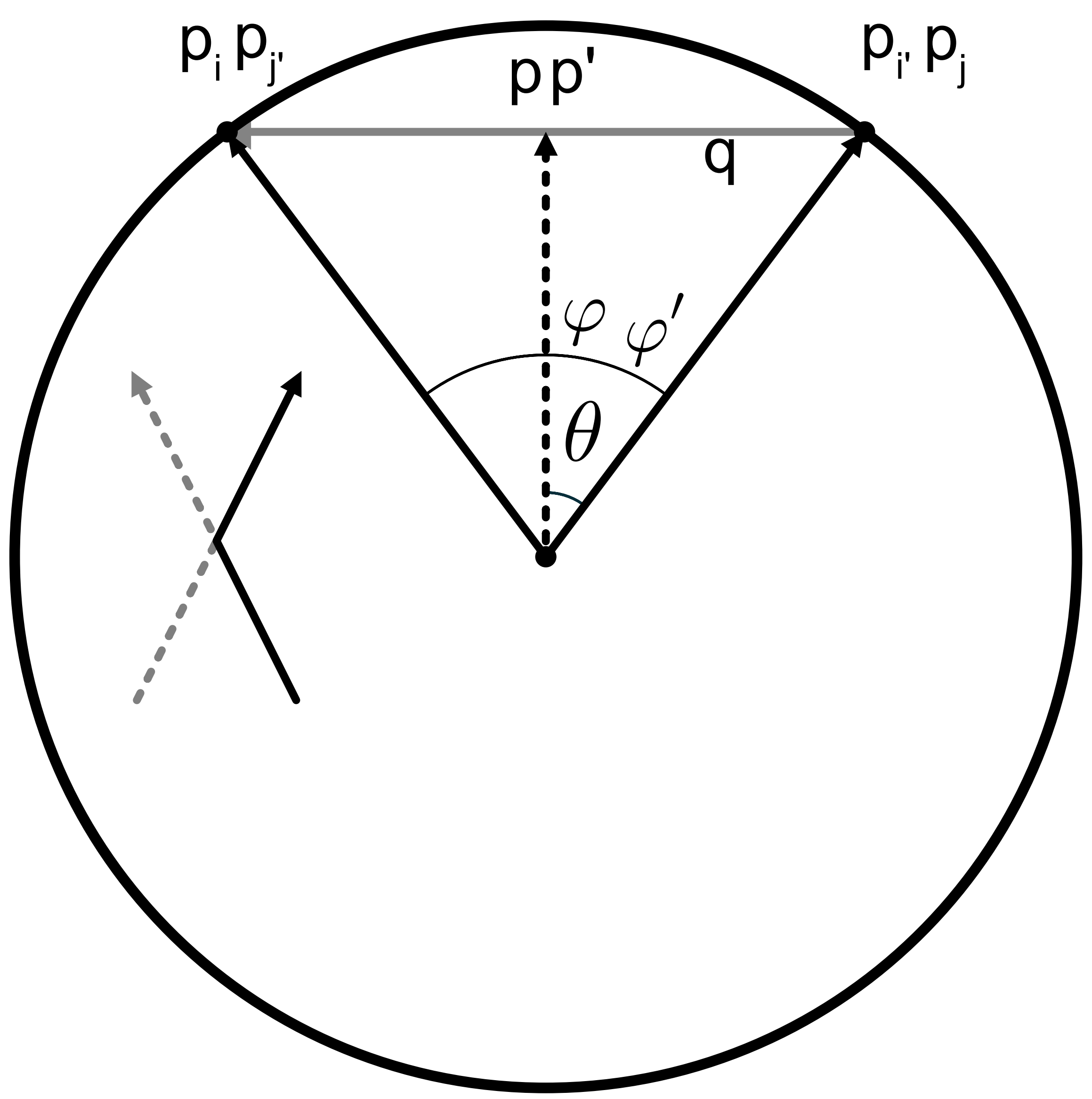
Allowed scattering processes

- At low temperatures collisions are restricted to the Fermi surface
- There are two types of allowed scattering processes



Head on

$$\frac{1}{\tau_2} \propto T^2/T_F$$



Collinear

$$0$$

For the charge channel

Laikhtman, PRB 45, 1259 (1992)

Ledwith, Guo, Levitov, Ann. of Phys. 411, 167913 (2019)

10.1103/PhysRevB.103.075422

Collision integral

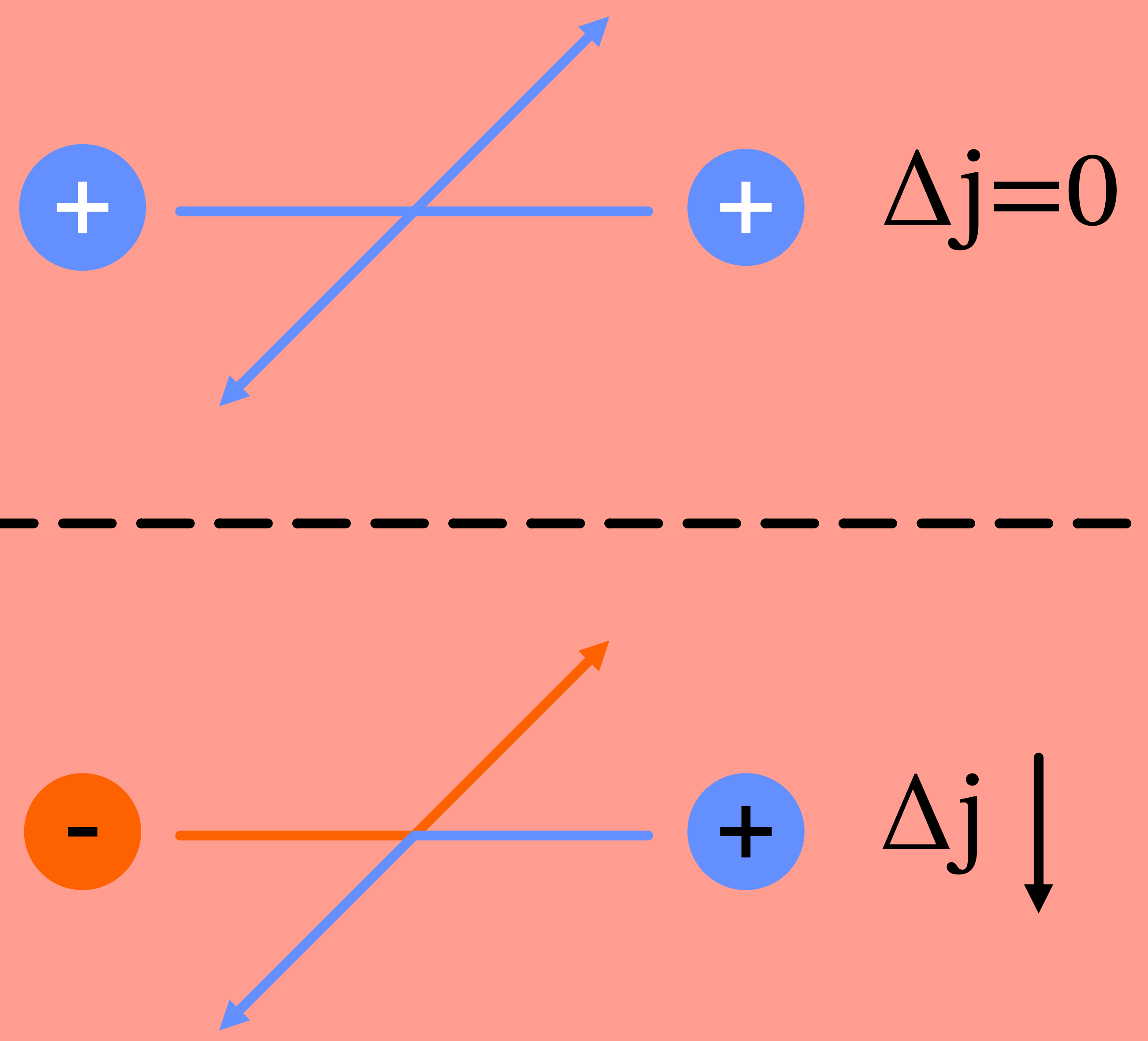
again by symmetry distinct channels

$$I(\mathbf{p}_i, \alpha) = -\frac{1}{T}\sum_{\beta\gamma\delta}\sum_{\mathbf{p}_j\mathbf{p}_{i'}\mathbf{p}_{j'}}(2\pi)^2\delta\Big(\sum_J{}^{'}\mathbf{p}_J\Big)2\pi\delta\Big(\sum_J{}^{'}\epsilon_J\Big)n_i n_j(1-n_{i'})(1-n_{j'})W^{\alpha\beta;\gamma\delta}_{ij;i'j'}\Big[\bar{\nu}_{i\alpha}+\bar{\nu}_{j\beta}-\bar{\nu}_{i'\gamma}-\bar{\nu}_{j'\delta}\Big]$$

W_+

Scattering of QP with same quantum number

Same as the usual case



Potentially different

Scattering of QP with different quantum number

W_-

10.1103/PhysRevB.103.075422

Collision integral

again by symmetry distinct channels

$$I(\mathbf{p}_i, \alpha) = -\frac{1}{T}\sum_{\beta\gamma\delta}\sum_{\mathbf{p}_j\mathbf{p}_{i'}\mathbf{p}_{j'}}(2\pi)^2\delta\Big({\sum_J}' \mathbf{p}_J\Big)2\pi\delta\Big({\sum_J}' \epsilon_J\Big)n_i n_j(1-n_{i'})(1-n_{j'})W^{\alpha\beta;\gamma\delta}_{ij;i'j'}\left[\bar{\nu}_{i\alpha}+\bar{\nu}_{j\beta}-\bar{\nu}_{i'\gamma}-\bar{\nu}_{j'\delta}\right]$$

W_+

Scattering of QP with same quantum number

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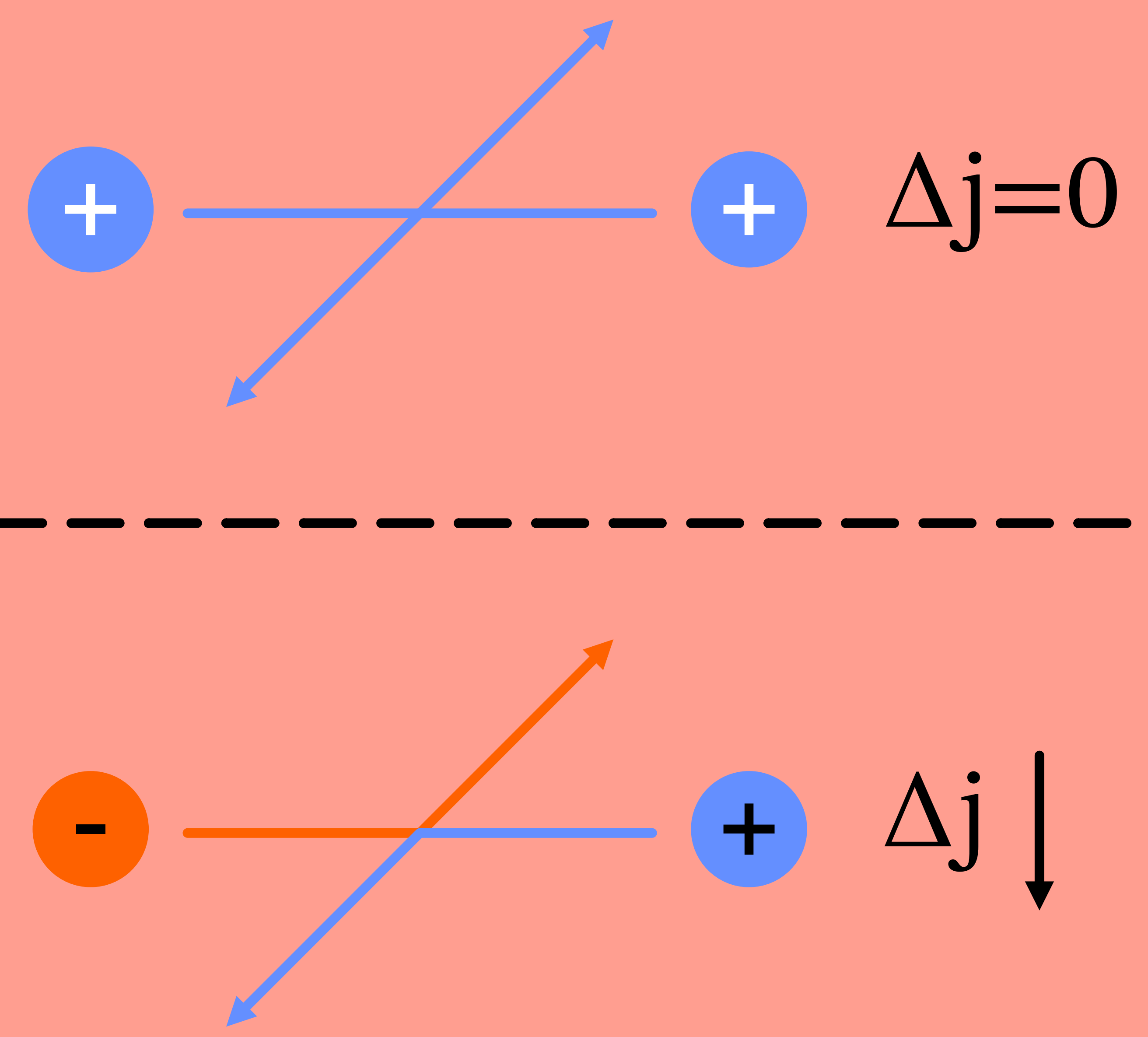


Potentially different

Scattering of QP with different quantum number

W_-

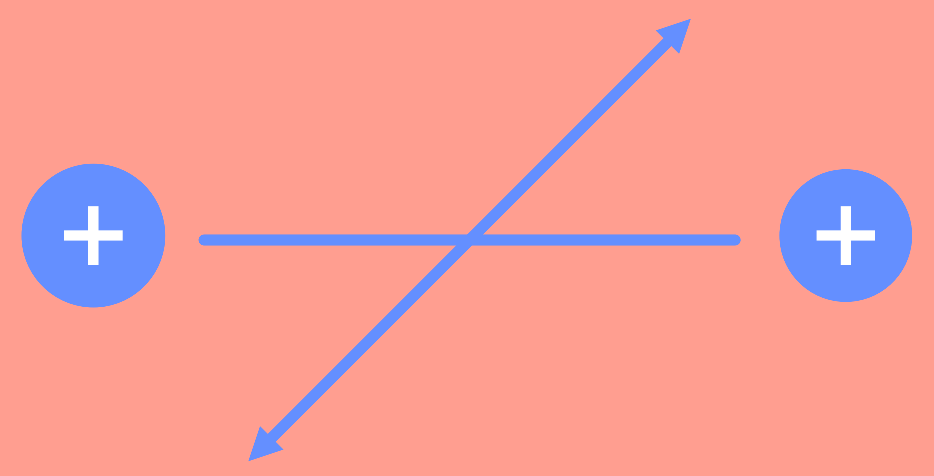
10.1103/PhysRevB.103.075422

Collision integral

again by symmetry distinct channels

$$I(\mathbf{p}_i, \alpha) = -\frac{1}{T} \sum_{\beta\gamma\delta} \sum_{\mathbf{p}_j \mathbf{p}_{i'} \mathbf{p}_{j'}} (2\pi)^2 \delta\Big(\sum_J^{\prime} \mathbf{p}_J\Big) 2\pi \delta\Big(\sum_J^{\prime} \epsilon_J\Big) n_i n_j (1-n_{i'})(1-n_{j'}) W^{\alpha\beta;\gamma\delta}_{ij;i'j'} \left[\bar{\nu}_{i\alpha} + \bar{\nu}_{j\beta} - \bar{\nu}_{i'\gamma} - \bar{\nu}_{j'\delta}\right]$$

W_+



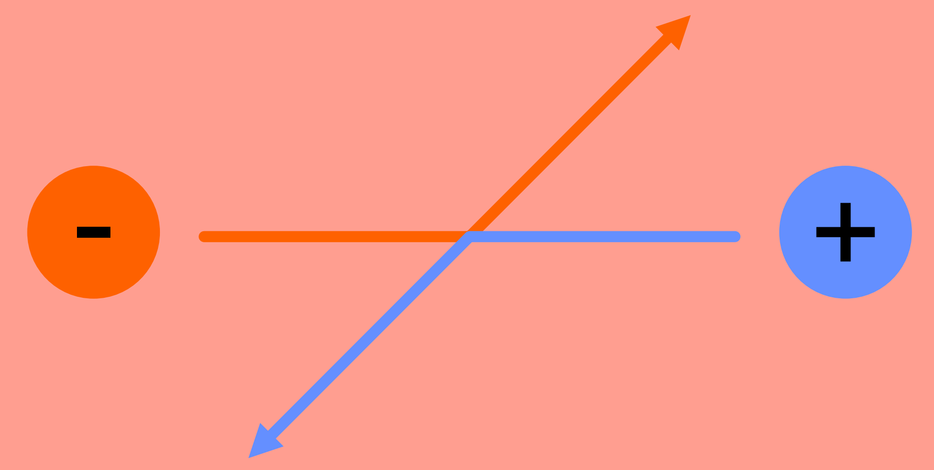
$\Delta j=0$

Scattering of QP with same quantum number

Same as the usual case

Potentially different

Scattering of QP with different quantum number



$\Delta j \downarrow$

W_-

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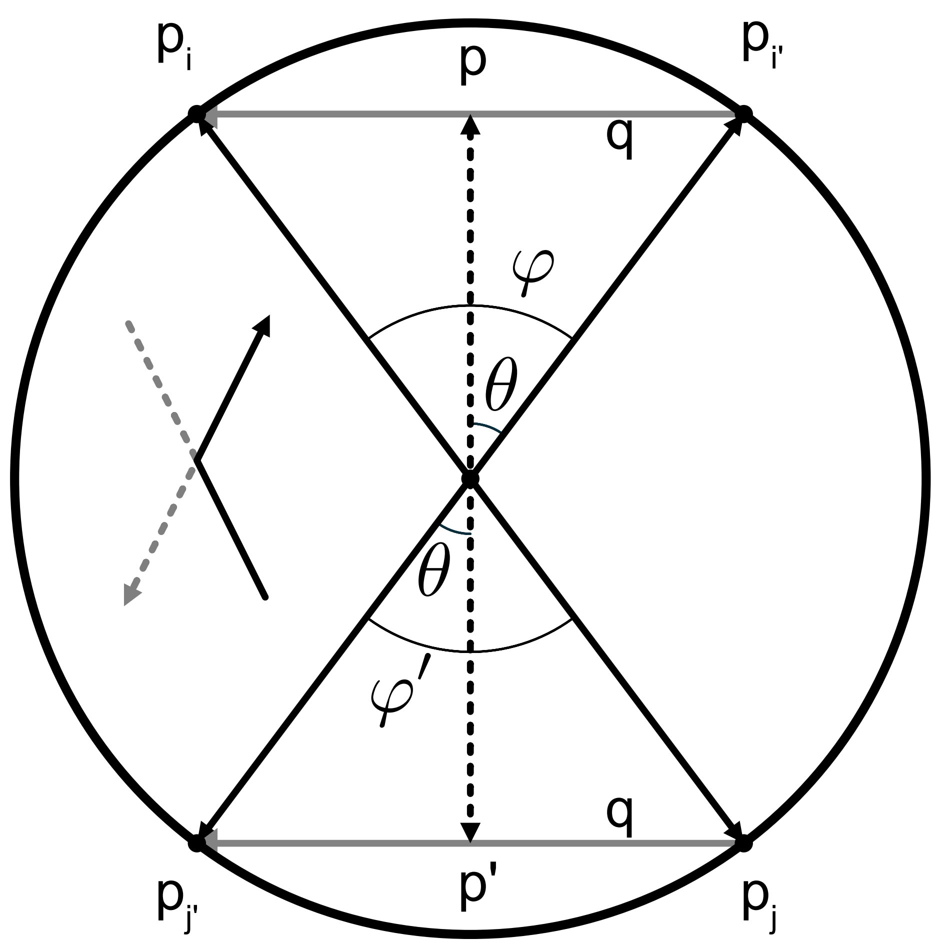
What about first sound

is there a hydrodynamic regime in neutral channels



$$\frac{1}{\tau_1} \ll \omega \ll \frac{1}{\tau_2} \propto T^2/T_F$$

Behaves similar to the charge channel

?

We have to evaluate the collision integral to know

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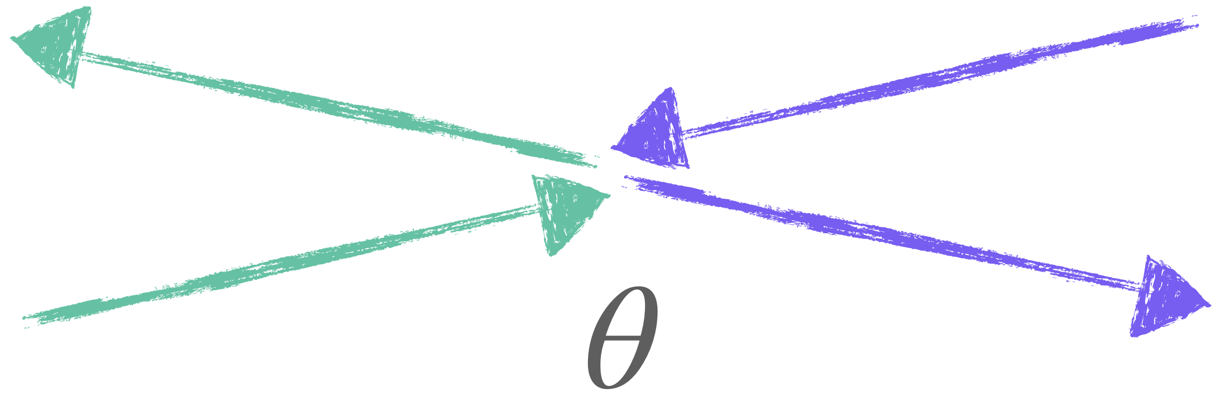
Neutral channels

Dominant contributions

$$\frac{1}{\tau^{\mu}_{tr}} \equiv \frac{1}{\tau^{\mu}_{1}} \approx \frac{1}{\tau^{\mu}_{1,\text{Backscatter}}} + \frac{1}{\tau^{\mu}_{1,\text{Forward}}}$$

Short range

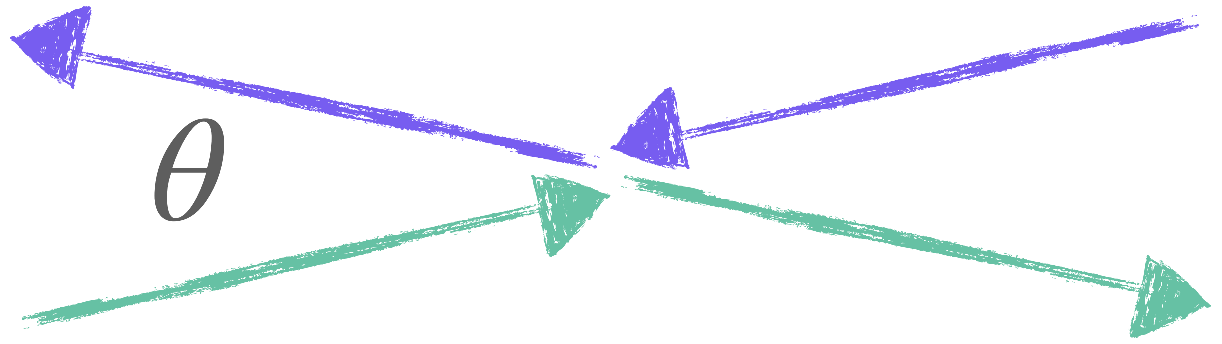
$$\frac{1}{\tau_{\text{Backscatter}}} \propto T^2 \ln \frac{\sqrt{\mu^2 - \Delta^2}}{T}$$



$$(2\pi)^2 \delta\left(\sum_J{}' \mathbf{p}_J\right) 2\pi\delta\left(\sum_J{}' \epsilon_J\right)$$

Long ranged (screened) Coulomb

$$\frac{1}{\tau_{\text{Forward}}} \propto T^2 \ln \frac{\sqrt{\mu^2 - \Delta^2}}{v q_{TF}}$$



$$V \sim \frac{1}{q + q_{TF}}$$

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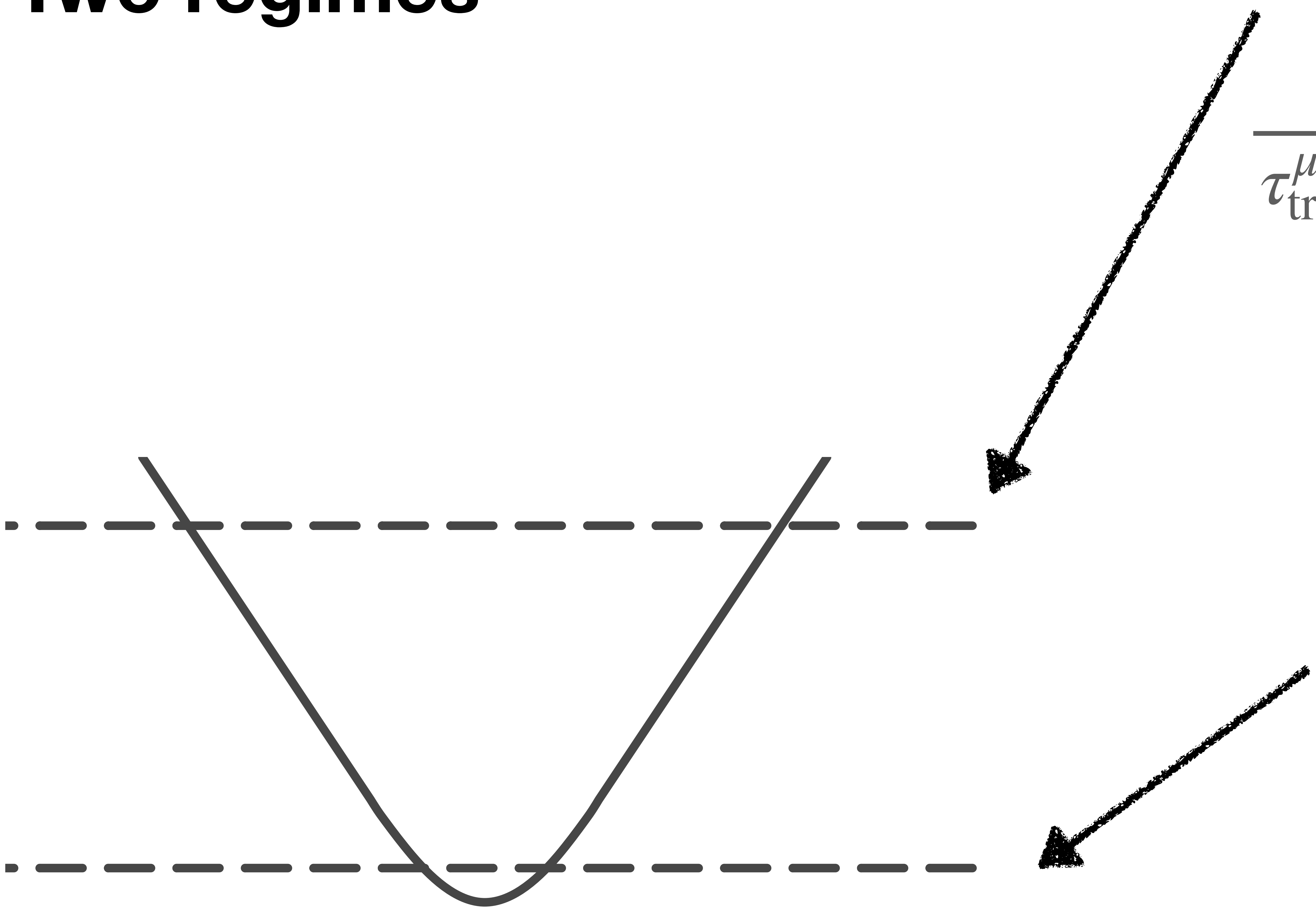
Transport rates

Two regimes

$\Delta \ll E_F$ **Relativistic**

$$\frac{1}{\tau_{\rm tr}^{\mu}(T)} \propto \frac{T^2}{E_F^2}\left\{g^2 \ln(E_F/T) + \alpha^2 \ln(E_F/vq_{TF})\right\}$$



$\Delta \sim E_F$ **Non-relativistic**

$$\frac{1}{\tau_{\rm tr}^{\mu}(T)} \propto \alpha^2 \frac{T^2}{E_F^2}\left(\ln(T/E_F) + \ln(E_F/vq_{TF})\right)$$

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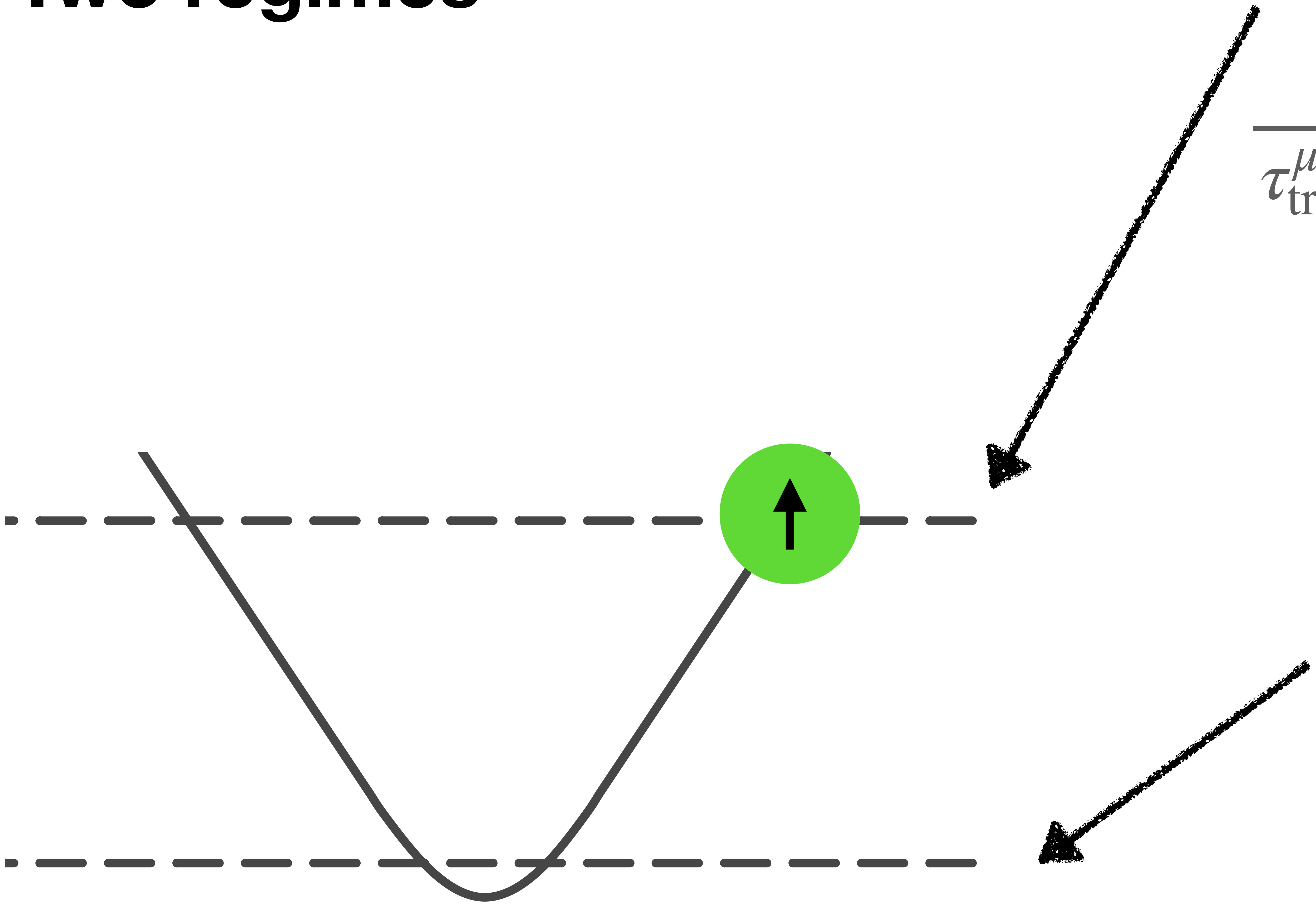

$\Delta \sim E_F$ **Non-relativistic**

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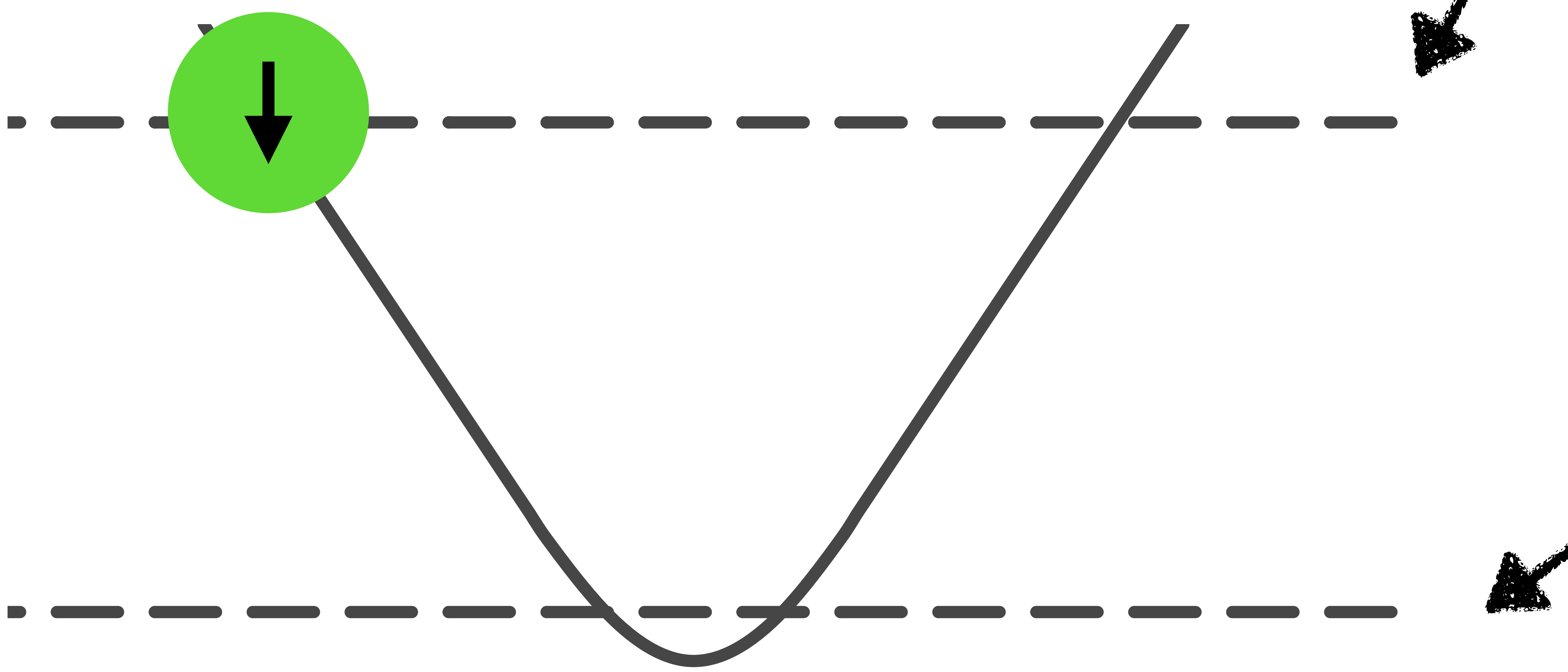
Transport rates

Two regimes

$$\Delta \ll E_F$$

Relativistic

$$\frac{1}{\tau_{\mathrm{tr}}^{\mu}(T)} \propto \frac{T^2}{E_F^2} \left\{ g^2 \ln(E_F/T) + \alpha^2 \ln(E_F/vq_{TF}) \right\}$$



$$\Delta \sim E_F$$

Non-relativistic

$$\frac{1}{\tau_{\mathrm{tr}}^{\mu}(T)} \propto \alpha^2 \frac{T^2}{E_F^2} \left(\ln(T/E_F) + \ln(E_F/vq_{TF}) \right)$$

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Transport rates

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$\Delta \ll E_F$ **Relativistic**

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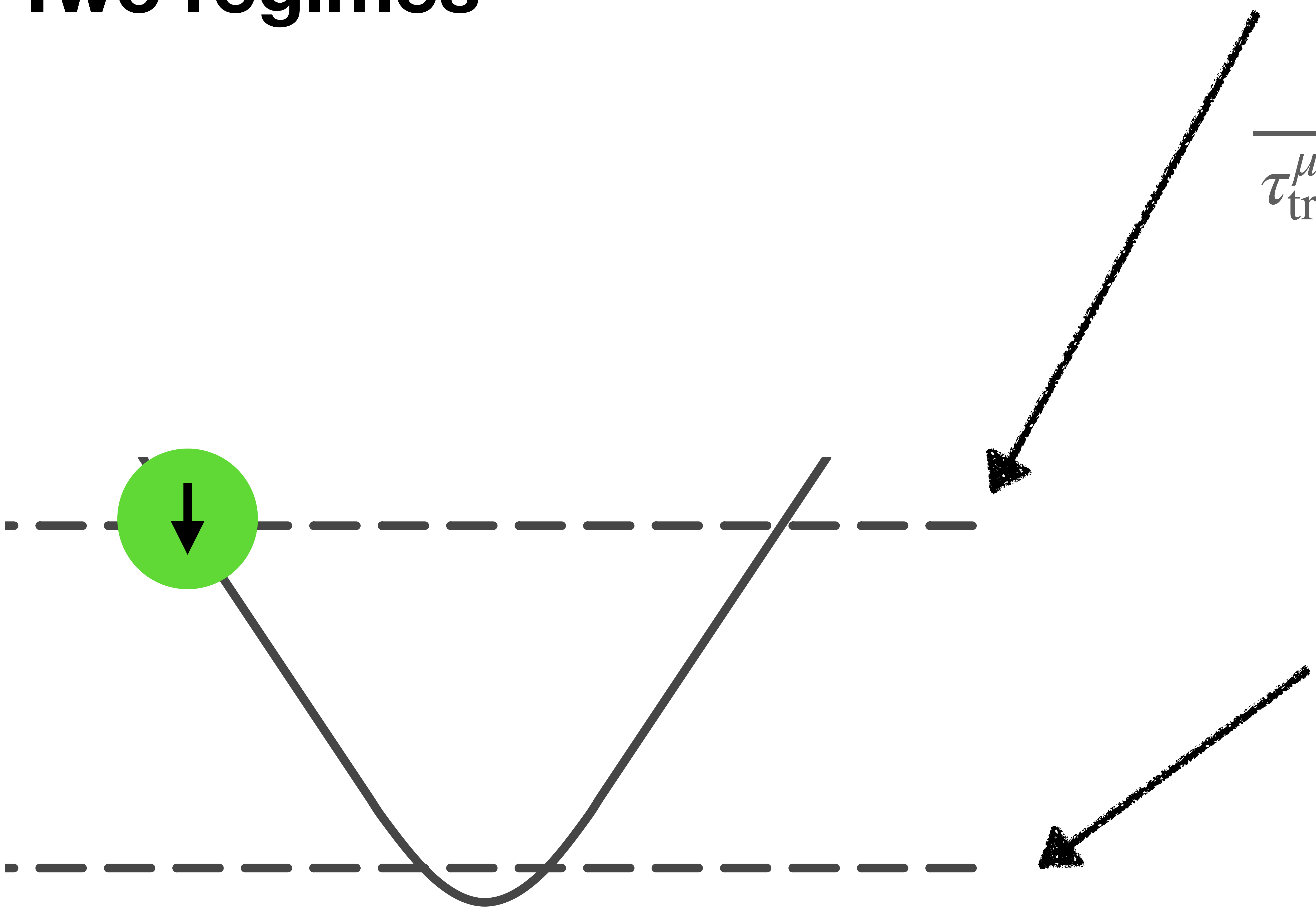
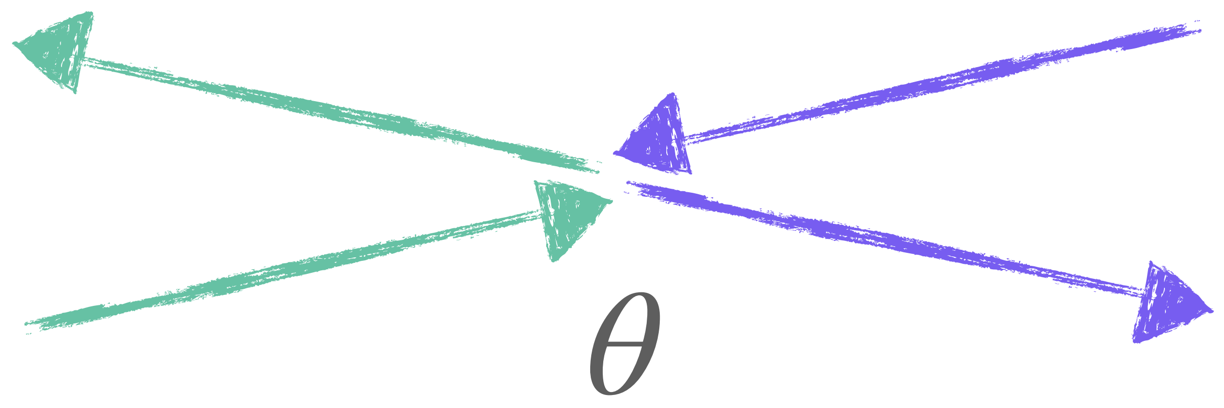


Δ breaks sub lattice symmetry

$\Delta \sim E_F$ **Non-relativistic**

$$\frac{1}{\tau^{\mu}_{\rm tr}(T)} \propto \alpha^2 \frac{T^2}{E_F^2}\left(\ln(T/E_F) + \ln(E_F/vq_{TF})\right)$$

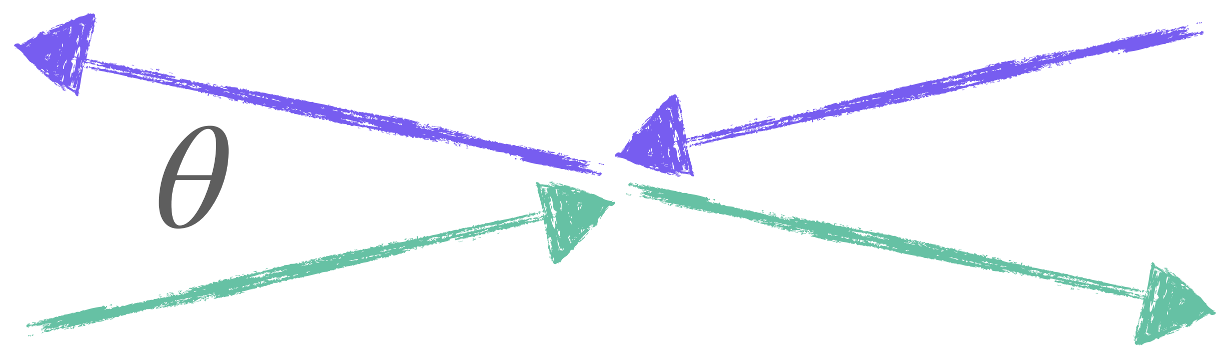
10.1103/PhysRevB.103.075422



$$\frac{1}{\tau_1} \ll \omega \ll \frac{1}{\tau_2}$$?

$$\frac{1}{\tau_1^{\mu}} \propto T^2 \left[\ln(E_F/T) + \ln(E_F/vq_{TF})\right]$$

$$\frac{1}{\tau_2^{\mu}} \propto T^2$$



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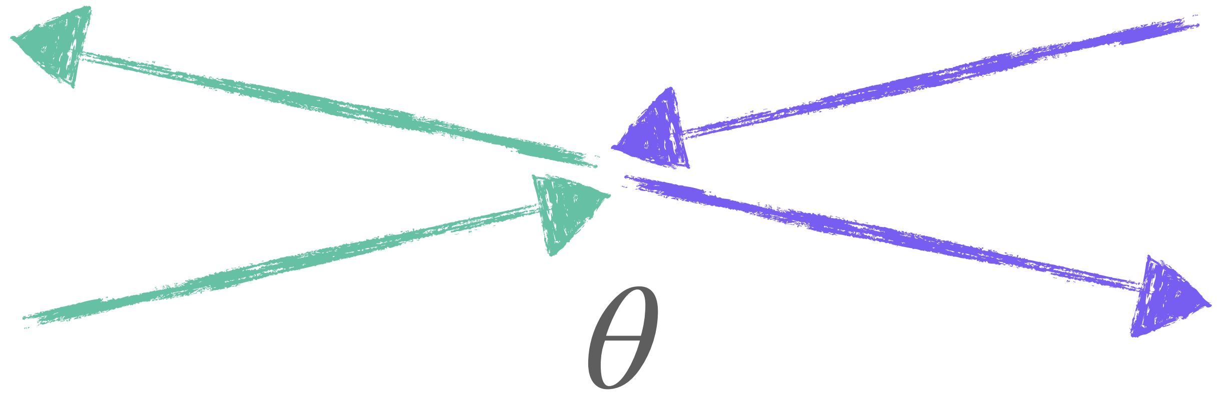


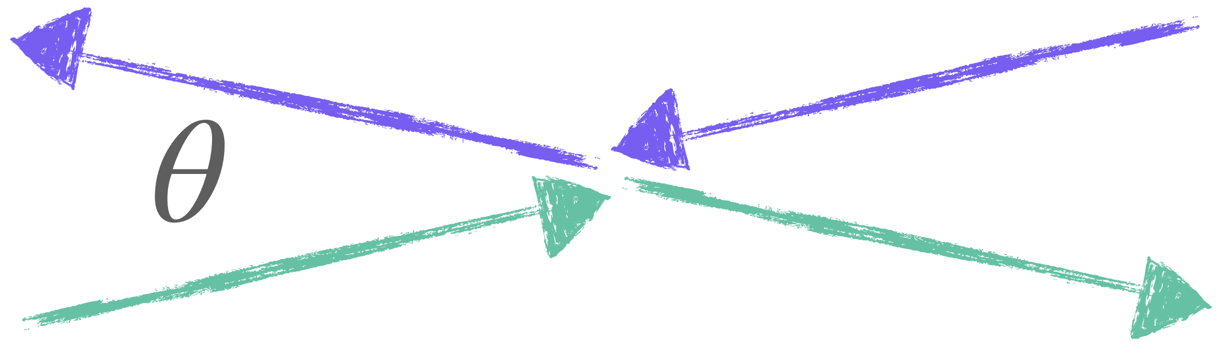
~~$\frac{1}{\tau_1} \ll \omega \ll \frac{1}{\tau_2}$~~ ?

$$\frac{1}{\tau_1^{\mu}} \propto T^2 \left[\ln(E_F/T) + \ln(E_F/vq_{TF})\right]$$

$\gtrsim$

First sound regime is "squeezed out"

$$\frac{1}{\tau_2^{\mu}} \propto T^2$$

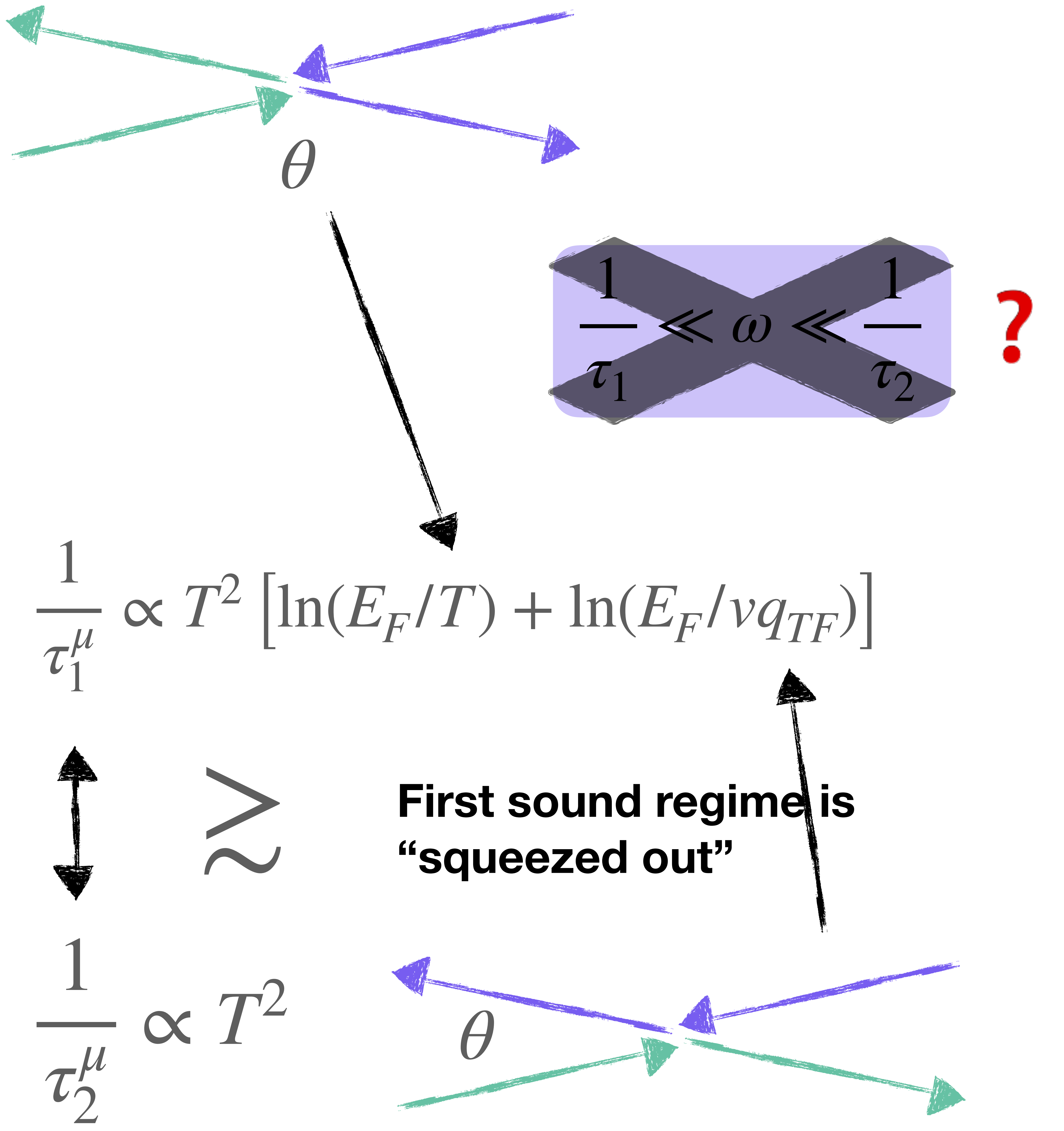


10.1103/PhysRevB.103.075422

No neutral first sound

Not hydrodynamics, but diffusion

- Always overdamped
- Finite temperature neutral transport is ultimately diffusive



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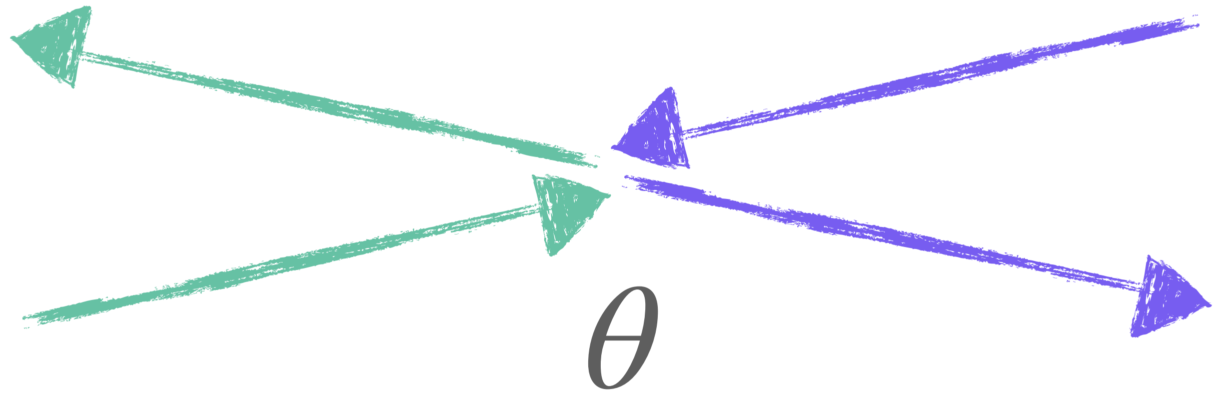


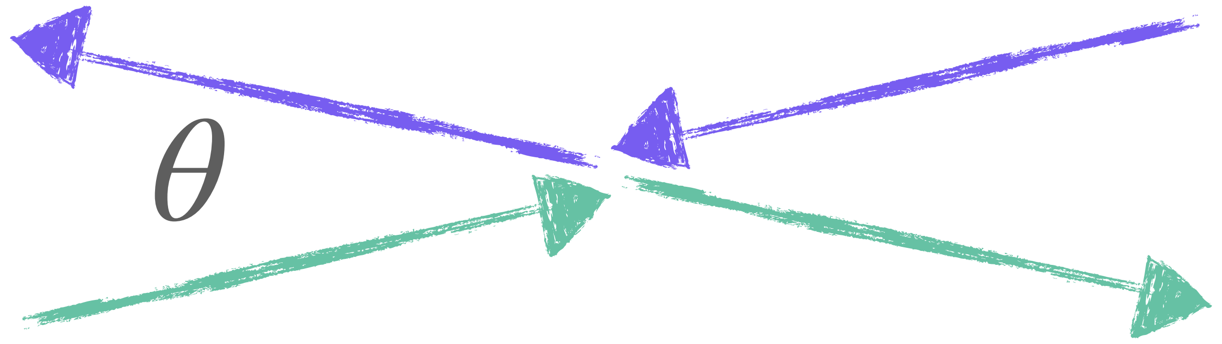
$$\frac{1}{\tau_1} \ll \omega \ll \frac{1}{\tau_2}$$?

$$\frac{1}{\tau_1^\mu} \propto T^2 \left[\ln(E_F/T) + \ln(E_F/vq_{TF})\right]$$

$$\gtrsim$$

First sound regime is “squeezed out”

$$\frac{1}{\tau_2^\mu} \propto T^2$$



$$\omega\tau_1 \ll 1$$

$$D^\mu \approx \frac{v_F^2}{2}\tau_{\text{tr}}^\mu(1 + F_0^\mu)$$

What about the collisionless limit?

10.1103/PhysRevB.103.075422

Zero sound

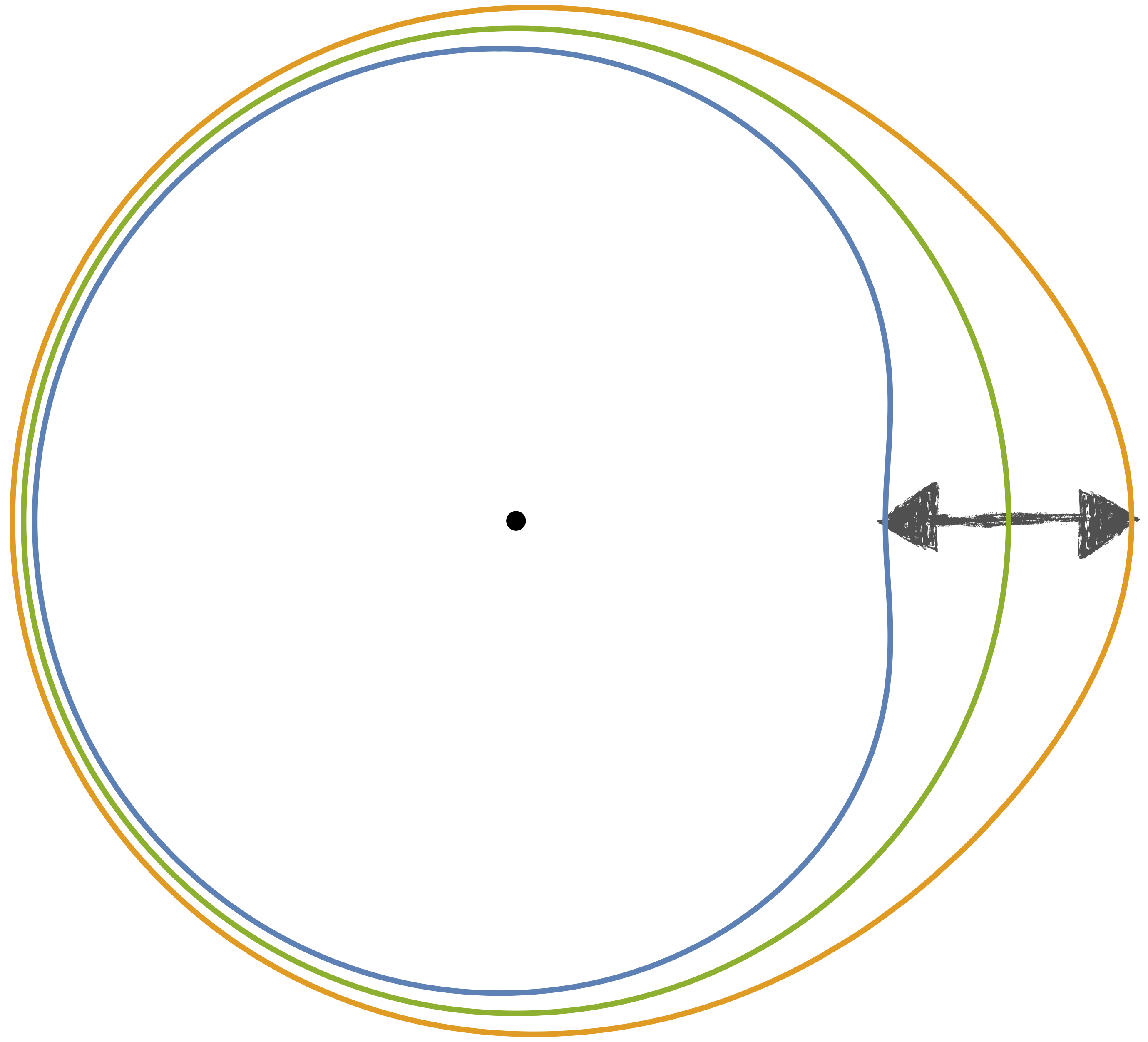
Collisionless equations for the uncharged channels

$$\frac{\partial \delta\rho^{\mu}(\mathbf{k},\mathbf{r})}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\delta\bar{\rho}^{\mu}(\mathbf{k},\mathbf{r}) = \cancel{\frac{1}{G_s G_v}\mathrm{tr}\,\hat{X}^{\mu}\hat{I}[\delta\hat{\rho}]}$$

$$\omega \gg \frac{1}{\tau}$$

occurs when

- Zero sound occurs in the collisionless limit
  - Sound oscillations are much faster than relaxation
  - e.g $T \to 0$ since $I \propto (T/E_F)^2$
- Relaxes through **Landau damping**



10.1103/PhysRevB.103.075422

Zero sound

Is it damped?

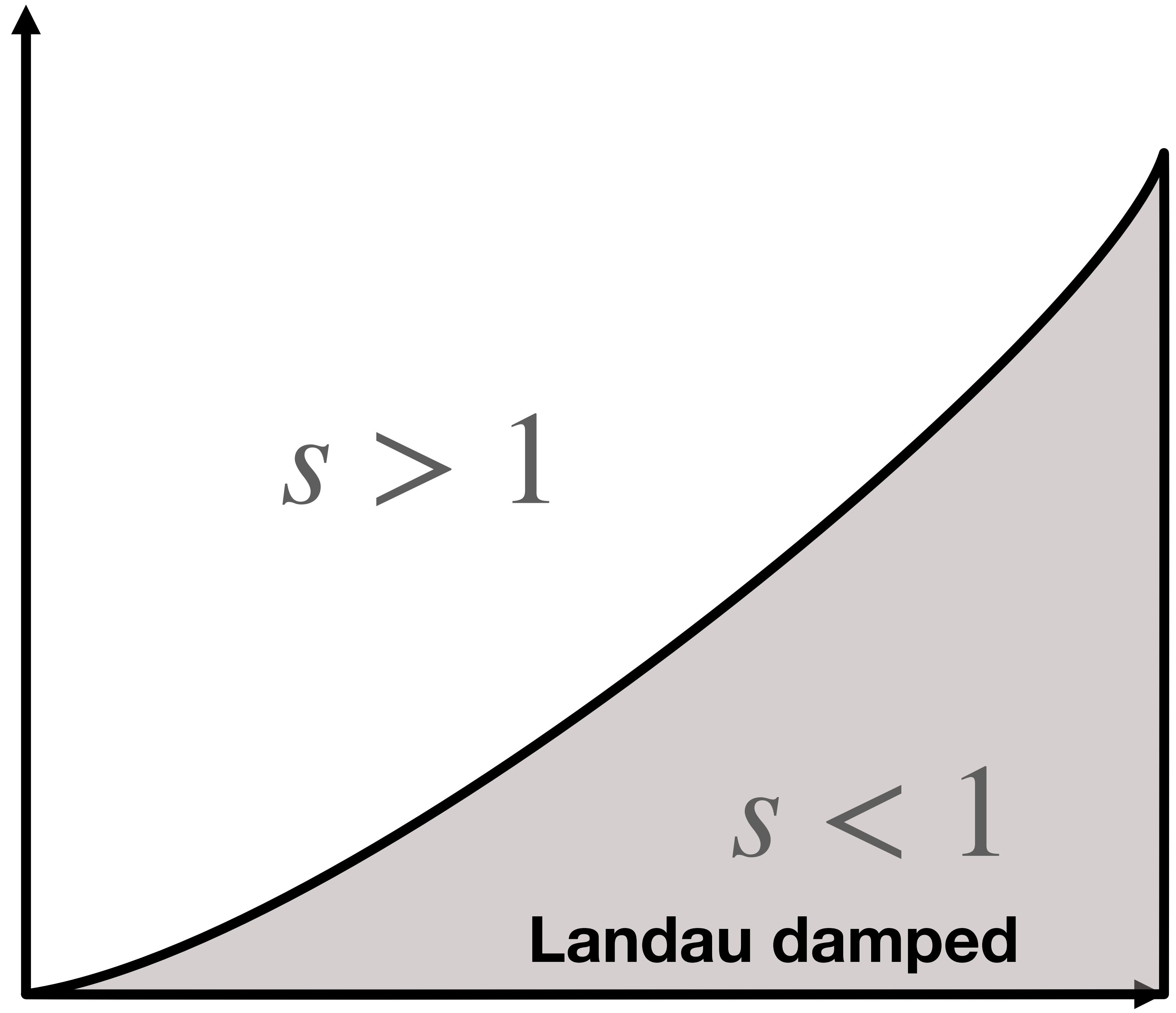
$\omega \gg \frac{1}{\tau}$

$$-i\omega\rho^{\mu}(\mathbf{k}, \mathbf{q}) + i\mathbf{v} \cdot \mathbf{q}\delta\bar{\rho}^{\mu}(\mathbf{k}, \mathbf{r}) = 0$$

- Natural independent variable

$$|s| \equiv \left| \frac{\omega}{v_F q} \right|$$

- Solutions for $s > 1$ undamped
- Solutions for $s < 1$ Landau damped



10.1103/PhysRevB.103.075422

Absence of zero sound

Simplest model

- For model of an attractive constant interaction only model it can be shown there is no zero sound
- Landau damped, $\omega < v_F q$

$$F_0^\mu < 0 \implies |s| \equiv \left| \frac{\omega}{v_F q} \right| < 1$$

Klein, Maslov, Pitaevskii, Chubukov, PRR 1, 033134 (2019)
Klein, Maslov, Chubukov, Npj Quantum Materials 5, 55 (2020)

10.1103/PhysRevB.103.075422

Absence of zero sound

Generic Landau damping

- At low temperature deviations of the occupation function are restricted to the Fermi surface
- This allows us to rephrase the zero sound equation as a self consistent integral expression

$$\delta\rho^{\mu}(\mathbf{p},\mathbf{r}) \equiv -\frac{\partial n}{\partial \epsilon}\bigg|_{\bar{\epsilon}} \nu^{\mu}(\phi,\mathbf{r})$$

What is f^{μ}?

$$\oint \frac{d\phi}{2\pi}(s-\cos\phi')[\nu^{\mu}(\phi)]^2 = \frac{G_s G_v p_F}{v_F}\oint\oint \frac{d\phi d\phi'}{2\pi}\nu^{\mu}(\phi)f^{\mu}(\phi-\phi')\nu^{\mu}(\phi')$$

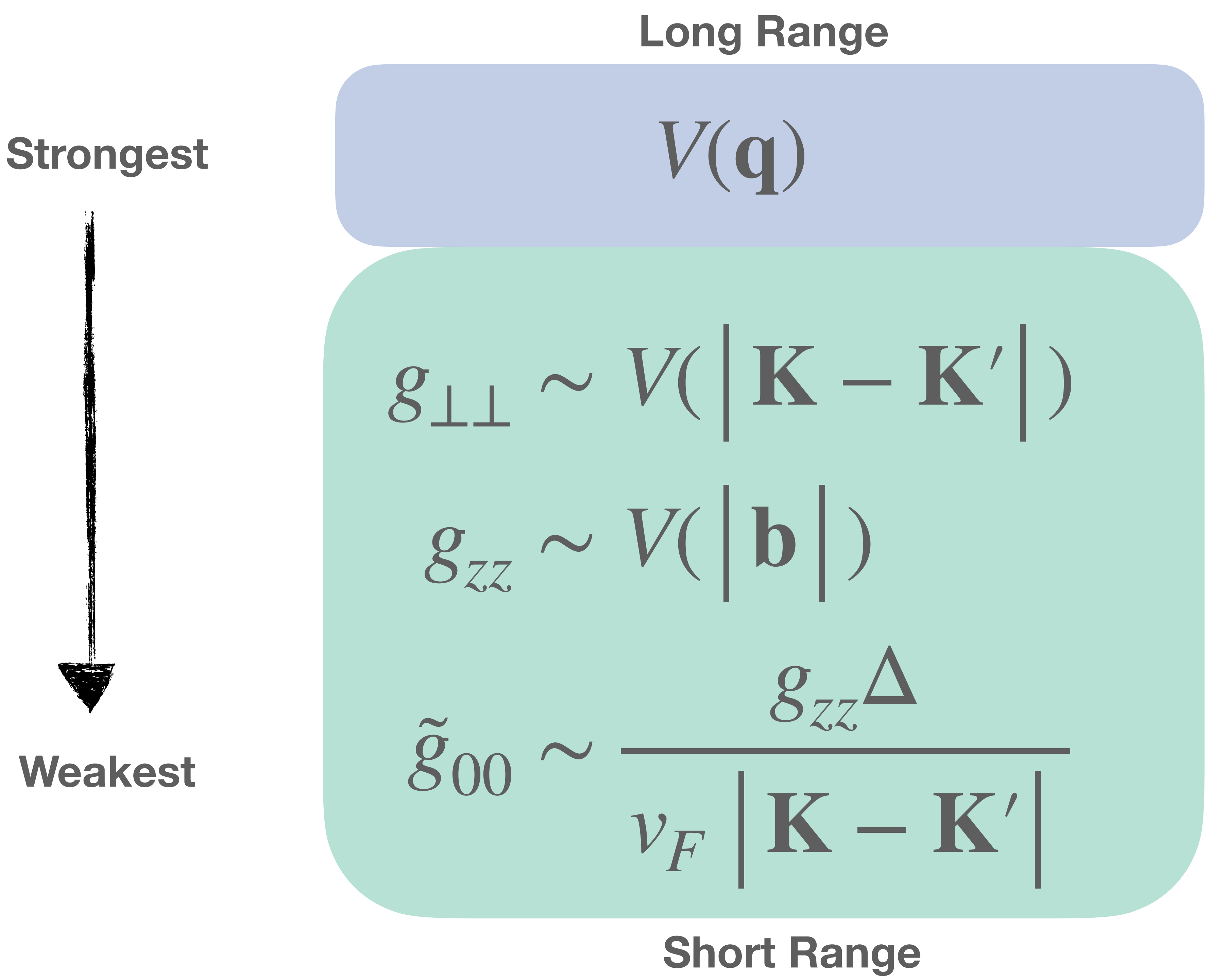
10.1103/PhysRevB.103.075422

Absence of zero sound

What is f?

- We recall that we estimated the interaction functions by taking matrix elements of the Coulomb potential

- The leading contribution to the interaction functions comes from the Coulomb potential



$$f^{\mu}(\theta) \sim -V\left[2k_F \sin\frac{\theta}{2}\right]$$

10.1103/PhysRevB.103.075422

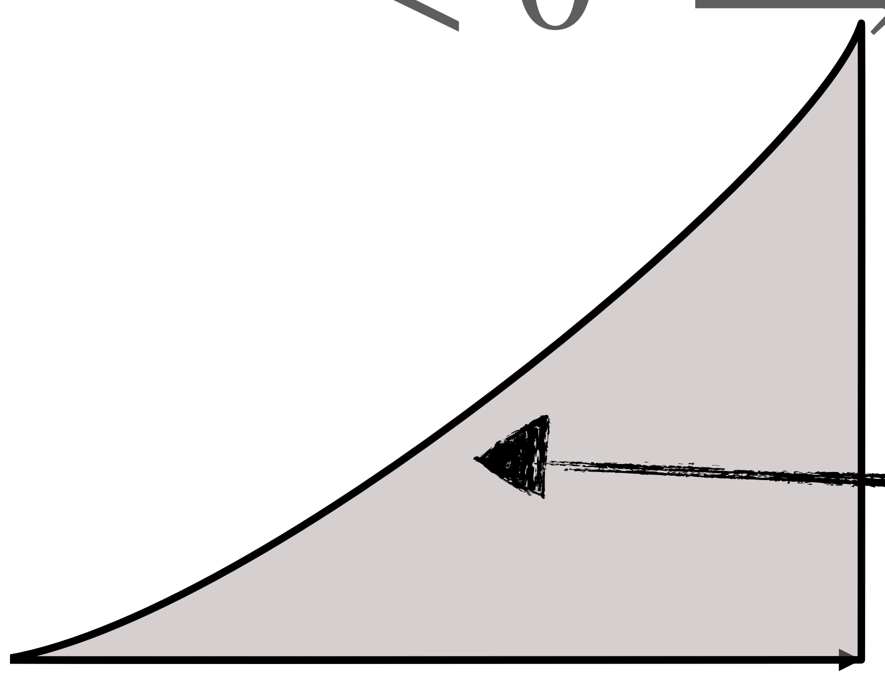
Absence of zero sound

Generic Landau damping

Due to the properties of the Coulomb interaction f^{μ} is negative definite

$$\oint \frac{d\phi}{2\pi}(s-\cos\phi')[\nu^{\mu}(\phi)]^2 = \frac{G_s G_v p_F}{v_F}\oint\oint \frac{d\phi d\phi'}{2\pi}\nu^{\mu}(\phi)f^{\mu}(\phi-\phi')\nu^{\mu}(\phi')$$

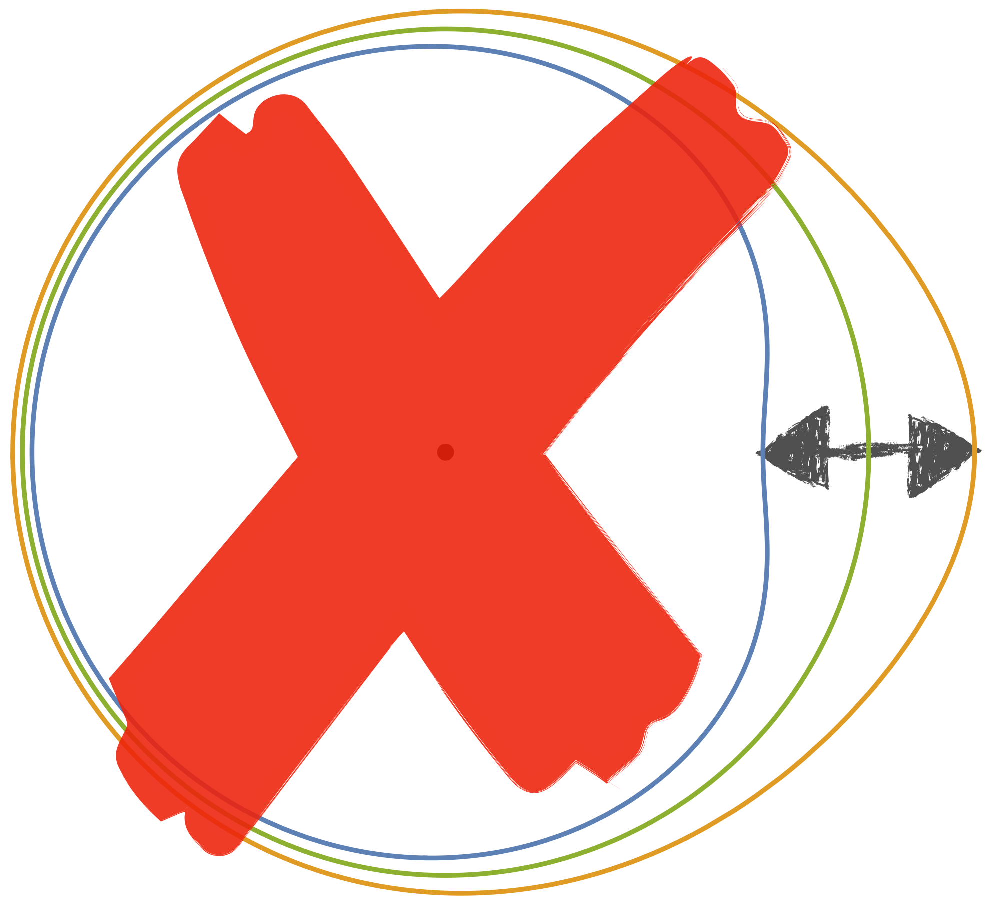
$< 0 \implies s < 1$ $\qquad$ < 0



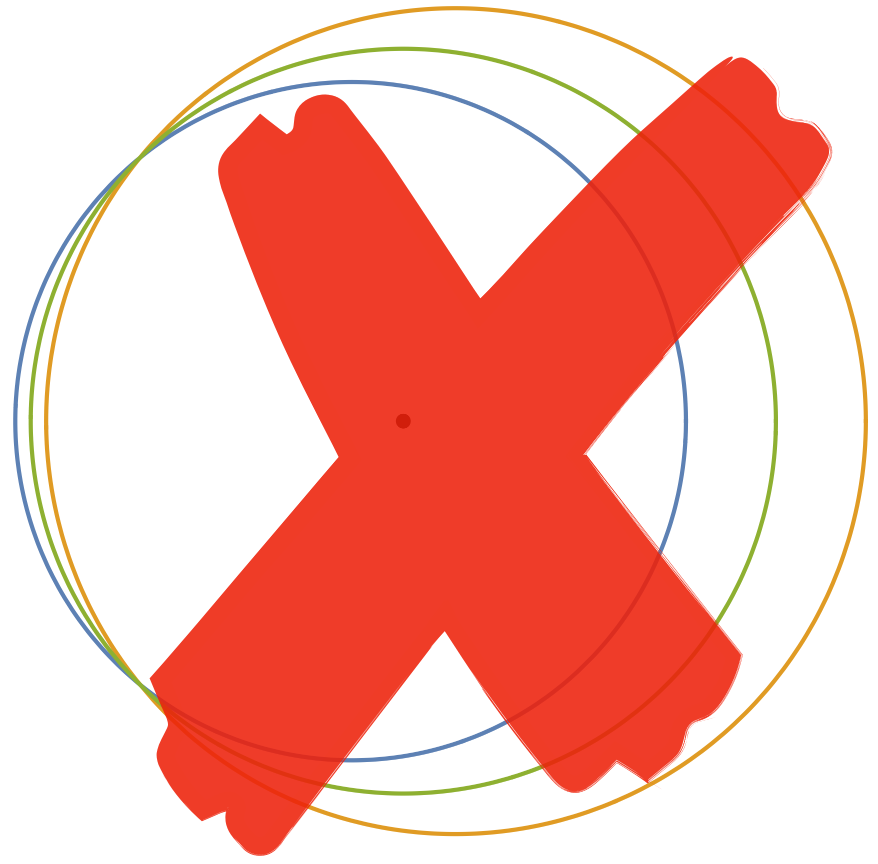
Landau damped, $\omega < v_F q$

Zero Sound

Killed by Landau damping



First Sound



Killed by collisional damping

There is no neutral sound in FL graphene

What if we break spin-rotation symmetry?

Why?

Absence of spin zero sound and Silin modes

- Arguments of the previous section generally apply to $SU(2)$ spin invariant Fermi liquids
- Magnetic field → undamped modes

SOVIET PHYSICS JETP VOLUME 6 (33), NUMBER 5 May, 1958

OSCILLATIONS OF A FERMI-LIQUID IN A MAGNETIC FIELD

V. P. SILIN

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Received by JETP editor May 6, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) 33, 1227-1234 (November, 1957)

A study is made of the spin oscillations of a paramagnetic Fermi-liquid (He^3) placed in a constant magnetic field at low temperatures, where collisions can be ignored.

Spin diffusion and spin echoes in liquid ^{3}He at low temperature

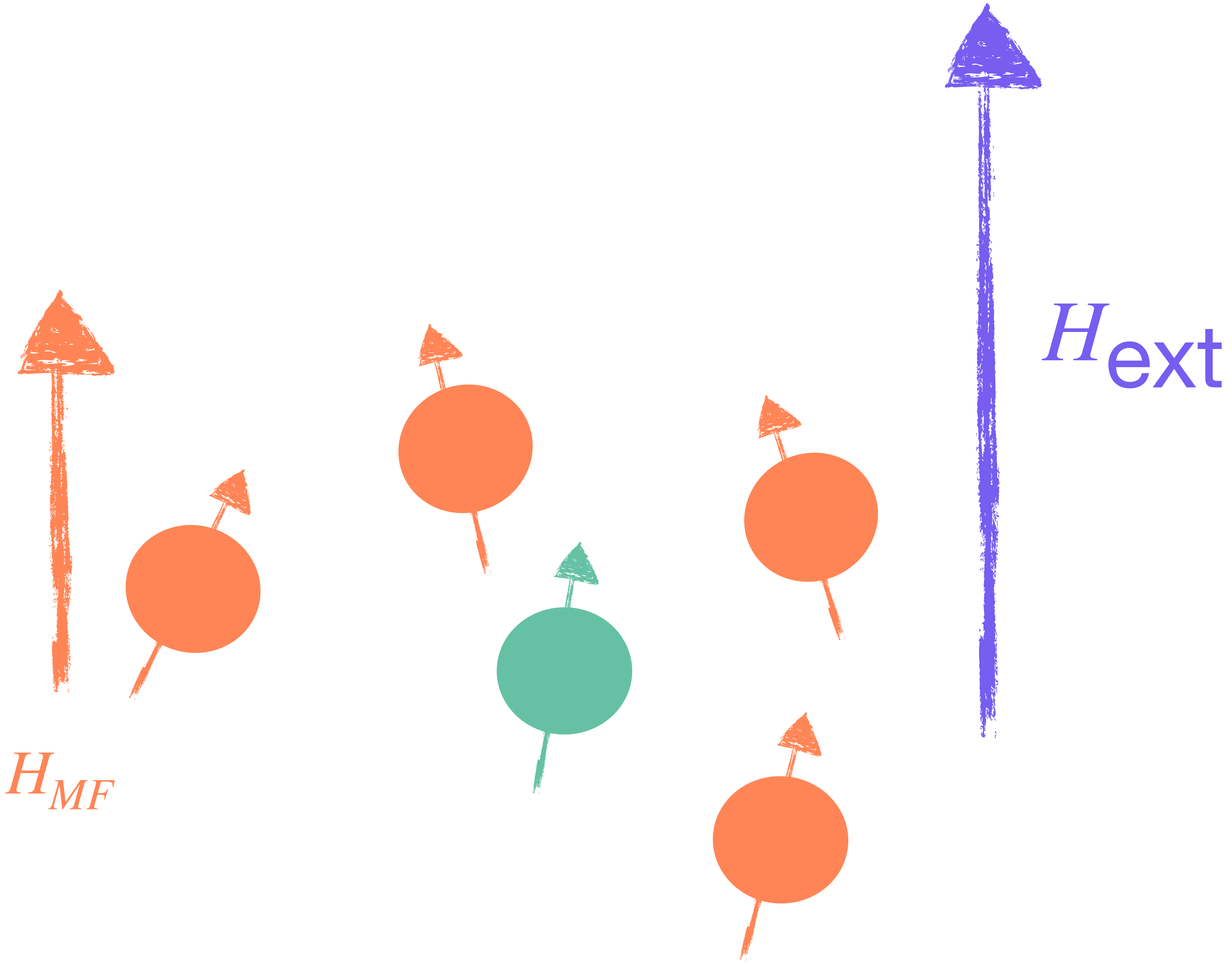
A. J. LEGGETT

School of Mathematical and Physical Sciences, University of Sussex, Falmer, Brighton

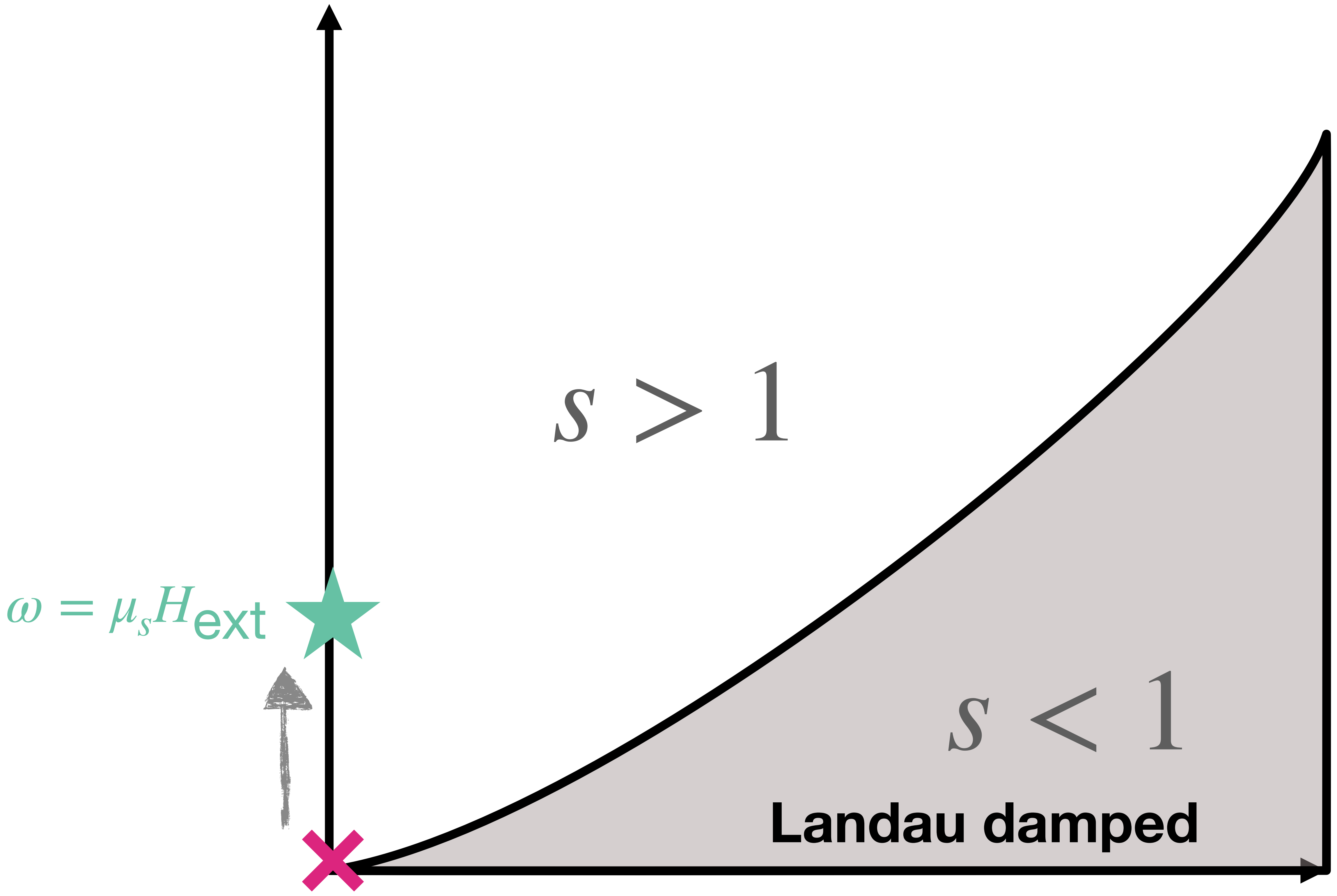
MS. received 3rd *July* 1969, *in revised form* 29*th September* 1969

Spin oscillations in magnetic field

Silin-Legget mode



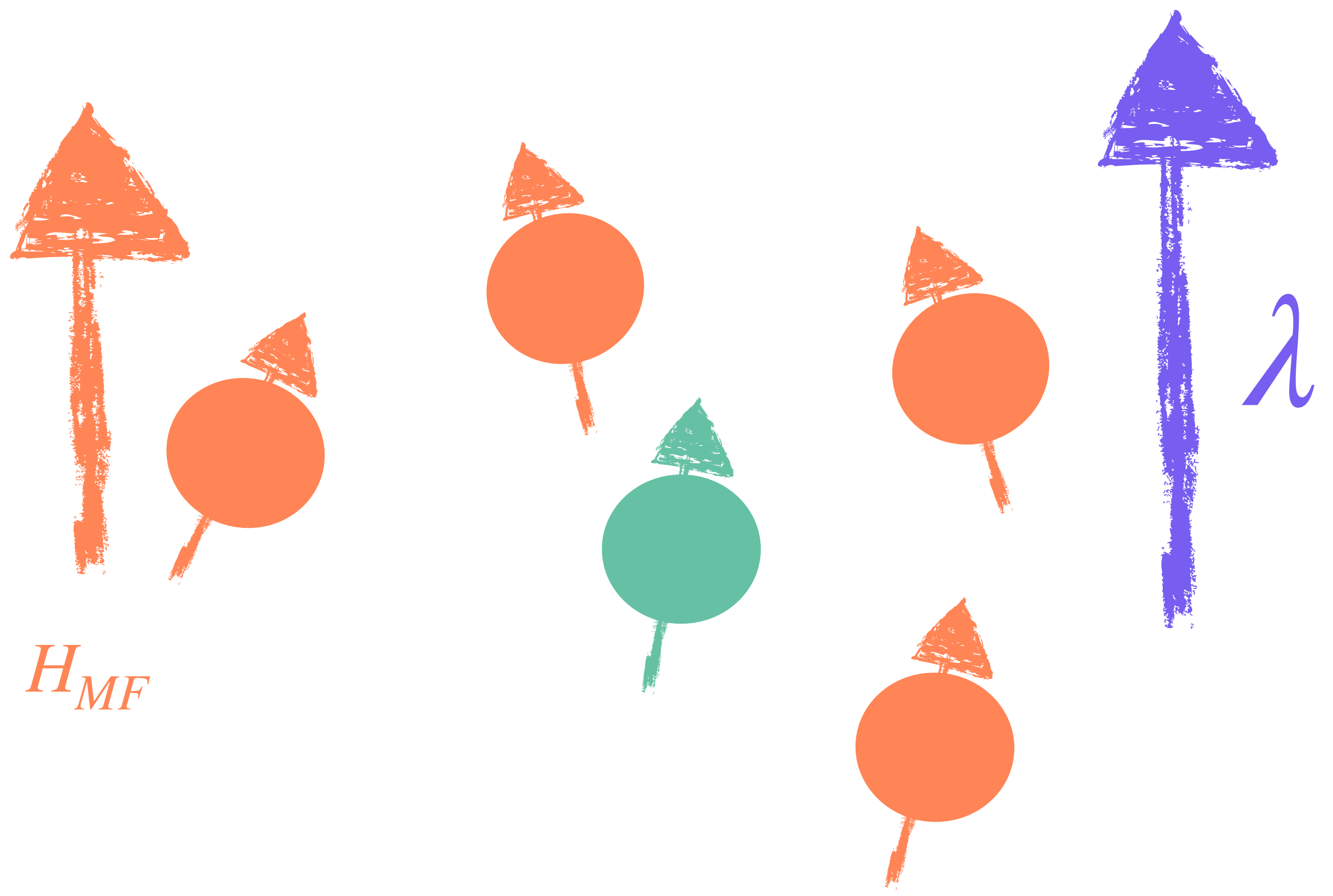
OSCILLATIONS OF A FERMI-LIQUID IN A MAGNETIC FIELD

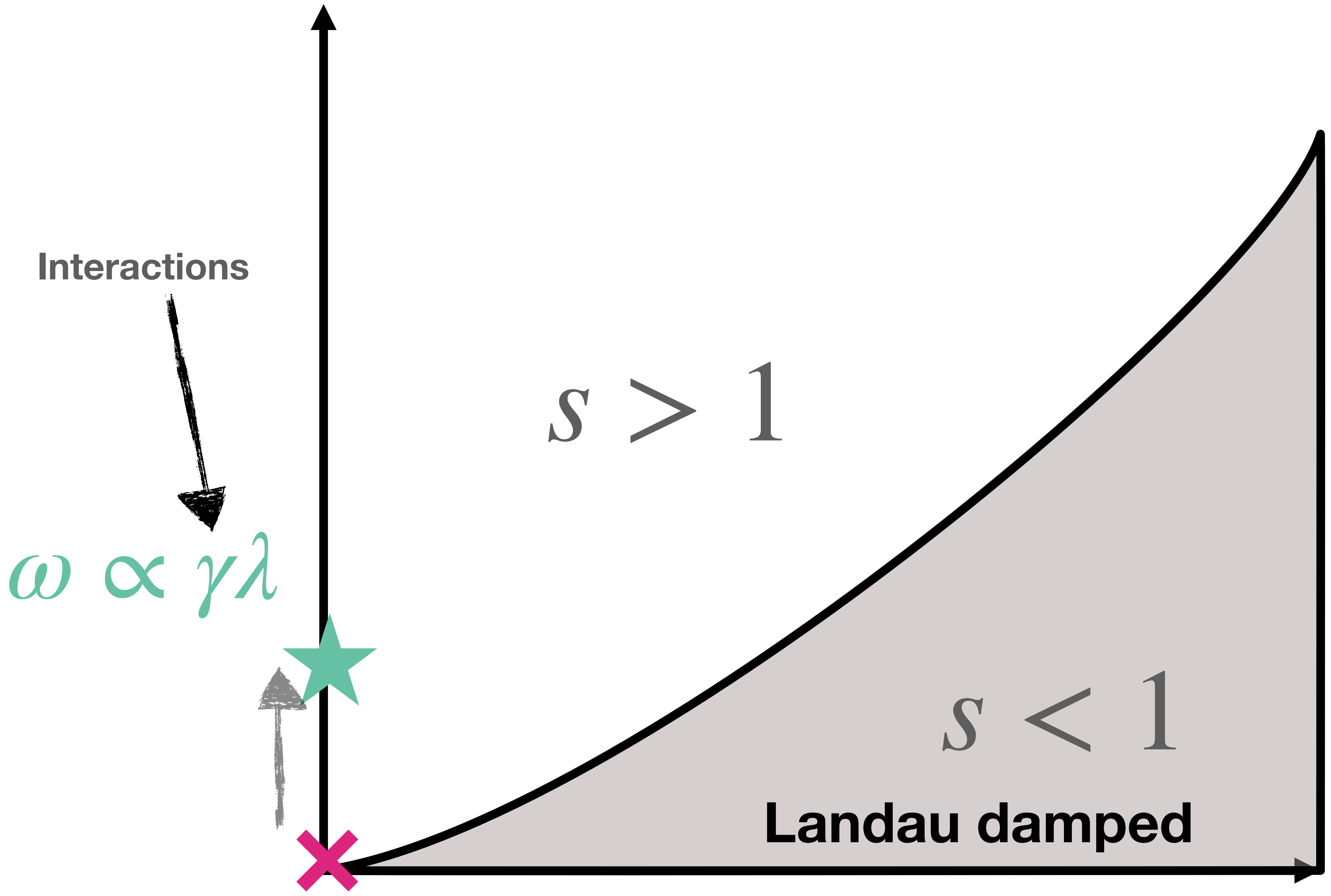


Kohn, PR 123, 1242 (1961)

Spin oscillations with SOC

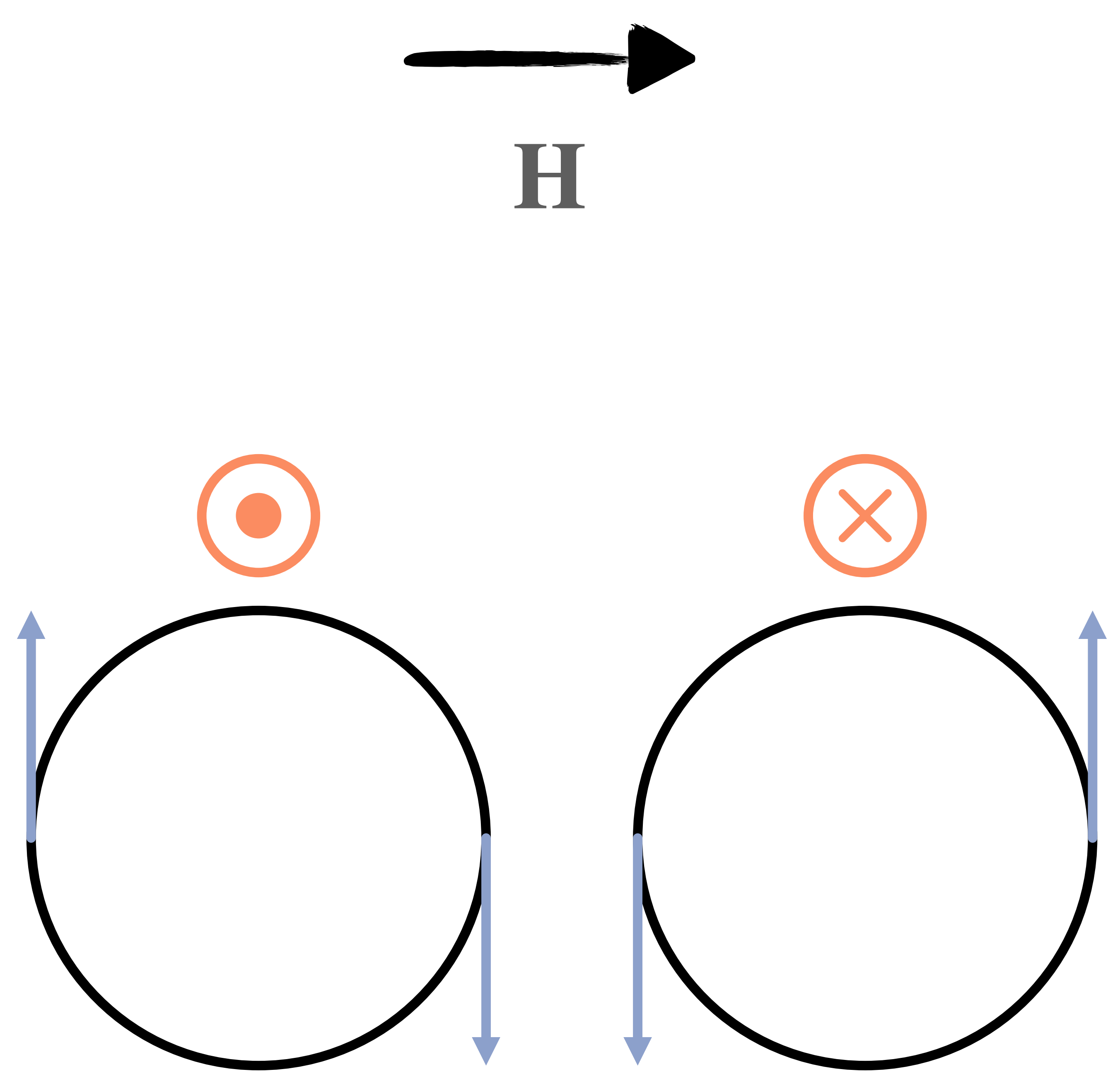
See e.g Shekhter, Khodas, Finkel'stein, PRB 71, 165329 (2005)
Ashrafi, Maslov, PRL 109, 227201 (2012)





Breaking the spin rotation symmetry

- Magnetic field (Silin✓)
- Spin orbit coupling
  - In the presence of SOC, electric field couples to spin
  - Coupling to $E \gg B$ due to SOC induced spin-flip transitions

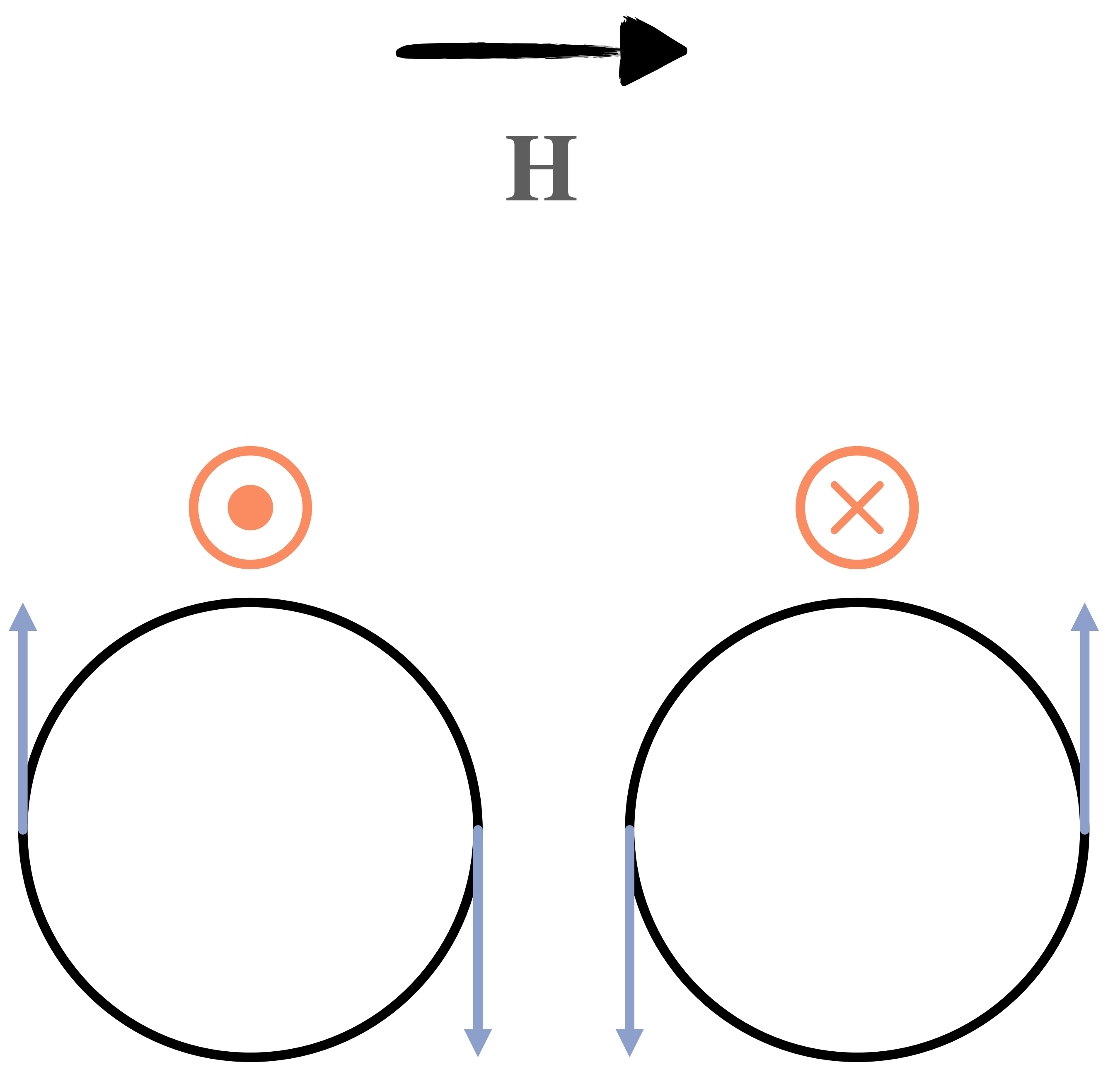


Rashba, Soviet Physics Uspekhi 7, 823 (1965)
Rashba, Efros, Phys Rev Lett 91, 126405 (2003)
Maiti, Zyuzin, Maslov, PRB 91, 035106 (2015)

SOC Fermi liquid graphene

with magnetic field

- Previous Fermi Liquid theory plus
    - Zeeman coupling to **in plane** magnetic field (we take $\mathbf{H} = H_0 \mathbf{e}_x$)
        - Berry Curvature $\boldsymbol{\Omega} \perp \mathbf{H}$
    - Extrinsic SOC



e.g. Wang et al., PRX 6, 041020 (2016)

$$H_p^+ = \epsilon_p + \alpha(p)\hat{z} \cdot (\sigma \times \mathbf{p}) + \Lambda(p)\hat{\sigma}_z \hat{\tau}_z$$

After upper band projection

Multicomponent Fermi Liquid theory

- Recall the form of our Fermi liquid theory in the $SU(2)$ invariant case
- Spin orbit coupling leads to different effective quasi-particle Hamiltonians for each channel but this is mostly bookkeeping

Free Energy

$$\mathscr{F} = \mathscr{F}_0 + \sum_k \xi_k \delta n_k + \frac{1}{2} \sum_{k,k'} f_{kk'} \delta n_k \delta n_{k'} + \cdots$$

e.g

$$\sum_k \epsilon_{ij} \delta n_{ji}(k)$$

e.g

$$\sum_{k,k'} \delta n_{ij}(k) f^{ij;lm}(k,k') \delta n_{lm}(k')$$

Bare Quasiparticle Energy

$$\xi_k \rightarrow \epsilon_{d;k}, \boldsymbol{\epsilon}_{s;k}, \boldsymbol{\epsilon}_{v;k}, \boldsymbol{\epsilon}_{mi;k}$$

Occupation Functions

$$\delta n_k \rightarrow \delta n_k, \delta \mathbf{s}_k, \delta \mathbf{Y}_k, \delta \overleftrightarrow{\mathbf{M}}_k$$

Symmetry distinguished channels

Landau Interaction Functions

$$f_{kk'} \rightarrow f^d_{kk'}, f^s_{kk'}, f^v_{kk';i}, f^m_{kk';i}$$

Symmetry constrained

Free Energy

$$\mathscr{F} = \mathscr{F}_0 + \sum_k \xi_k \delta n_k + \frac{1}{2}\sum_{k,k'} f_{kk'}\delta n_k \delta n_{k'} + \cdots$$

e.g

$$\sum_k \epsilon_{ij}\delta n_{ji}(k)$$

e.g

$$\sum_{k,k'} \delta n_{ij}(k) f^{ij;lm}(k,k')\delta n_{lm}(k')$$

Multicomponent Fermi Liquid theory

- Recall the form of our Fermi liquid theory in the $SU(2)$ invariant case
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Bare Quasiparticle Energy

$$\xi_k \to \epsilon_{d;k}, \boldsymbol{\epsilon}_{s;k}, \boldsymbol{\epsilon}_{v;k}, \boldsymbol{\epsilon}_{mi;k}$$

Symmetry distinguished channels

Occupation Functions

$$\delta n_k \to \delta n_k, \delta \mathbf{s}_k, \delta \mathbf{Y}_k, \delta \overleftrightarrow{\mathbf{M}}_k$$

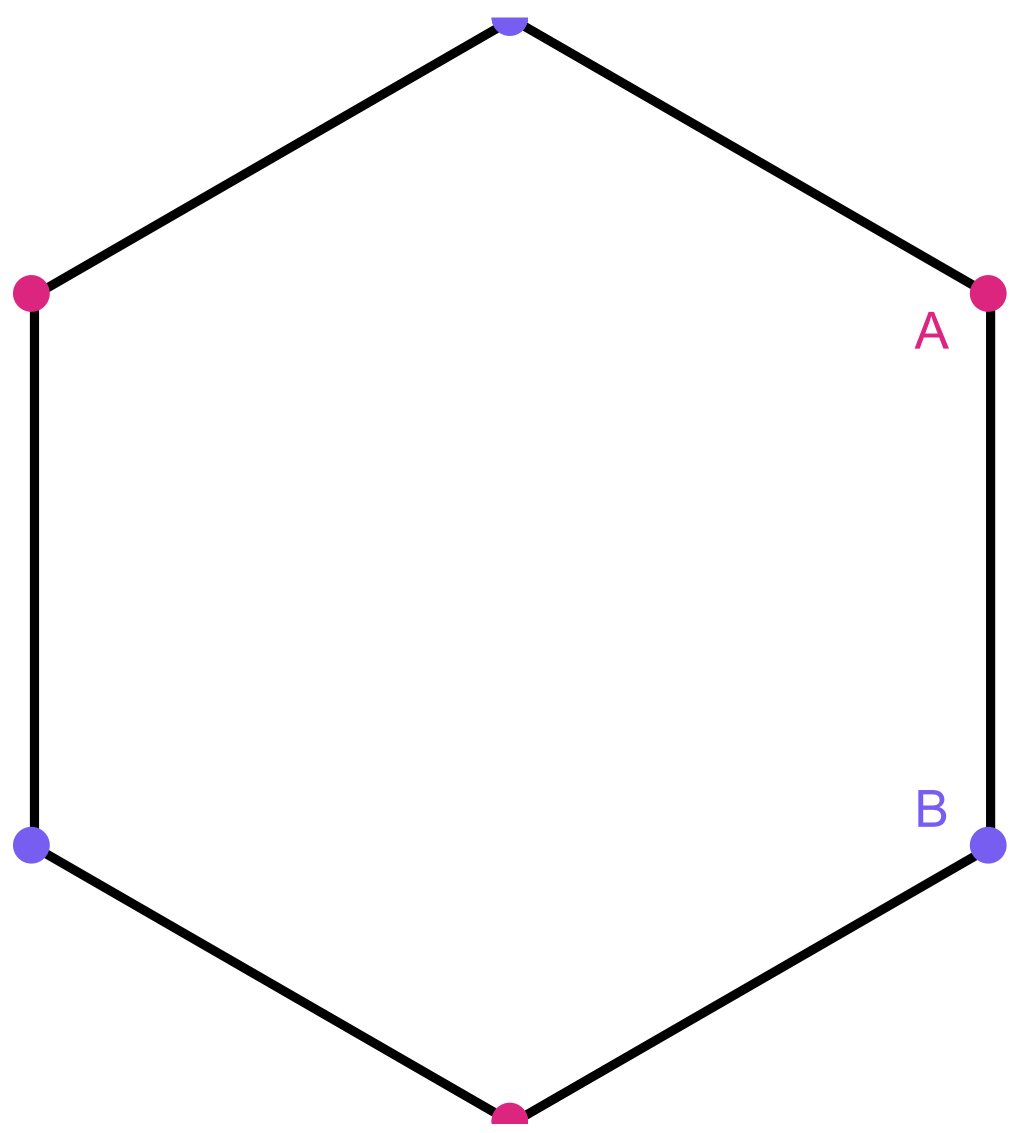
Landau Interaction Functions

$$f_{kk'} \to f^d_{kk'}, f^s_{kk'}, f^v_{kk';i}, f^m_{kk';i}$$

Symmetry constrained

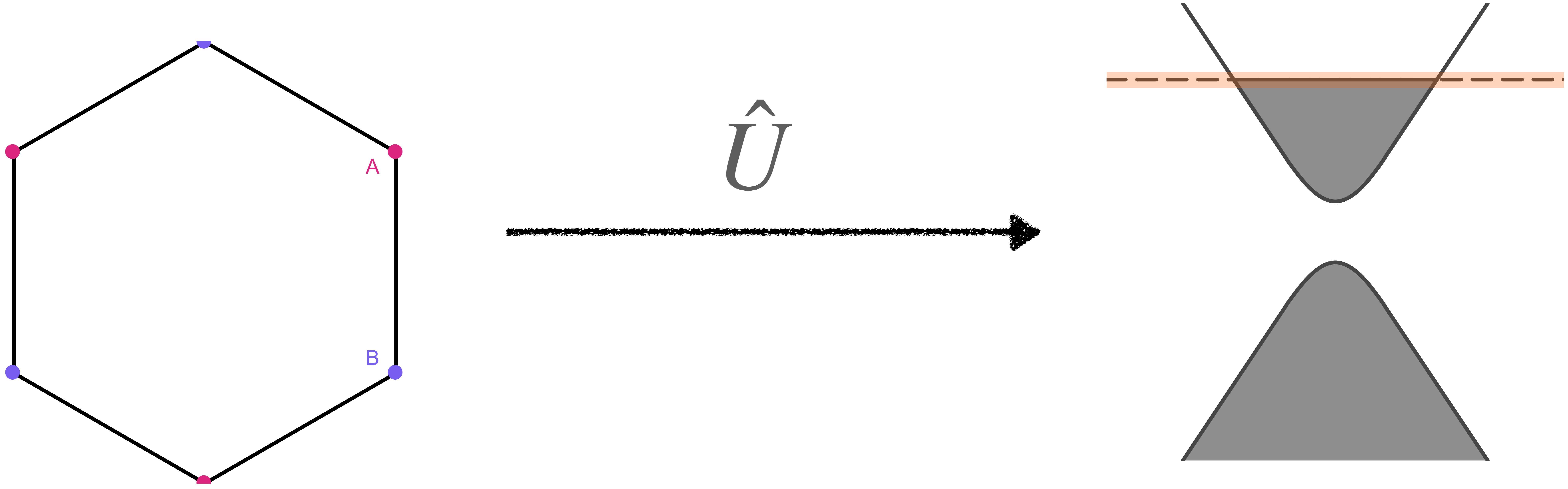
What else is different?

Berry connection



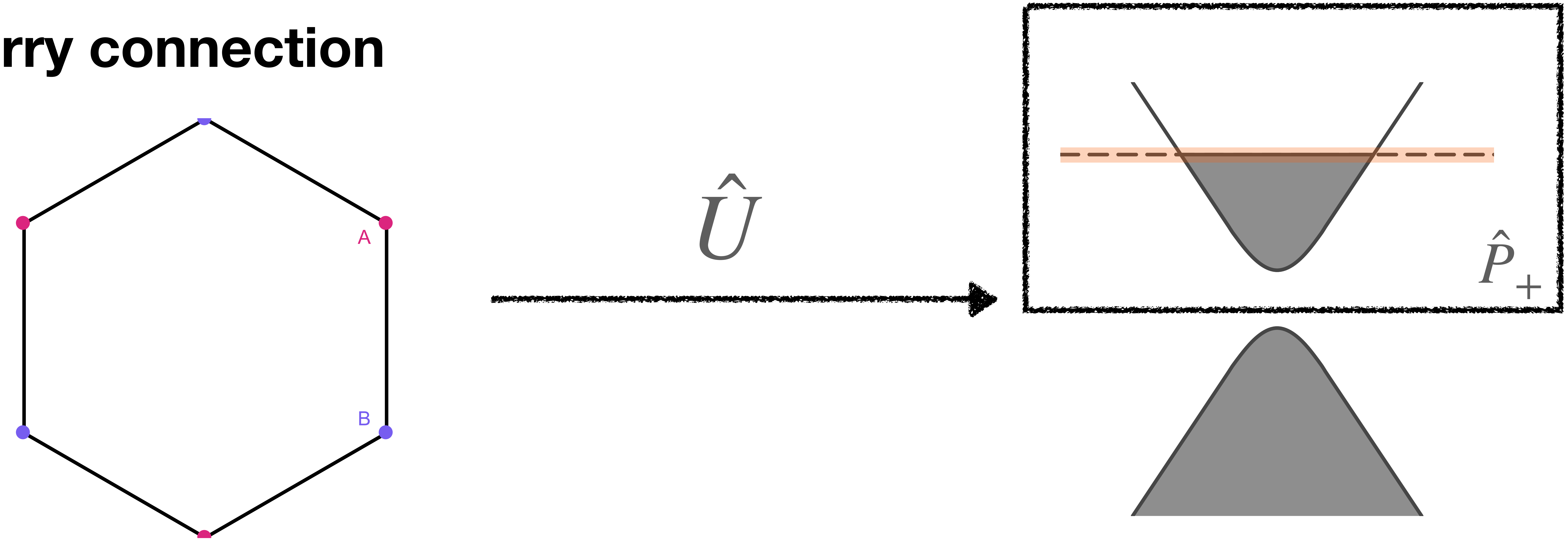
What else is different?

Berry connection



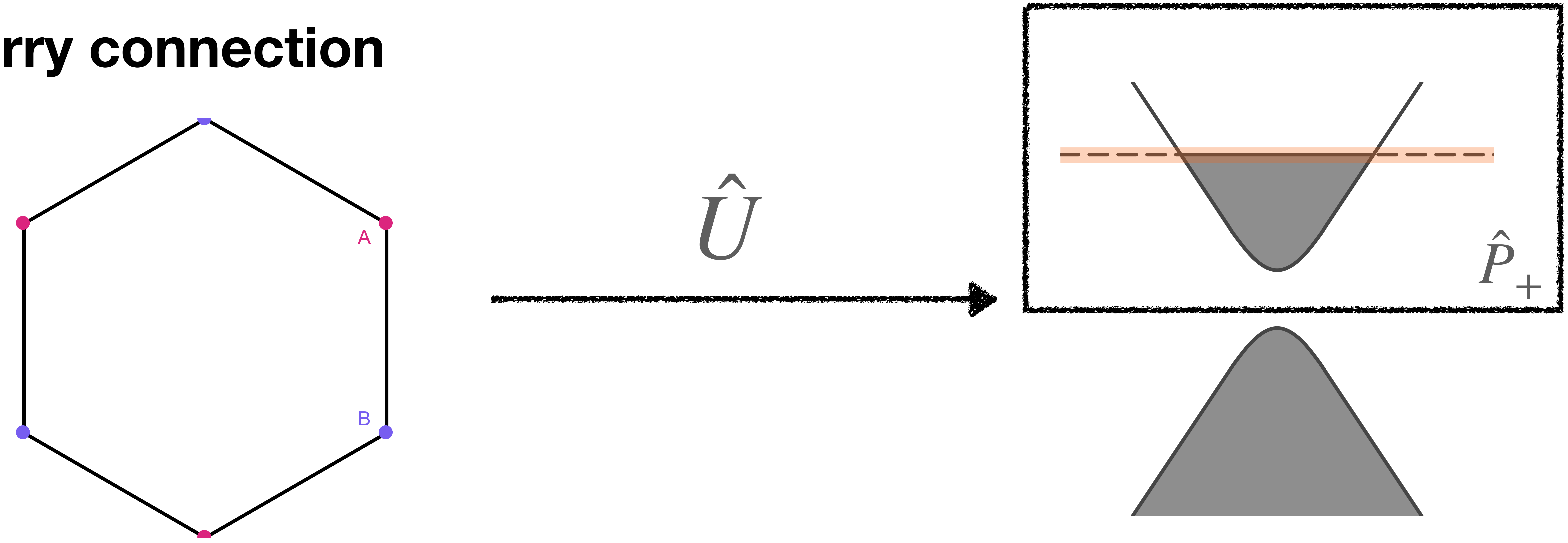
What else is different?

Berry connection



What else is different?

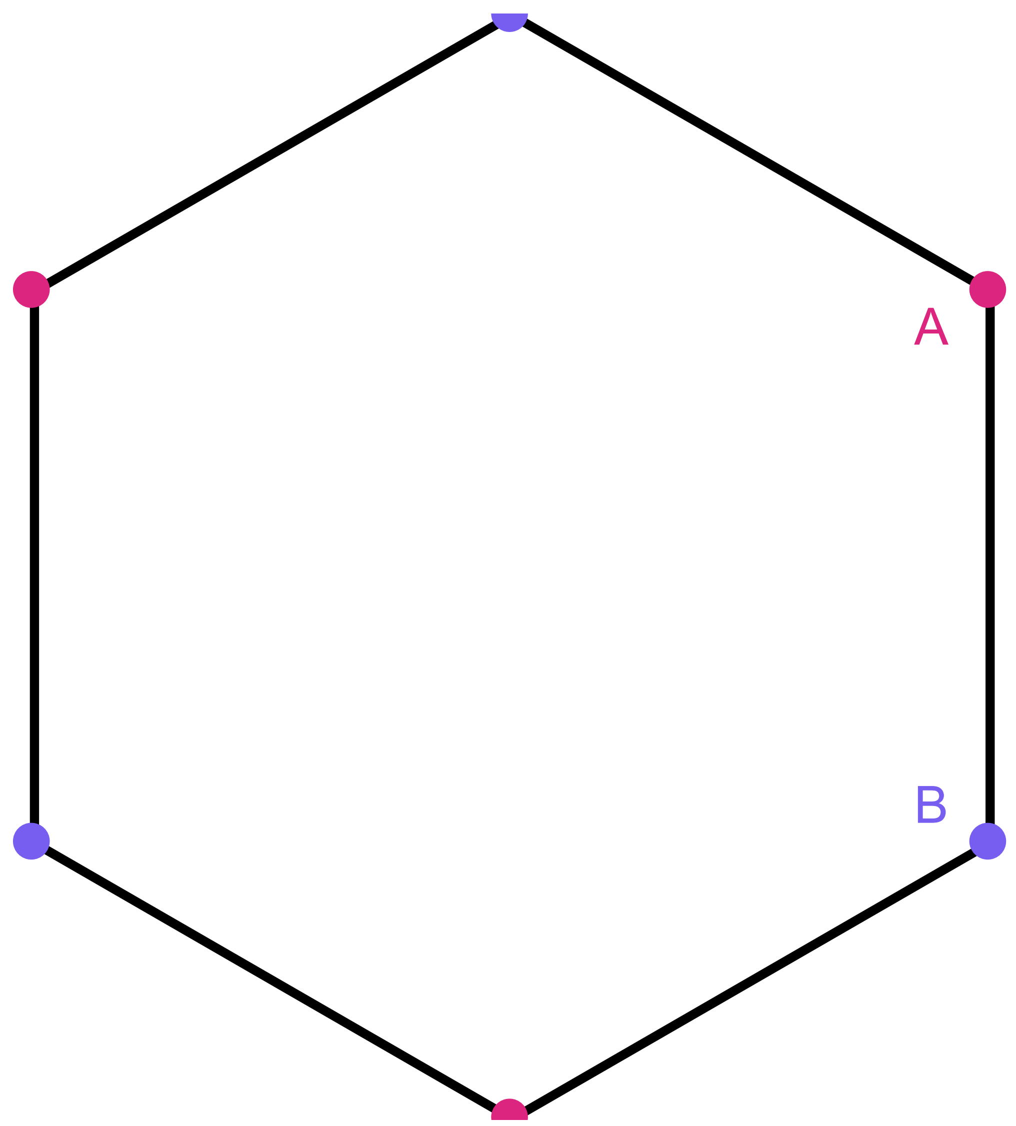
Berry connection

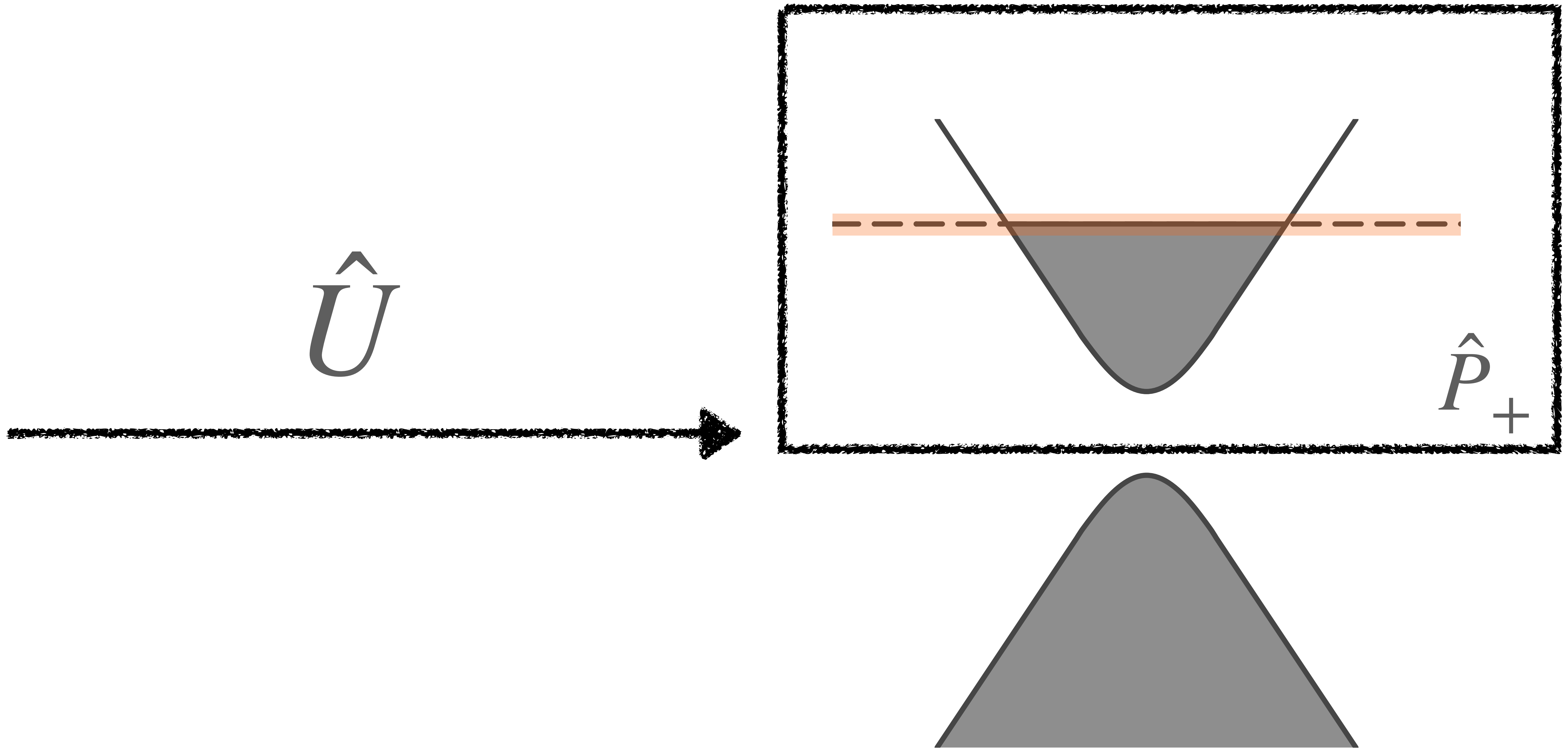


$$\hat{r} \rightarrow \hat{P}_+ \hat{r} + i\hat{P}_+ \hat{U}^\dagger \partial_p \hat{U} \hat{P}_+$$

What else is different?

Berry connection





"Gauged" position

$$\hat{r} \rightarrow \hat{P}_+ \left(\hat{r} + \hat{\mathscr{A}} \right)$$

Berry connection

$$\hat{\mathscr{A}} \hat{P}_+ = i \hat{P}_+ \hat{U}^\dagger \frac{\partial}{\partial \mathbf{p}} \hat{U} \hat{P}_+$$

Motivating the kinetic equation

Recall

$$\frac{\partial n(\mathbf{k},\mathbf{r})}{\partial t} + \frac{\partial \epsilon}{\partial \mathbf{k}} \cdot \frac{\partial}{\partial \mathbf{r}} n(\mathbf{k},\mathbf{r}) - \frac{\partial \epsilon}{\partial \mathbf{r}} \cdot \frac{\partial}{\partial \mathbf{k}} n(\mathbf{k},\mathbf{r}) = \hat{I}[n]$$

Motivating the kinetic equation

Recall

$$\frac{\partial n(\mathbf{k},\mathbf{r})}{\partial t} + \frac{\partial \epsilon}{\partial \mathbf{k}} \cdot \frac{\partial}{\partial \mathbf{r}} n(\mathbf{k},\mathbf{r}) - \frac{\partial \epsilon}{\partial \mathbf{r}} \cdot \frac{\partial}{\partial \mathbf{k}} n(\mathbf{k},\mathbf{r}) = \hat{I}[n]$$

Berry connection

$$\hat{\mathscr{A}}\hat{P}_+ = i\hat{P}_+\hat{U}^\dagger \frac{\partial}{\partial \mathbf{p}} \hat{U}\hat{P}_+$$

"Gauged" position

$$\hat{r} \to \hat{P}_+ \left(\hat{r} + \hat{\mathscr{A}} \right)$$

Berry Covariant Kinetic equation

The "obvious" generalization

$$\partial_t \hat{\rho} + \frac{1}{2}\left[\frac{\partial}{\partial \mathbf{r}}\hat{\rho}; \mathscr{D}\hat{\epsilon}\right]_+ - \frac{1}{2}\left[\mathscr{D}\hat{\rho}; \frac{\partial}{\partial \mathbf{r}}\hat{\epsilon}\right]_+ + i\left[\hat{\epsilon}, \hat{\rho}\right]_- = 0$$

Berry Gauge Covariant derivative

$$\mathscr{D}\hat{g} = \frac{\partial}{\partial \mathbf{p}}\hat{g} - \imath[\hat{\mathscr{A}}, \hat{g}]$$

Bettelheim, J. Phys. A 50, 415303 (2017)

Silin, JETP 6, 945 (1958)
single particle von Neumann

Symmetrize

Berry Covariant Kinetic equation

Kinetic eqn

$$\partial_t \hat{\rho} + \frac{1}{2}[\frac{\partial}{\partial \mathbf{r}}\hat{\rho}; \hat{\mathbf{v}}]_+ + \frac{1}{2}[\mathscr{D}\hat{\rho}; \hat{\boldsymbol{F}}]_+ + i\left[\hat{\epsilon}, \hat{\rho}\right]_- = 0$$

- Described by semiclassical EOM as in wave packet dynamics

EOM

$$\hat{\mathbf{v}} \approx \mathscr{D}\hat{\epsilon}, \quad \hat{\boldsymbol{F}} \approx e\mathbf{E} - \frac{\partial}{\partial \mathbf{r}}\hat{\epsilon}$$

$$\mathscr{D}\hat{g} = \frac{\partial}{\partial \mathbf{p}}\hat{g} - \imath[\hat{\mathscr{A}}, \hat{g}]$$

Covariant derivative

Xiao, Chang, Niu, RMP 82, 1959 (2010)

Bettelheim, J. Phys. A 50, 415303 (2017)

Motivating the kinetic equation

Another way

Recall

$$\frac{\partial n(\mathbf{k},\mathbf{r})}{\partial t} + \frac{\partial \epsilon}{\partial \mathbf{k}} \cdot \frac{\partial}{\partial \mathbf{r}} n(\mathbf{k},\mathbf{r}) - \frac{\partial \epsilon}{\partial \mathbf{r}} \cdot \frac{\partial}{\partial \mathbf{k}} n(\mathbf{k},\mathbf{r}) = 0$$

Motivating the kinetic equation

Another way

Recall

$$\frac{\partial n(\mathbf{k},\mathbf{r})}{\partial t} + \frac{\partial \epsilon}{\partial \mathbf{k}} \cdot \frac{\partial}{\partial \mathbf{r}} n(\mathbf{k},\mathbf{r}) - \frac{\partial \epsilon}{\partial \mathbf{r}} \cdot \frac{\partial}{\partial \mathbf{k}} n(\mathbf{k},\mathbf{r}) = 0$$

Motivating the kinetic equation

Another way

Recall

$$\frac{\partial n(\mathbf{k},\mathbf{r})}{\partial t} + \frac{\partial \epsilon}{\partial \mathbf{k}} \cdot \frac{\partial}{\partial \mathbf{r}} n(\mathbf{k},\mathbf{r}) - \frac{\partial \epsilon}{\partial \mathbf{r}} \cdot \frac{\partial}{\partial \mathbf{k}} n(\mathbf{k},\mathbf{r}) = 0$$

Poisson Bracket

$$\{A, B\} = \frac{\partial}{\partial \mathbf{r}} A \cdot \frac{\partial}{\partial \mathbf{p}} B - \frac{\partial}{\partial \mathbf{p}} A \cdot \frac{\partial}{\partial \mathbf{r}} B$$

Motivating the kinetic equation

Another way

Recall

$$\frac{\partial n(\mathbf{k},\mathbf{r})}{\partial t} + \frac{\partial \epsilon}{\partial \mathbf{k}} \cdot \frac{\partial}{\partial \mathbf{r}} n(\mathbf{k},\mathbf{r}) - \frac{\partial \epsilon}{\partial \mathbf{r}} \cdot \frac{\partial}{\partial \mathbf{k}} n(\mathbf{k},\mathbf{r}) = 0$$

Poisson Bracket

$$\{A, B\} = \frac{\partial}{\partial \mathbf{r}} A \cdot \frac{\partial}{\partial \mathbf{p}} B - \frac{\partial}{\partial \mathbf{p}} A \cdot \frac{\partial}{\partial \mathbf{r}} B$$

Liouville Equation

$$\frac{\partial n}{\partial t} + \{n, H\} = 0$$

Motivating the kinetic equation

another way

Poisson Bracket

$$\{A,B\} = \frac{1}{2}\left[\frac{\partial}{\partial \mathbf{r}}\hat{A};\mathscr{D}\hat{B}\right]_{+} - \frac{1}{2}\left[\mathscr{D}\hat{A};\frac{\partial}{\partial \mathbf{r}}\hat{B}\right]_{+} - i[\hat{A},\hat{B}]$$

$$\mathscr{D}\hat{g} = \frac{\partial}{\partial \mathbf{p}}\hat{g} - \imath[\hat{\mathscr{A}},\hat{g}]$$

Liouville Equation

$$\frac{\partial\hat{\rho}}{\partial t} + \{\hat{\rho},\hat{\epsilon}\} = 0$$

Motivating the kinetic equation

another way

An Aside

Modified symplectic structure / non-canonical coordinates

Stat mech on an enlarged phase space

$$\mathbb{R}^{2n} \times SU(N)$$

Poisson Bracket

$$\{A, B\} = \frac{1}{2}\left[\frac{\partial}{\partial \mathbf{r}}\hat{A}; \mathscr{D}\hat{B}\right]_+ - \frac{1}{2}\left[\mathscr{D}\hat{A}; \frac{\partial}{\partial \mathbf{r}}\hat{B}\right]_+ - i[\hat{A}, \hat{B}]$$

$$\mathscr{D}\hat{g} = \frac{\partial}{\partial \mathbf{p}}\hat{g} - \imath[\hat{\mathscr{A}}, \hat{g}]$$

Liouville Equation

$$\frac{\partial \hat{\rho}}{\partial t} + \{\hat{\rho}, \hat{\epsilon}\} = 0$$

Berry Covariant Kinetic equation

Linearized Kinetic Equation

$$\partial_t \delta\hat{\rho} + \frac{1}{2}\nabla \cdot \left[[\hat{\mathbf{v}}, \delta\hat{\rho}]_+ - [\delta\hat{\epsilon}, \mathscr{D}\hat{\rho}_{\rm eq}]_+ \right] + i[\hat{\epsilon}_{\rm eq}, \delta\hat{\bar{\rho}}] = - e\mathbf{E} \cdot \mathscr{D}\hat{\rho}_{\rm eq}$$

- A good testbed for understanding the role of band geometry on
  - the kinetic theory
  - spin-valley collective modes

$$\hat{\mathbf{v}} \approx \mathscr{D}\hat{\epsilon}, \quad \hat{\boldsymbol{F}} \approx e\mathbf{E} - \nabla\hat{\epsilon}$$

$$\mathscr{D}\hat{g} = \frac{\partial}{\partial \mathbf{p}}\hat{g} - \imath[\hat{\mathscr{A}}, \hat{g}]$$

Linearized transport equation

Consider the uniform limit: $q \to 0$

Plasmon $n(\mathbf{r}, \mathbf{p}) = \frac{1}{G_s G_v} \operatorname{tr} \hat{\sigma}_0 \hat{\tau}_0 \hat{\rho}(\mathbf{r}, \mathbf{p})$

Linearized kinetic equation

$$\partial_t \delta\hat{\rho} + i[\hat{\epsilon}_{\text{eq}}, \delta\hat{\bar{\rho}}] = -e\mathbf{E}\cdot\mathcal{D}\hat{\rho}_{\text{eq}}$$

Silin mode sector

Modes separate into two sectors

Optically inactive

$$\mathbf{s}(\mathbf{r}, \mathbf{p}) = \frac{1}{G_s G_v} \operatorname{tr} \hat{\boldsymbol{\sigma}} \hat{\rho}(\mathbf{r}, \mathbf{p})$$

$$M_i^j(\mathbf{r}, \mathbf{p}) = \frac{1}{G_s G_v} \operatorname{tr} \hat{\tau}_i \hat{\sigma}_j \hat{\rho}(\mathbf{r}, \mathbf{p})$$

$$\mathbf{Y}(\mathbf{r}, \mathbf{p}) = \frac{1}{G_s G_v} \operatorname{tr} \hat{\boldsymbol{\tau}} \hat{\rho}(\mathbf{r}, \mathbf{p})$$

$\mathbf{s}$ and M_i^j with $j = z$ → Silin mode sector

M_i^j with $j = \parallel$ and $\mathbf{Y}$ → Optically inactive

Linearized transport equation

Consider the uniform limit: $q \to 0$

Linearized kinetic equation

$$\partial_t \delta\hat{\rho} + i[\hat{\epsilon}_{\text{eq}}, \delta\hat{\bar{\rho}}] = -e\mathbf{E}\cdot\mathcal{D}\hat{\rho}_{\text{eq}}$$

Plasmon $n(\mathbf{r},\mathbf{p}) = \frac{1}{G_s G_v}\,\mathrm{tr}\,\hat{\sigma}_0\hat{\tau}_0\hat{\rho}(\mathbf{r},\mathbf{p})$

Silin mode sector

Modes separate into two sectors

Optically inactive

$$\mathbf{s}(\mathbf{r},\mathbf{p}) = \frac{1}{G_s G_v}\,\mathrm{tr}\,\hat{\boldsymbol{\sigma}}\hat{\rho}(\mathbf{r},\mathbf{p})$$

$j = z$

$$M_i^j(\mathbf{r},\mathbf{p}) = \frac{1}{G_s G_v}\,\mathrm{tr}\,\hat{\tau}_i\hat{\sigma}_j\hat{\rho}(\mathbf{r},\mathbf{p})$$

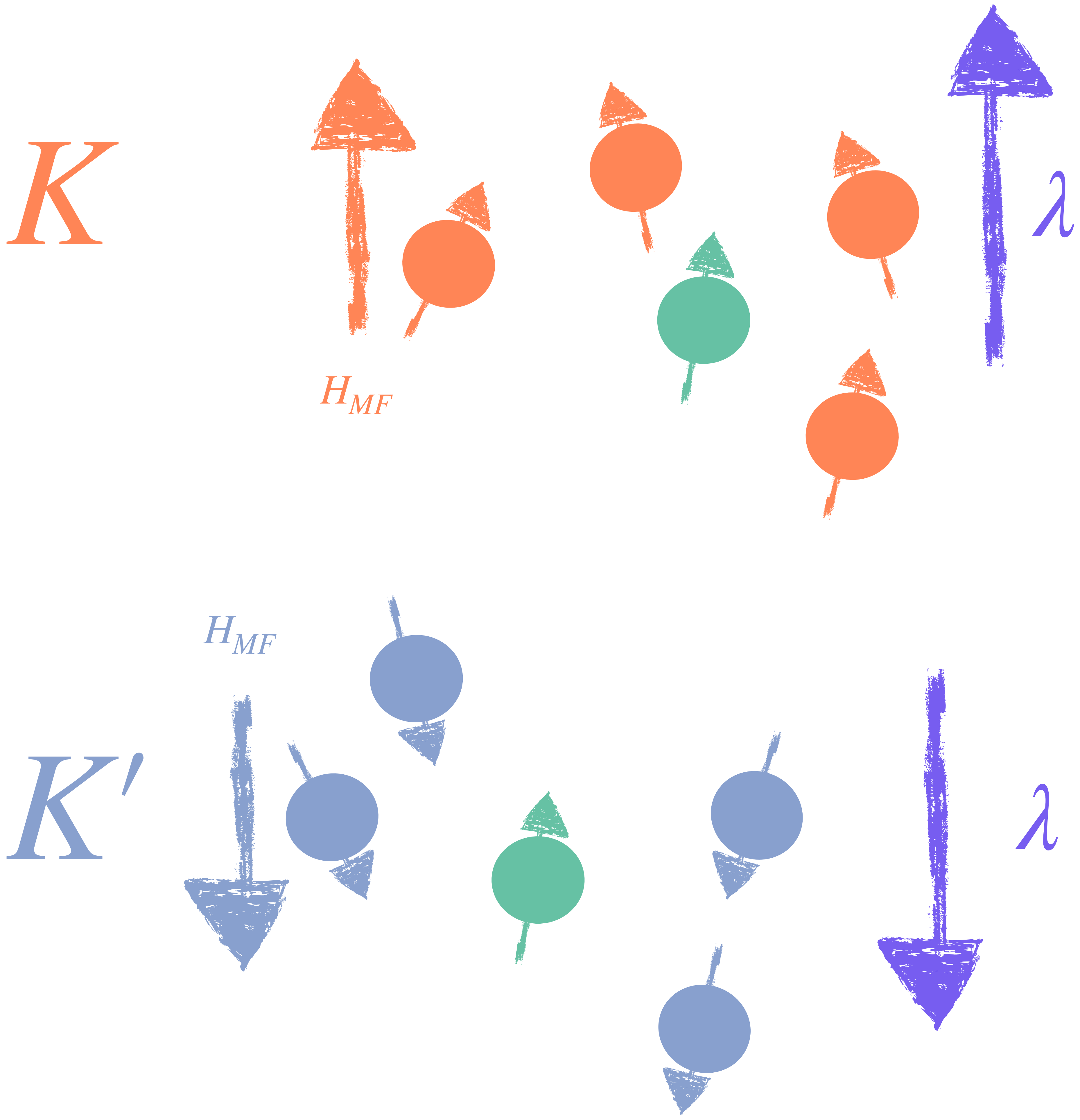
$j = \parallel$

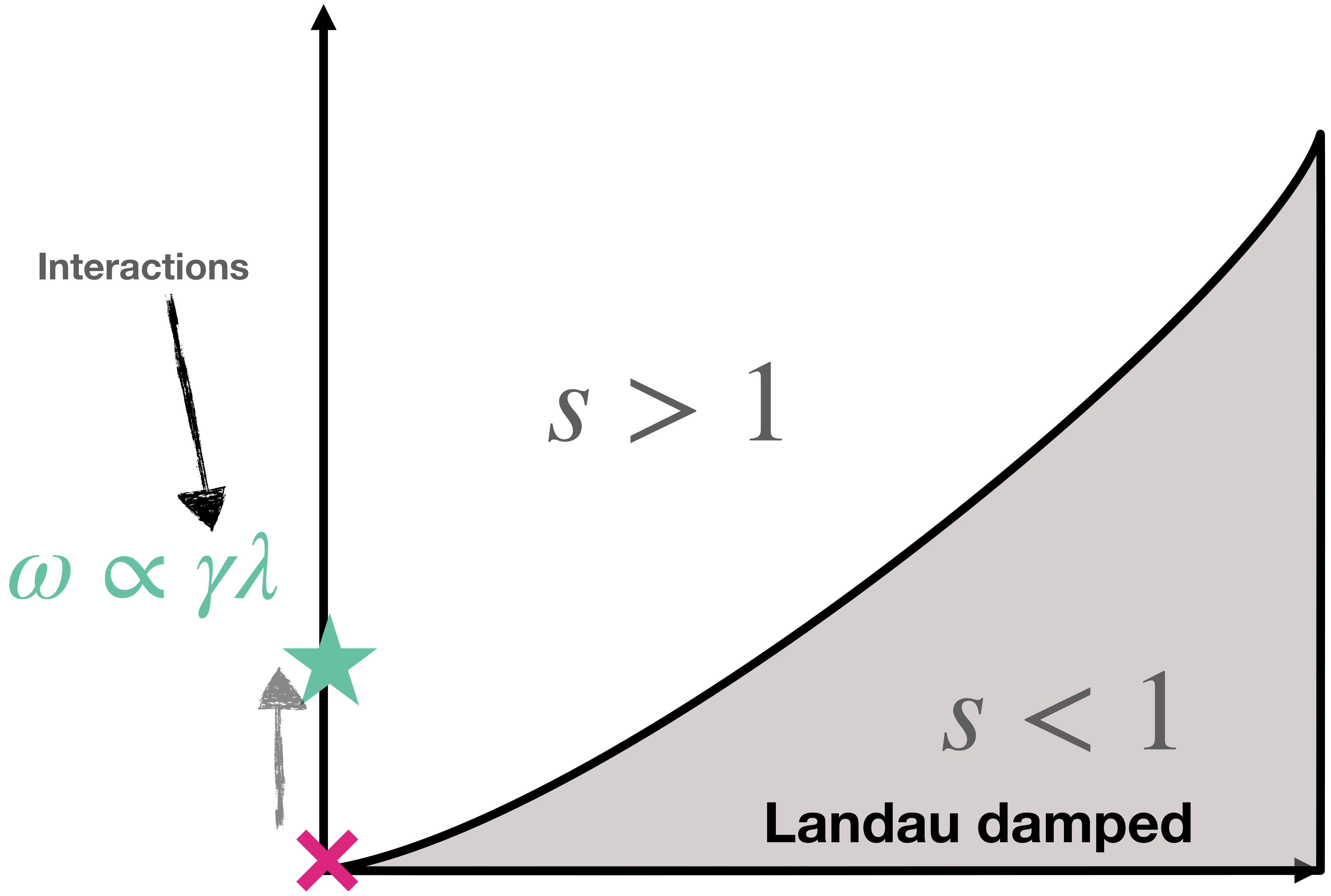
$$\mathbf{Y}(\mathbf{r},\mathbf{p}) = \frac{1}{G_s G_v}\,\mathrm{tr}\,\hat{\boldsymbol{\tau}}\hat{\rho}(\mathbf{r},\mathbf{p})$$

Spin oscillations with valley SOC

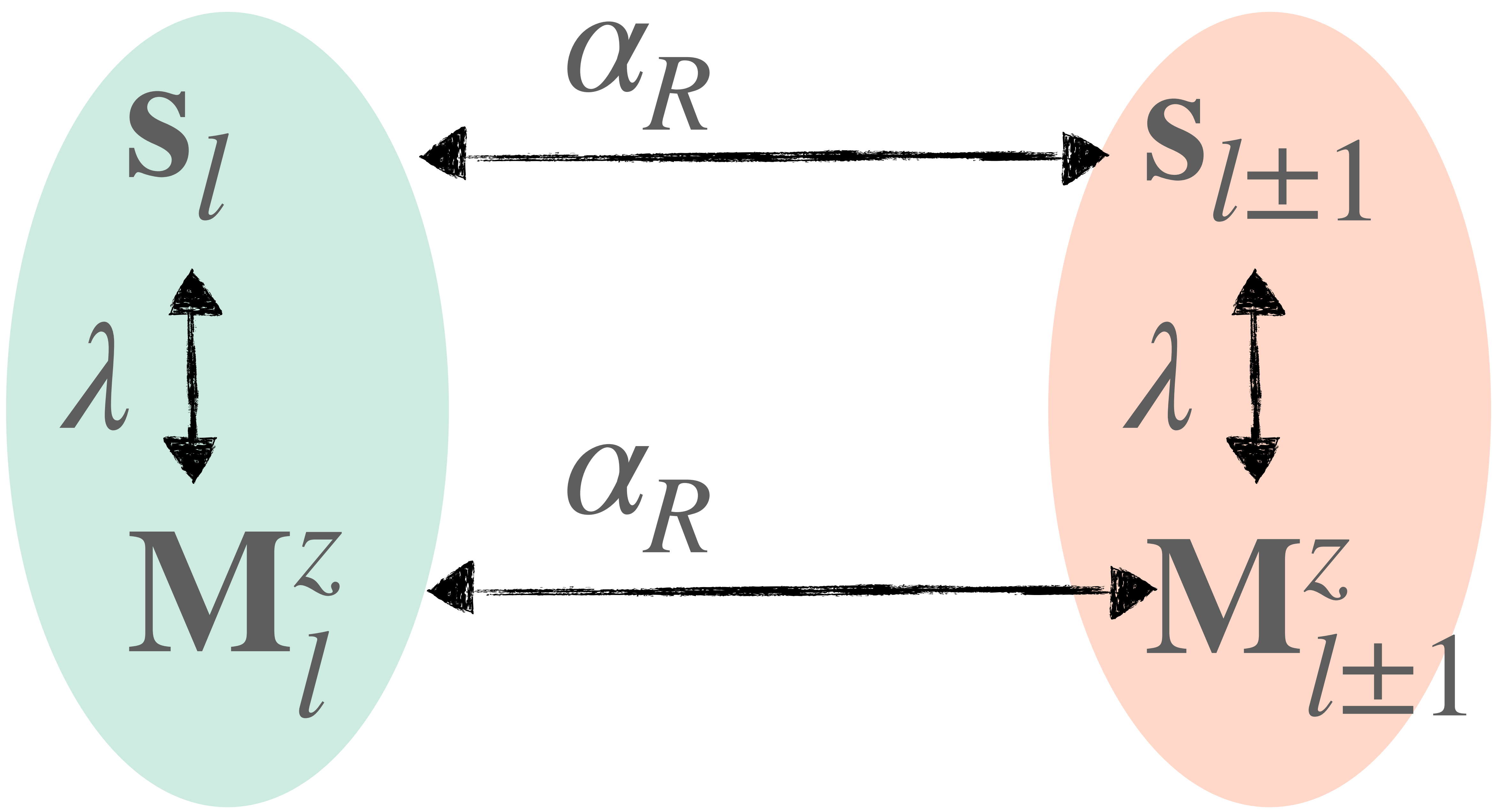
Valley staggered Silin mode

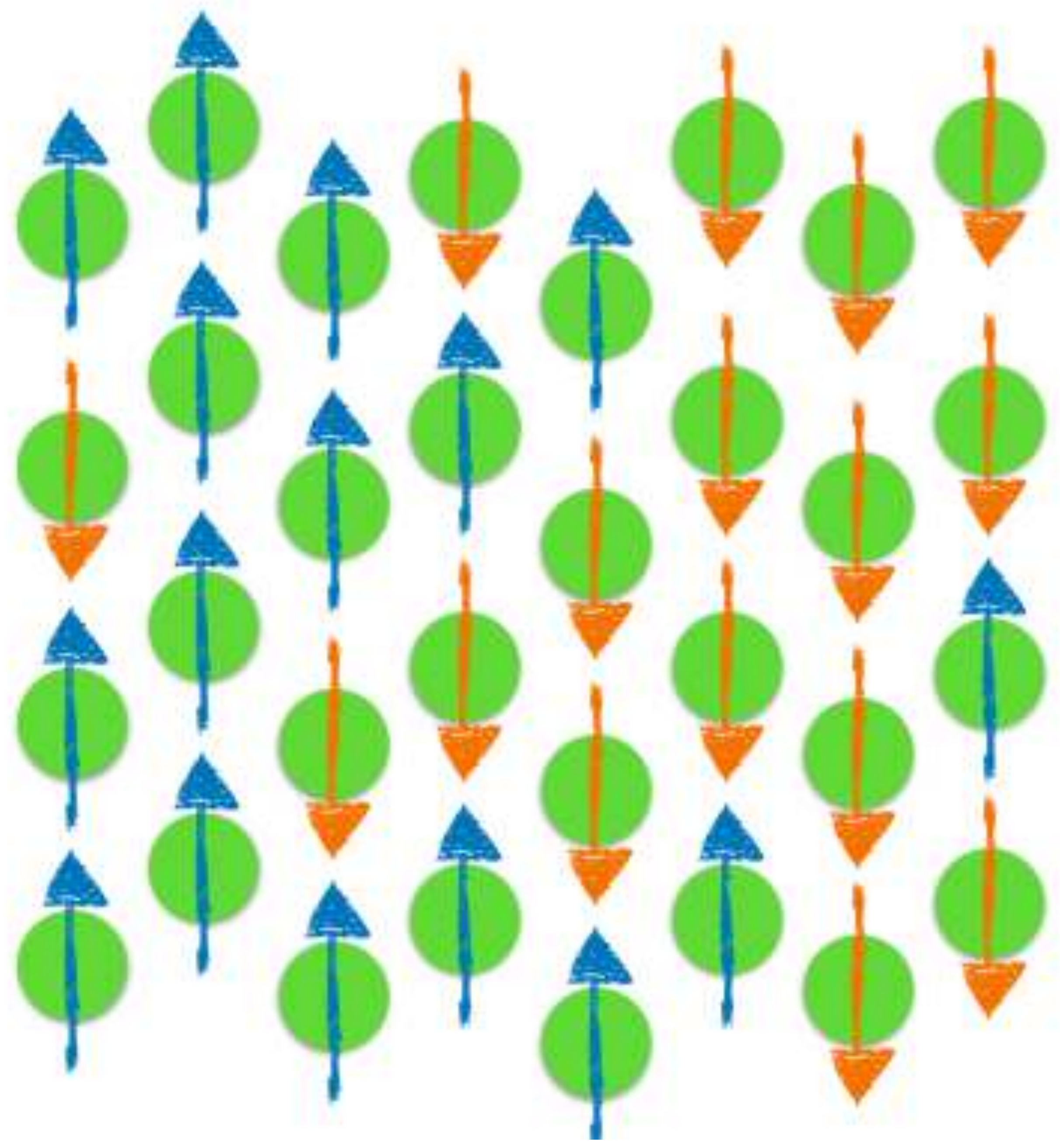
$$M_z^j(\mathbf{r},\mathbf{p}) = \frac{1}{G_s G_v} \operatorname{tr} \hat{\tau}_z \hat{\sigma}_j \hat{\rho}(\mathbf{r},\mathbf{p})$$





Silin mode sector





Silin mode sector (without SOC)

$\mathbf{s}_l$ $\quad$ $\mathbf{s}_{l\pm1}$

$\mathbf{M}_l^z$ $\quad$ $\mathbf{M}_{l\pm1}^z$

$$\partial_t \delta \mathbf{s}_l - \omega_{sl} \mathbf{e}_x \times \delta \mathbf{s}_l = 0$$
$$\partial_t \delta \mathbf{M}_l^z - \omega_{ml} \mathbf{e}_x \times \delta \mathbf{M}_l^z = 0$$

x **axis defined by** $\mathbf{H}_0$

Silin mode sector

$\mathbf{S}_l$ Spin mode

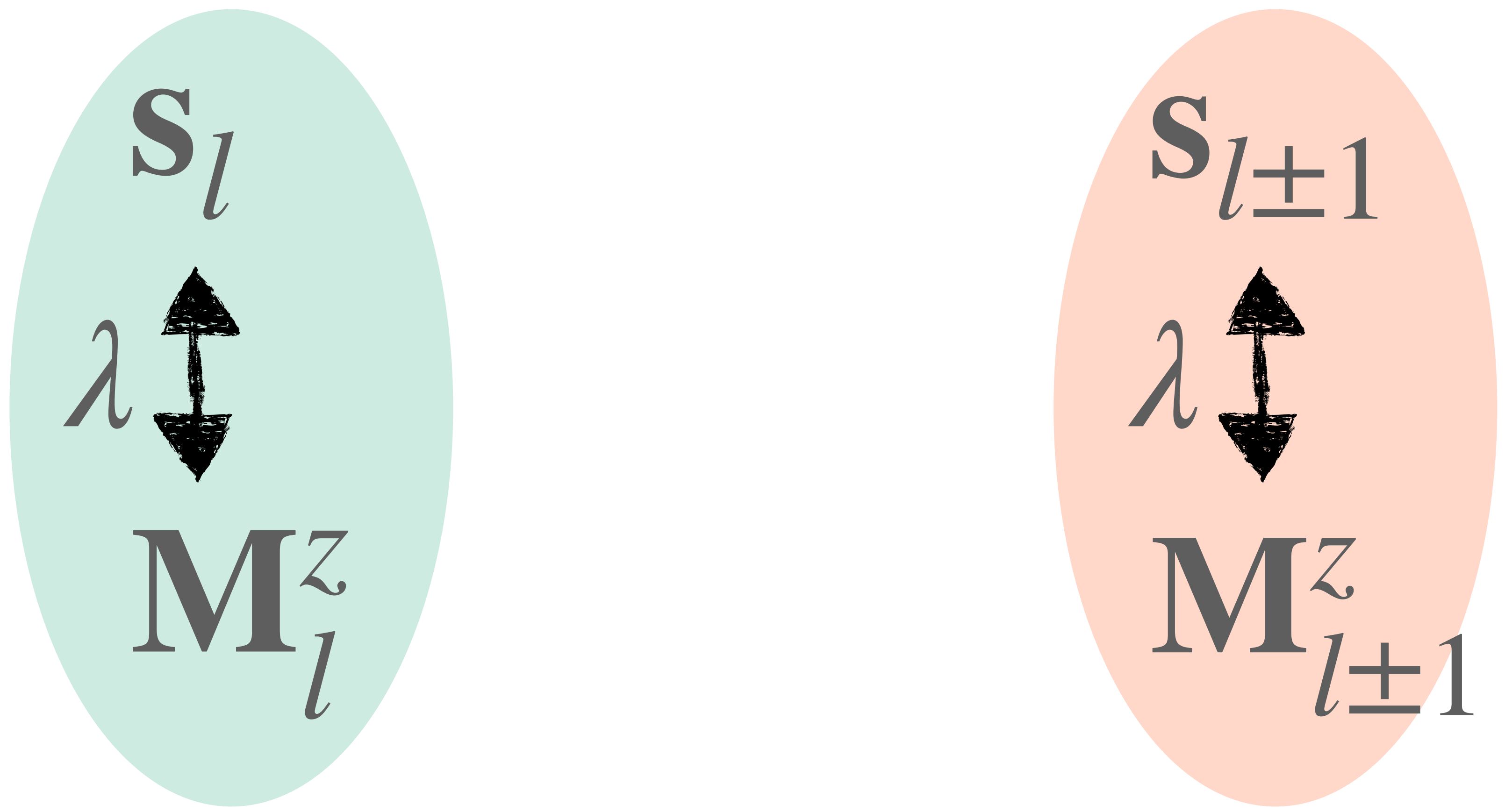
$\mathbf{M}_l^z$ Valley staggered spin mode

$$\omega_{sl} = \mu_s H_0 \frac{1+F_0^s}{1+F_1^s}, \quad \omega_m = \gamma_l \omega_{sl}, \quad \gamma \equiv \frac{1+F_l^{mz}}{1+F_l^s}$$

Renormalized by Interactions

Silin Mode Sector (without Rashba)

$\alpha_R \to 0$



$$\frac{\partial \mathbf{s}_0}{\partial t} - \omega_s \hat{x} \times \mathbf{s}_0 + 2\lambda \hat{z} \times \mathbf{M}_0^z = 0$$

$$\frac{\partial \mathbf{M}_0^z}{\partial t} - \omega_m \hat{x} \times \mathbf{M}_0^z + 2\gamma\lambda \hat{z} \times \mathbf{s}_0 = 0$$

x axis defined by $\mathbf{H}_0$

$$\omega_s = \mu_s H_0, \quad \omega_m = \gamma \omega_s, \quad \gamma \equiv \frac{1 + F_0^{mz}}{1 + F_0^s}$$

Renormalized by Interactions

Valley-spin Silin modes

- There are two eigenmodes adiabatically connected to
  - the spin mode $\mathbf{s}$
  - and valley-staggered spin mode $\mathbf{M}^z$

$$\omega_1 = |\mathbf{b}_1| = \sqrt{\omega_s^2 + 4\lambda^2\gamma^{-1}}$$
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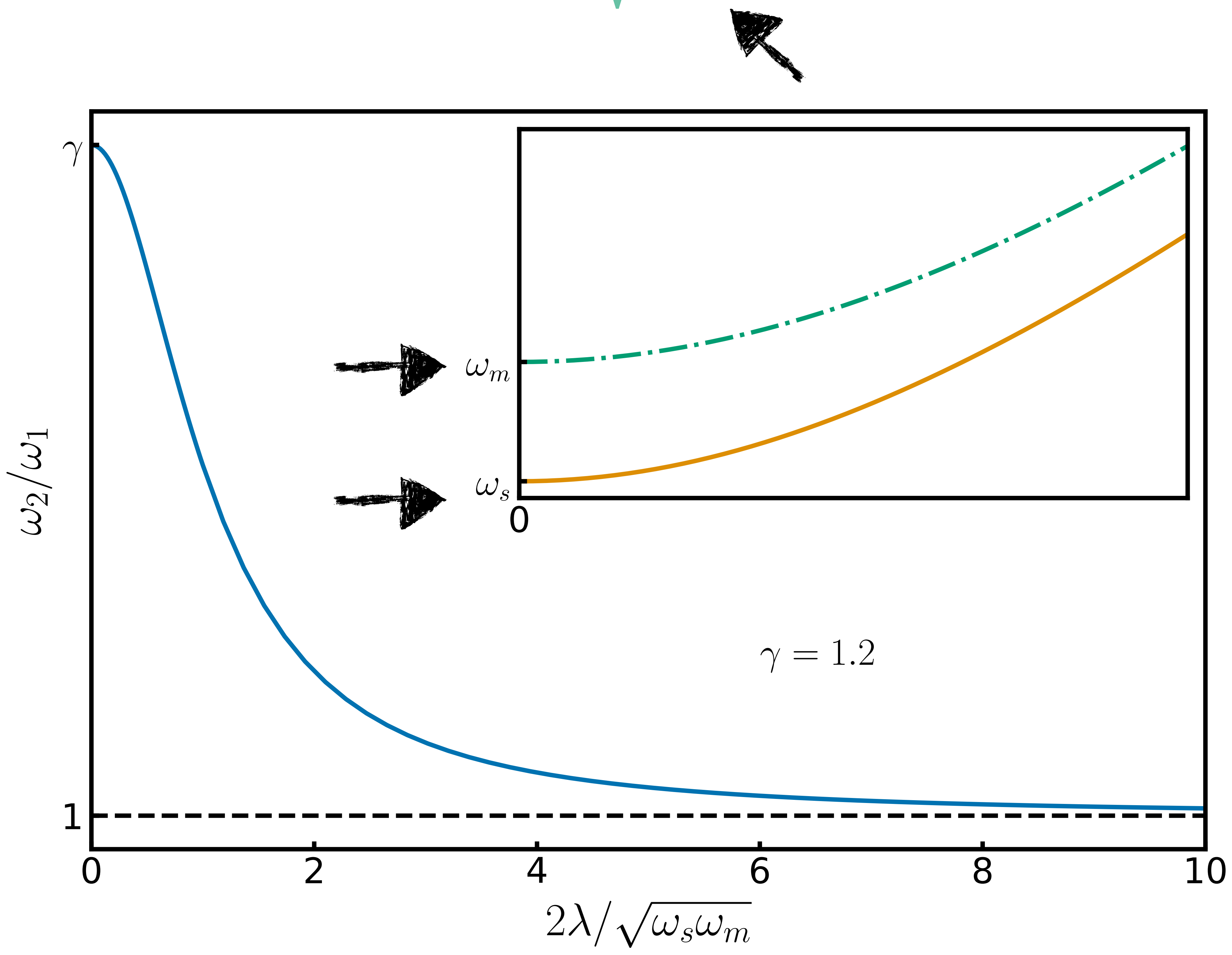

Valley-spin Silin modes

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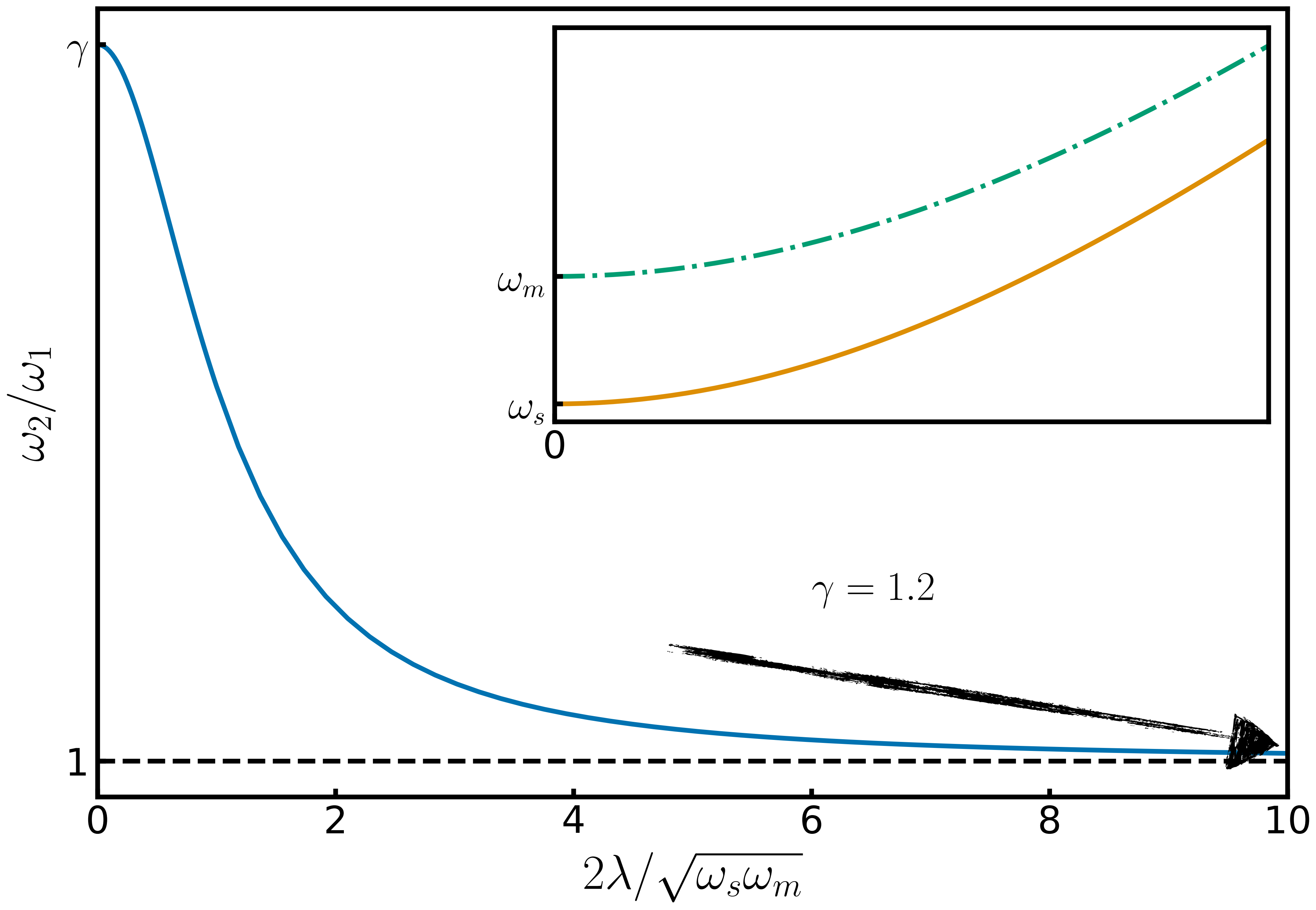
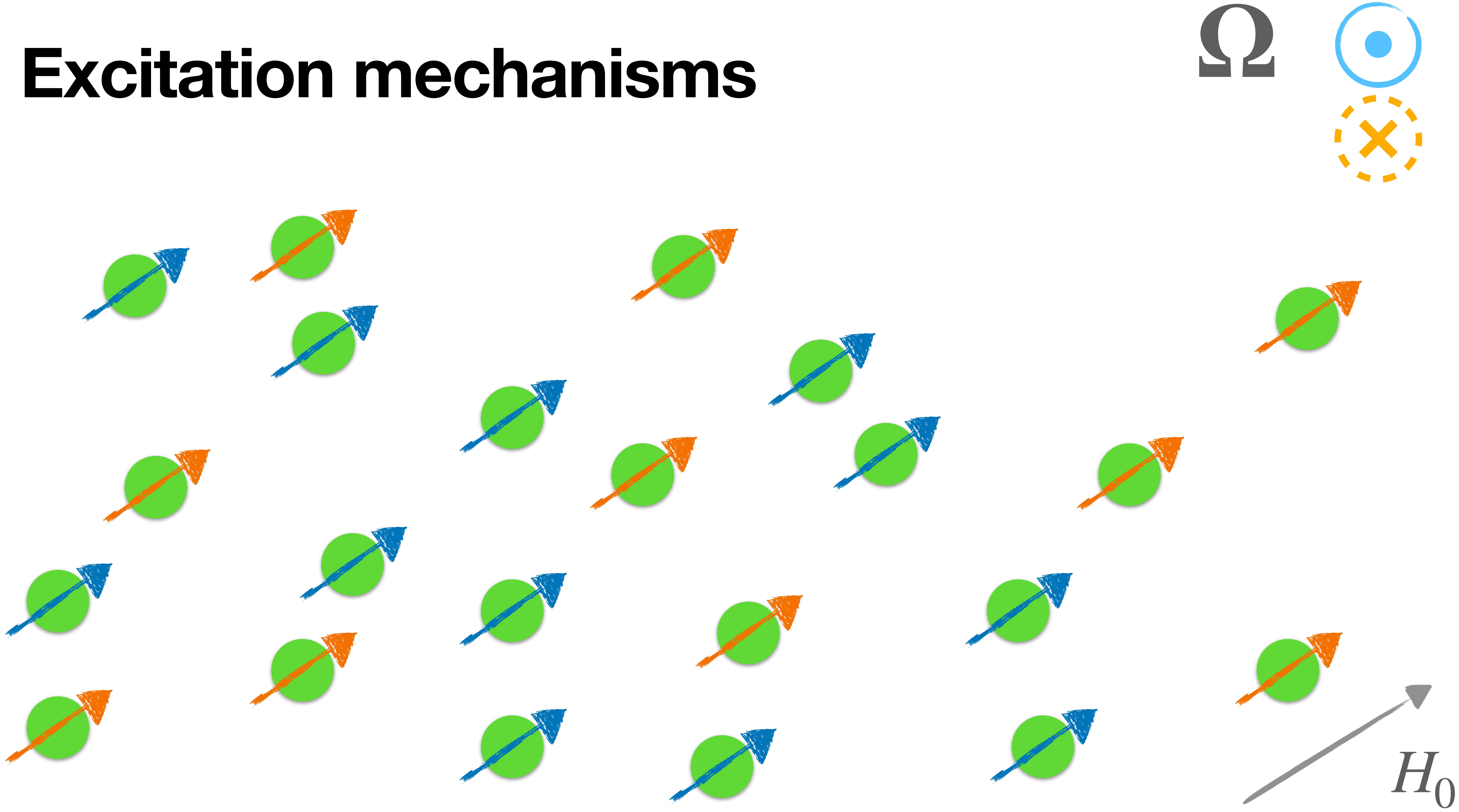
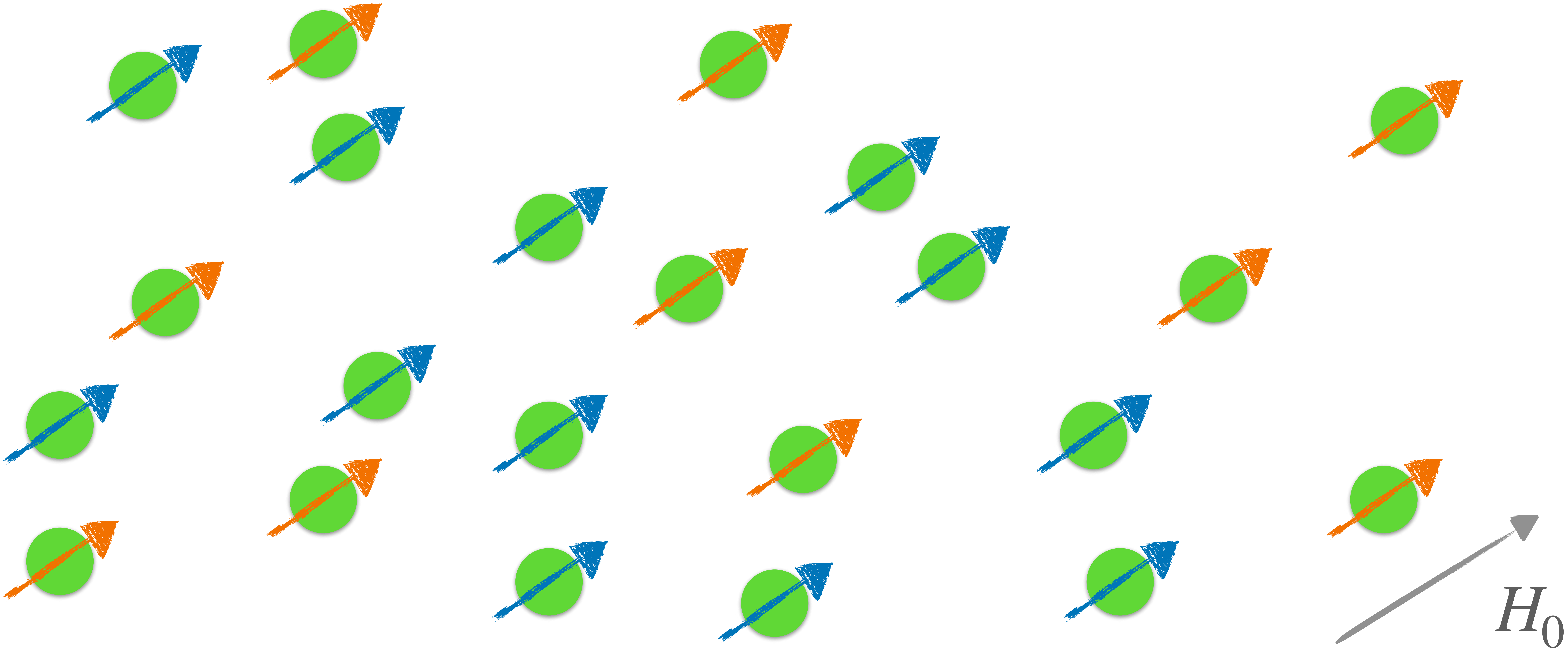


Excitation mechanisms



Excitation mechanisms

Spin mode (Rashba torque)



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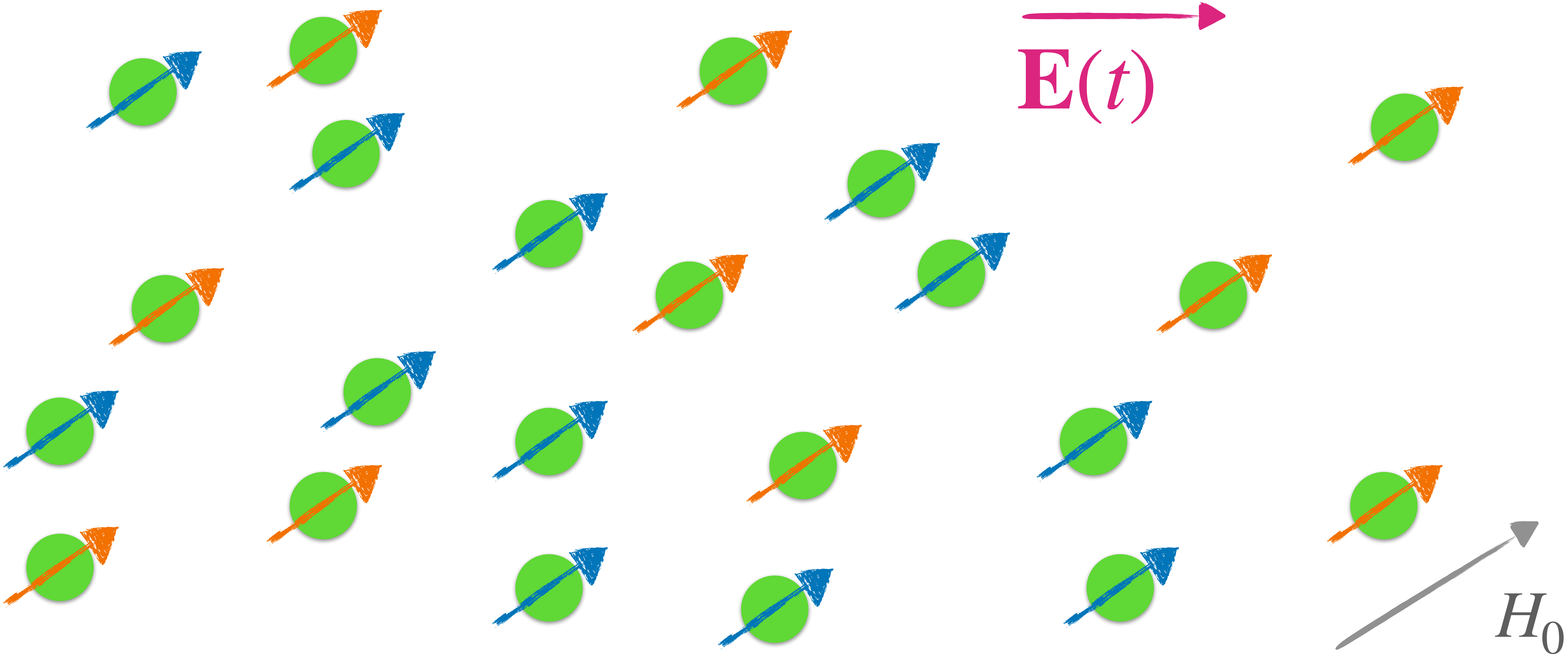

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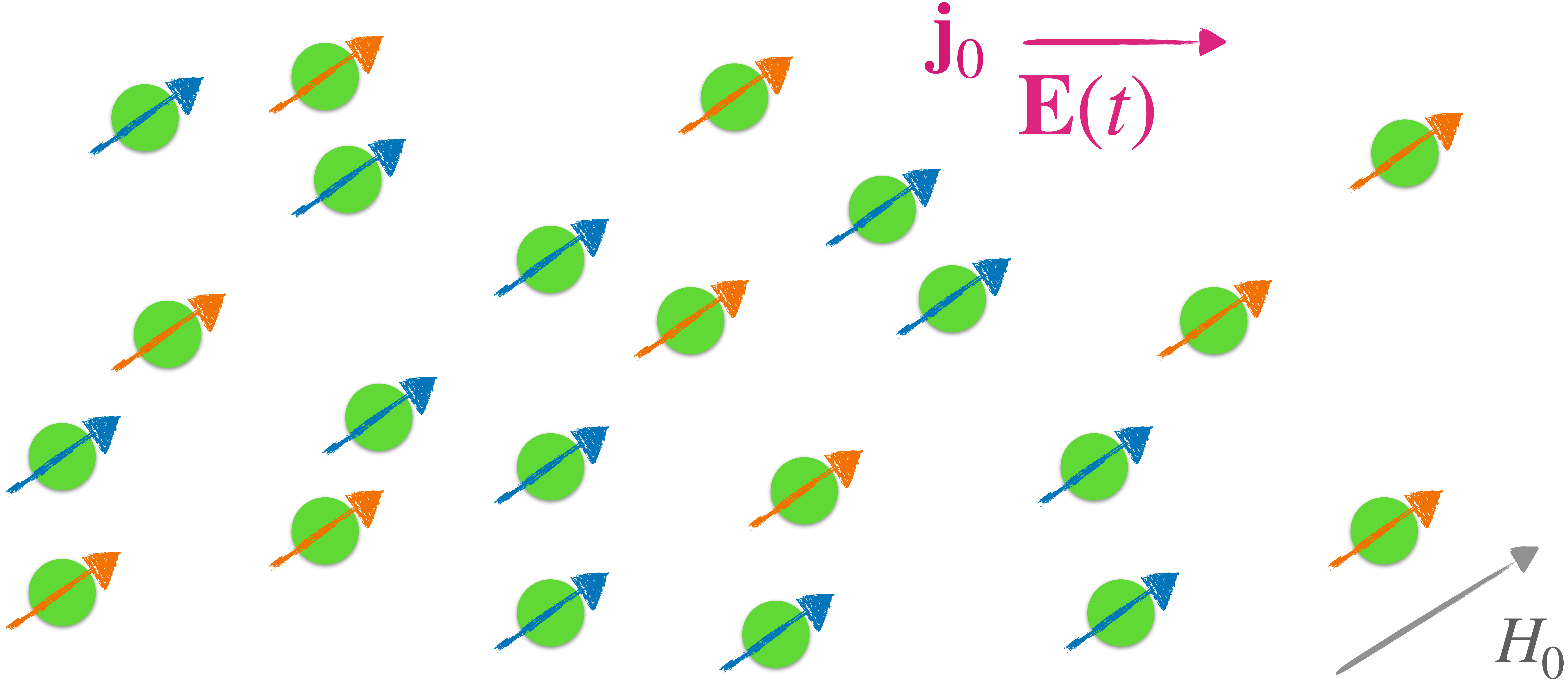

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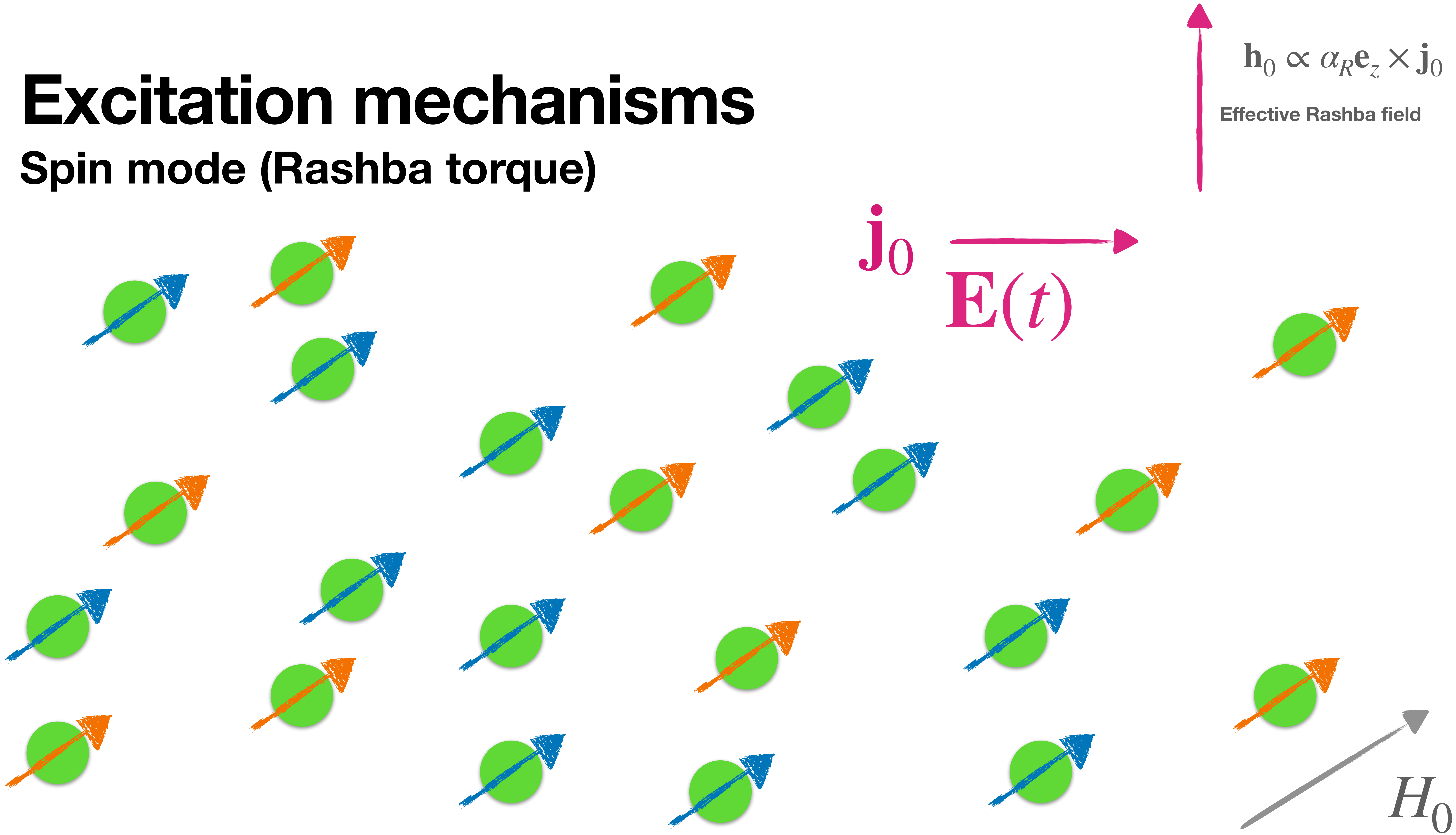
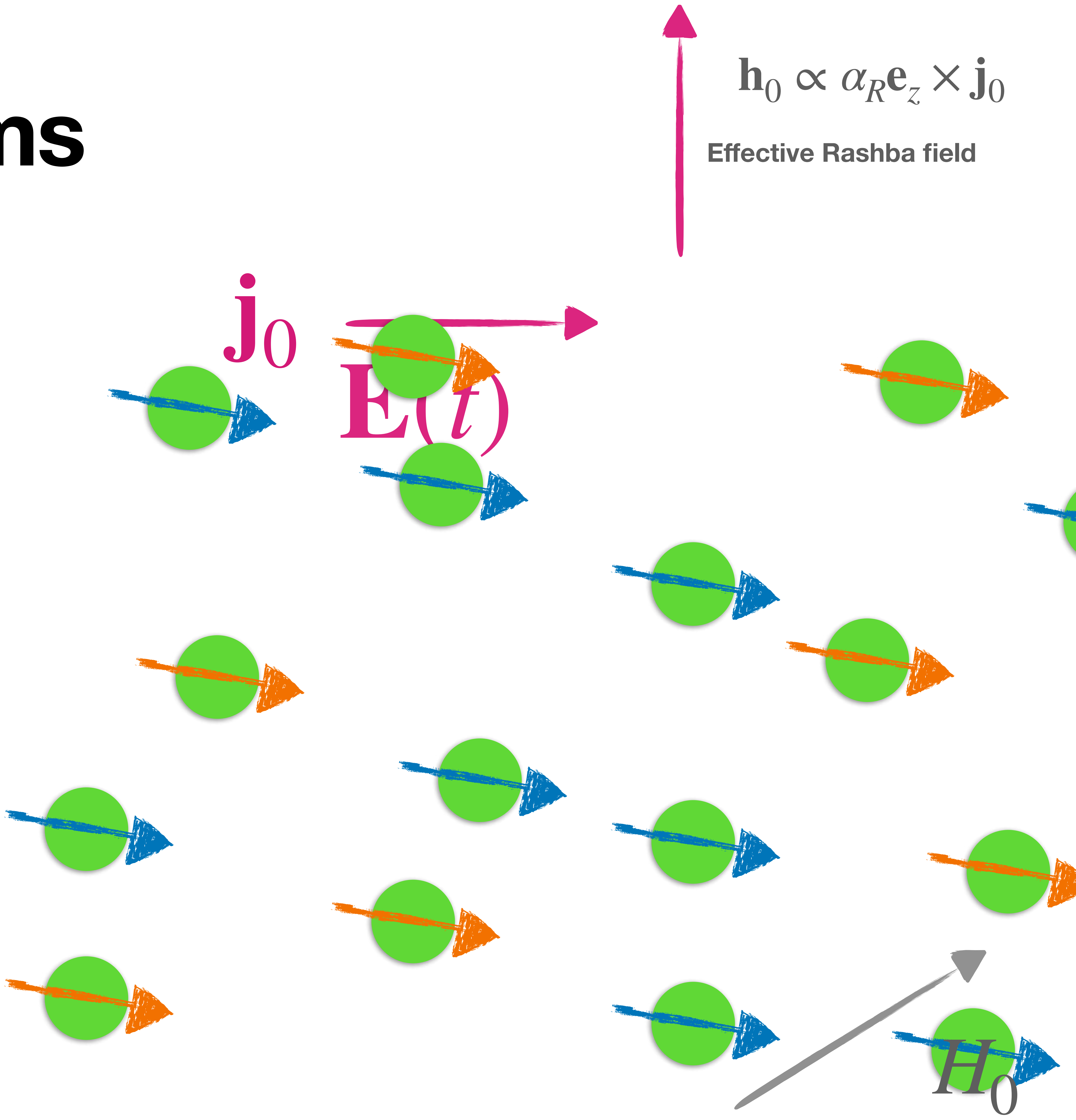


Excitation mechanisms

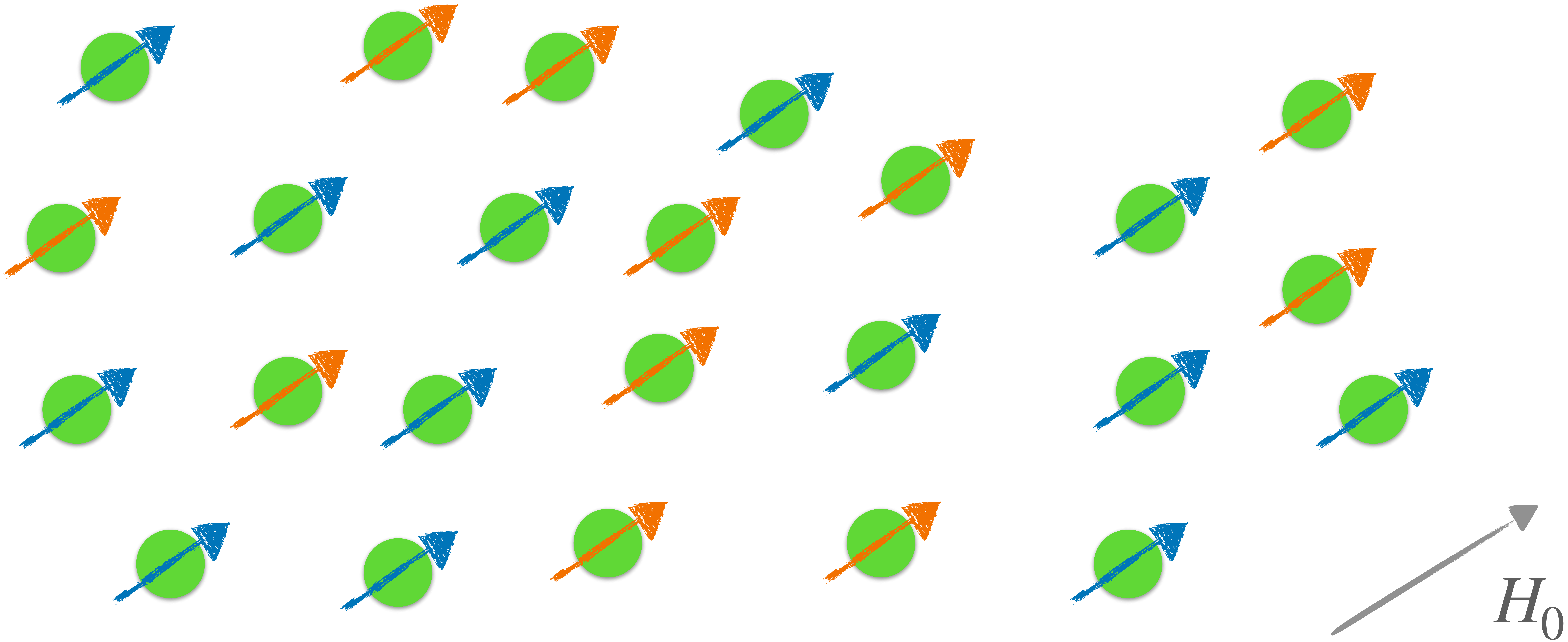
Spin mode (Rashba torque)

s mode



Excitation mechanisms

Valley Spin mode (anomalous Rashba torque)



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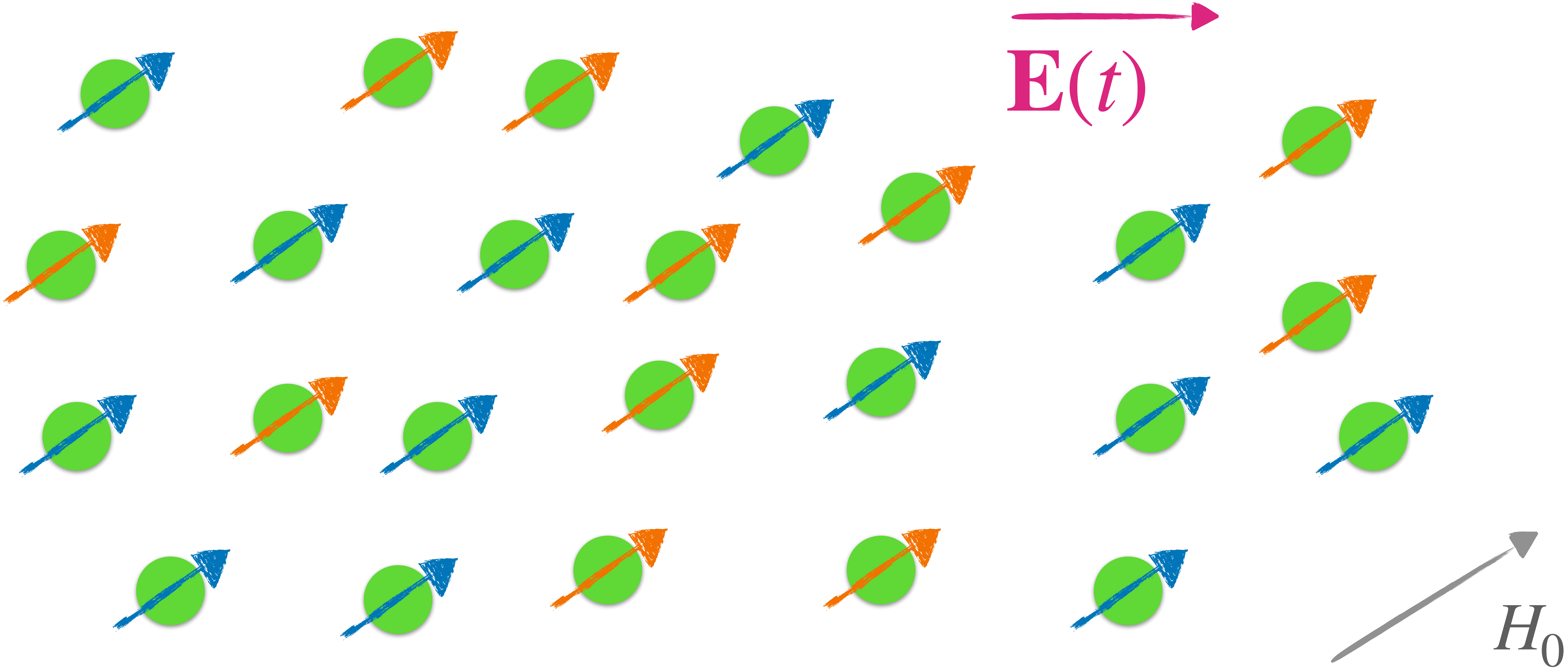

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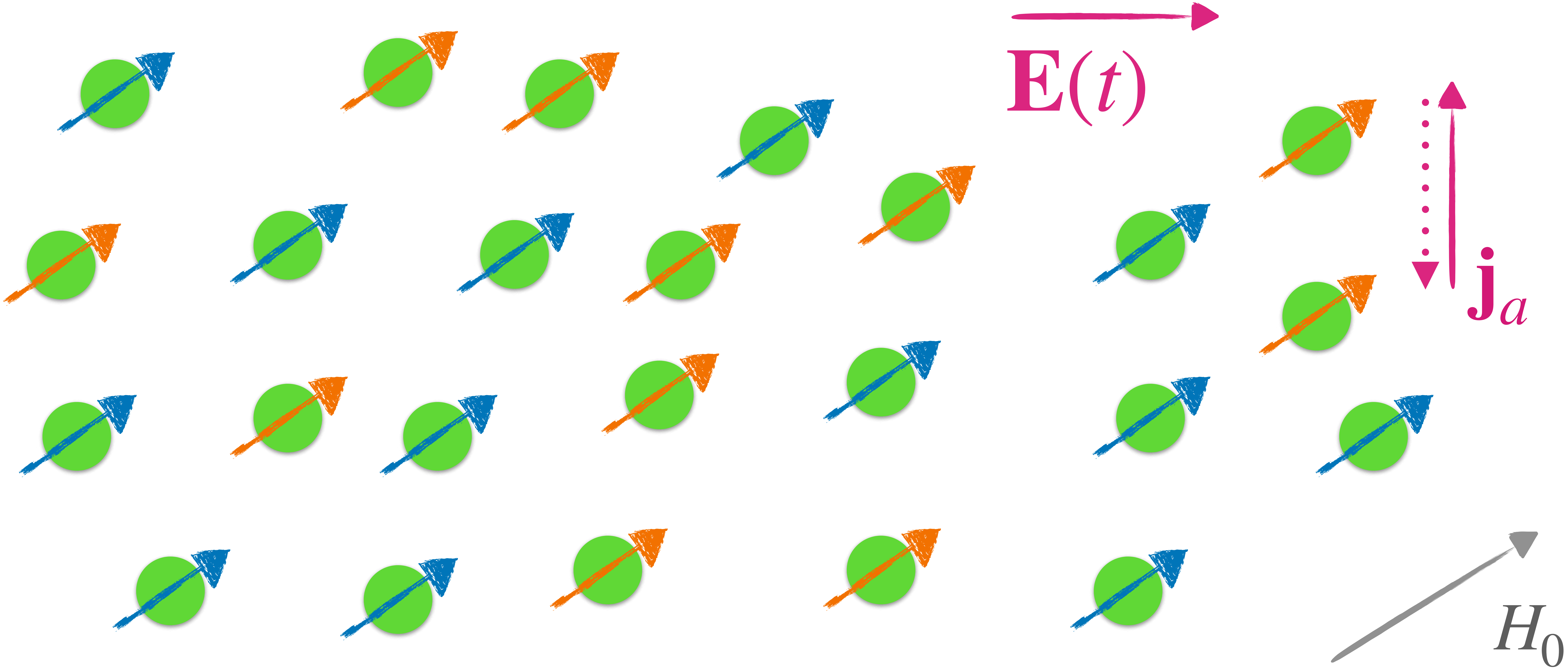

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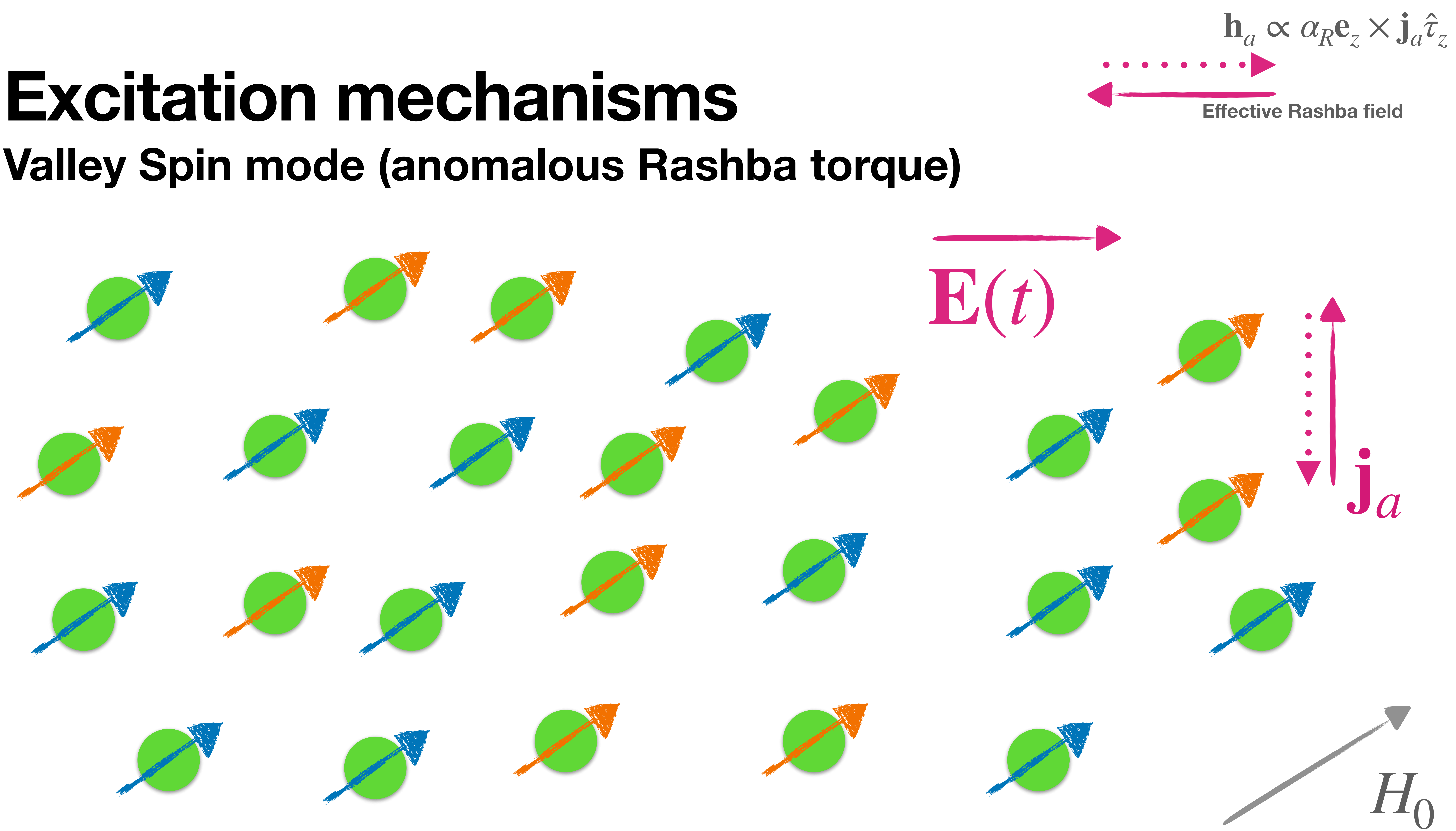

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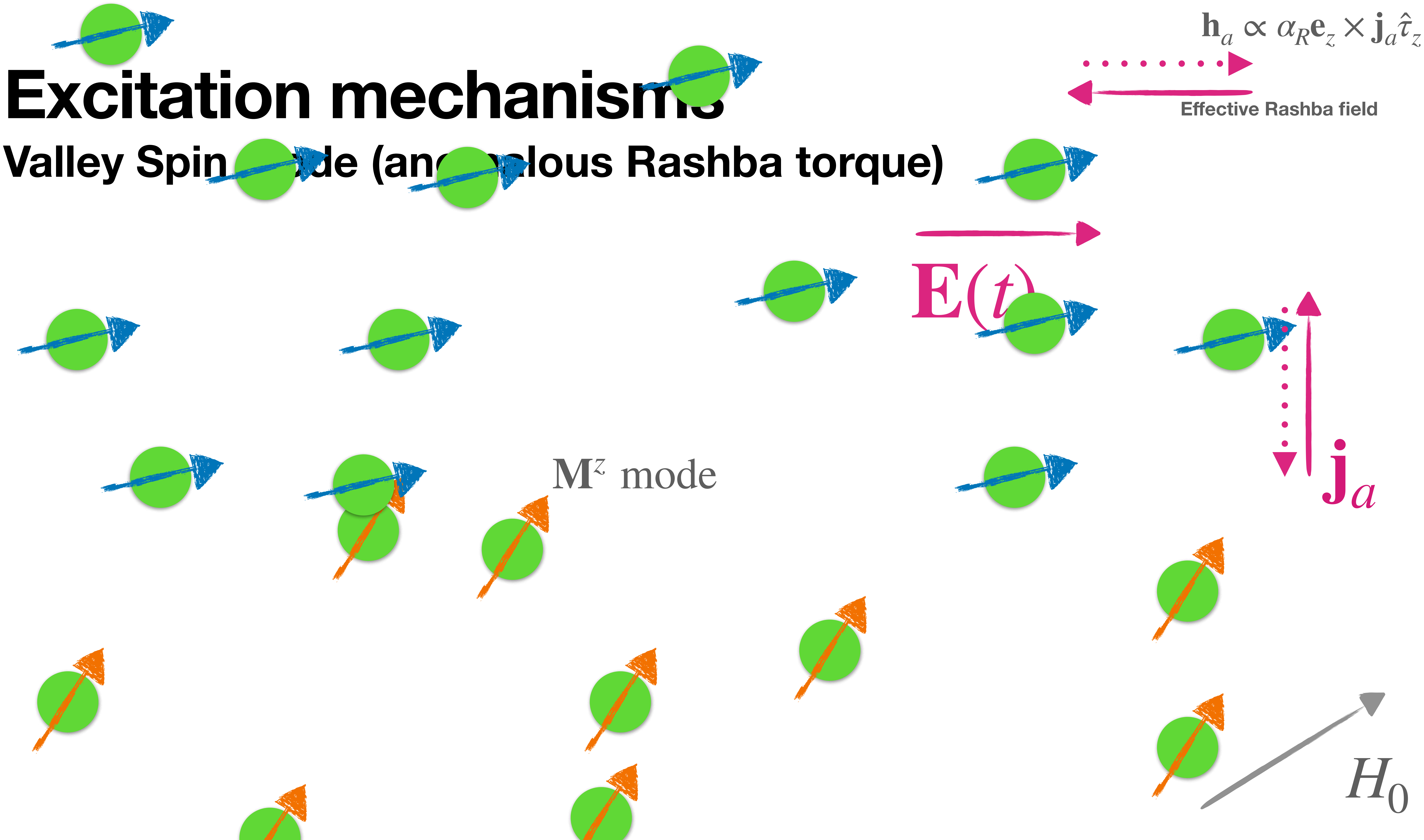

How does one see these modes?

How does one see these modes?

One way is absorption peaks in the optical conductivity $\Re\sigma_{xx}(\omega), \Re\sigma_{yy}(\omega)$

Dissipative current from kinetic equation

$$\partial_t \delta\hat{\rho} + \frac{1}{2}\nabla \cdot \left[[\hat{\mathbf{v}}, \delta\hat{\rho}]_+ - [\delta\hat{\epsilon}, \mathscr{D}\hat{\rho}_{\text{eq}}]_+ \right] + i[\hat{\epsilon}_{\text{eq}}, \delta\hat{\bar{\rho}}] = -e\mathbf{E} \cdot \mathscr{D}\hat{\rho}_{\text{eq}}$$

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$$\sum_{\mathbf{p}} \mathrm{tr} \;\Big\downarrow$$

$$e\sum_{\mathbf{p}}\mathrm{tr}\,\partial_t\delta\hat{\rho} + \nabla\cdot e\sum_{\mathbf{p}}\mathrm{tr}\left(\hat{\mathbf{v}}_{\mathbf{p}}\delta\hat{\rho} - \delta\hat{\epsilon}\mathscr{D}\hat{\rho}_{\rm eq}\right) = 0$$ **Conservation equation**

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$$\partial_t n$$

Dissipative current from kinetic equation

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$\partial_t n$ $\qquad$ $\nabla \cdot \mathbf{j}$

Dissipative current from kinetic equation

$$\partial_t \delta\hat{\rho} + \frac{1}{2}\nabla\cdot\left[[\hat{\mathbf{v}},\delta\hat{\rho}]_+ - [\delta\hat{\epsilon}, \mathscr{D}\hat{\rho}_{\mathrm{eq}}]_+\right] + i[\hat{\epsilon}_{\mathrm{eq}},\delta\hat{\bar{\rho}}] = -e\mathbf{E}\cdot\mathscr{D}\hat{\rho}_{\mathrm{eq}}$$

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Conservation equation

$\partial_t n$ $\qquad$ $\nabla\cdot\mathbf{j}$

$$\downarrow$$

Dissipative Current

$$\mathbf{j} = e\sum_{\mathbf{p}}\mathrm{tr}\left(\hat{\mathbf{v}}_{\mathbf{p}}\delta\hat{\rho} - \delta\hat{\epsilon}\mathscr{D}\hat{\rho}_{\mathrm{eq}}\right)$$

- Given a solution for $\delta\hat{\rho}[\mathbf{E}]$ we thus have the longitudinal conductivity

Dissipative current from the transport eqn

$$\mathbf{j} = e\sum_{\mathbf{p}} \mathrm{tr}\left(\hat{\mathbf{v}}_{\mathbf{p}}\delta\hat{\rho} - \delta\hat{\epsilon}\mathscr{D}\hat{\rho}_{\mathrm{eq}}\right)$$

$$\mathbf{j}_{\nabla} = e\sum_{\mathbf{p}} \mathrm{tr}\,\nabla^{(\mathbf{p})}\hat{\epsilon}_{\mathrm{eq}}\delta\hat{\rho}$$

$$\mathbf{j}_{\delta} = -e\sum_{\mathbf{p}} \mathrm{tr}\,\delta\hat{\epsilon}\,\nabla^{(\mathbf{p})}\hat{\rho}_{\mathrm{eq}}$$

$$\mathbf{j}_{\mathscr{A}} = -ie\sum_{\mathbf{p}} \mathrm{tr}[\hat{\mathscr{A}}, \hat{\rho}_{\mathrm{eq}}]\delta\hat{\bar{\rho}}$$

Geometric effects

- Given a solution for $\delta\hat{\rho}[\mathbf{E}]$ we thus have the longitudinal conductivity

$$\zeta = 1 + \frac{E_F - \sqrt{E_F^2 - \Delta^2}}{E_F}$$

Contributions to conductivity

- Spin-valley modes contribute resonant peaks to the real part of the conductivity

$$\Re\sigma_1^{ii} = \frac{1}{2}\pi e^2 G_s G_v \nu_F \tilde{\alpha}_R^2 W_1^{ii} A_{m1}(\omega^2),$$

$$\Re\sigma_0^{ii} = \frac{1}{2}\pi e^2 G_s G_v \nu_F \tilde{\alpha}_R^2 W_0^{ii} A_{s0}(\omega^2),$$

$$\Re\sigma_2 = \frac{1}{2}\pi e^2 G_s G_v \nu_F \tilde{\alpha}_R^2 W_2 A_{s2}(\omega^2)$$

!

$$W_1^{xx} = 2\tilde{\lambda}(1 + F_1^s) 2\pi\nu_F |\Omega_0^z| \left(1 + \frac{1}{2}\gamma_1^{-1}\frac{1 + F_1^s}{1 + F_0^{mz}}\right)(1 - \gamma_1),$$

$$W_1^{yy} = 2\tilde{\lambda}(1 + F_1^s) 2\pi\nu_F |\Omega_0^z|, \left[\frac{\omega_{s1} - \omega_{m1}}{\omega_{m0} - \omega_{m1}} + (1 - \gamma_1)\left(1 + \frac{1}{2}\gamma_1^{-1}\frac{1 + F_1^s}{1 + F_0^{mz}}\right)\right]$$

$$W_0^{xx} = \zeta\frac{\omega_{1s}}{2\omega_s} + 2\tilde{\lambda}(2\pi\nu_F |\Omega_0^z|)\left(1 + F_1^s - \zeta\right)$$

$$W_0^{yy} = 2\pi\nu_F |\Omega_0^z| \left\{ 2\pi\nu_F |\Omega_0^z| \frac{\omega_{m0}^2}{1 + F_0^{mz}} + 2\tilde{\lambda}\left[(1 + F_1^{mz})\frac{\omega_{s1} - \omega_{m0}}{\omega_{m0} - \omega_{m1}} - \frac{1}{1 + F_0^{mz}}\zeta\right]\right\};$$

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Dissipative optical conductivity

$$A_{\mu l}(\omega^2) \equiv \omega_{\mu l}\delta(\omega^2 - \omega_{\mu l}^2)$$

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Spectral function

$$\Re\sigma_l^{ii} = \frac{1}{2}\pi e^2 G_s G_v \nu_F \tilde{\alpha}_R^2 W_l^{ii} A_l(\omega^2)$$

EDSR energy scale

Spectral weight

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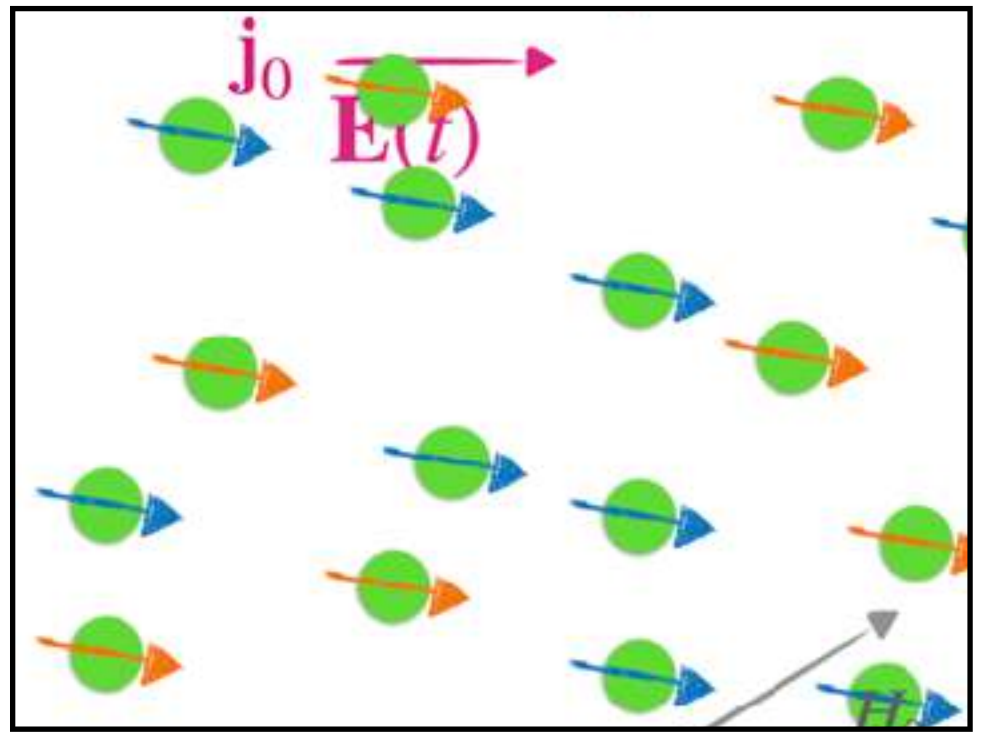
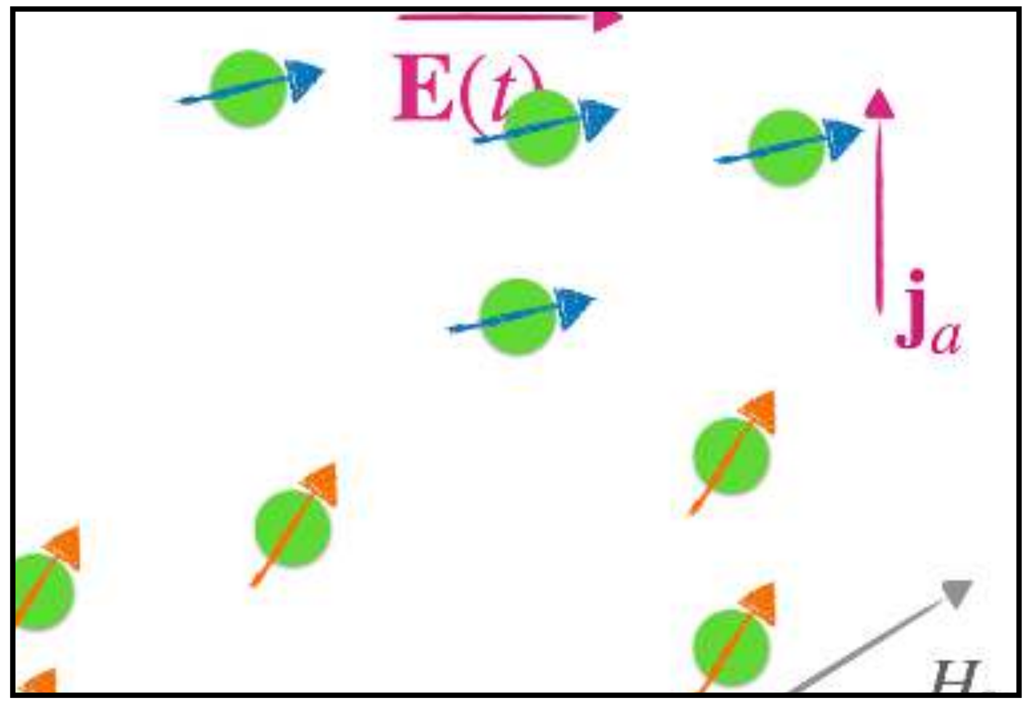
$\mathbf{S}$





$\mathbf{M}^z$

$\mathbf{E}\cdot\mathbf{H}$ $\quad$ $|\mathbf{E}\times\mathbf{H}|$

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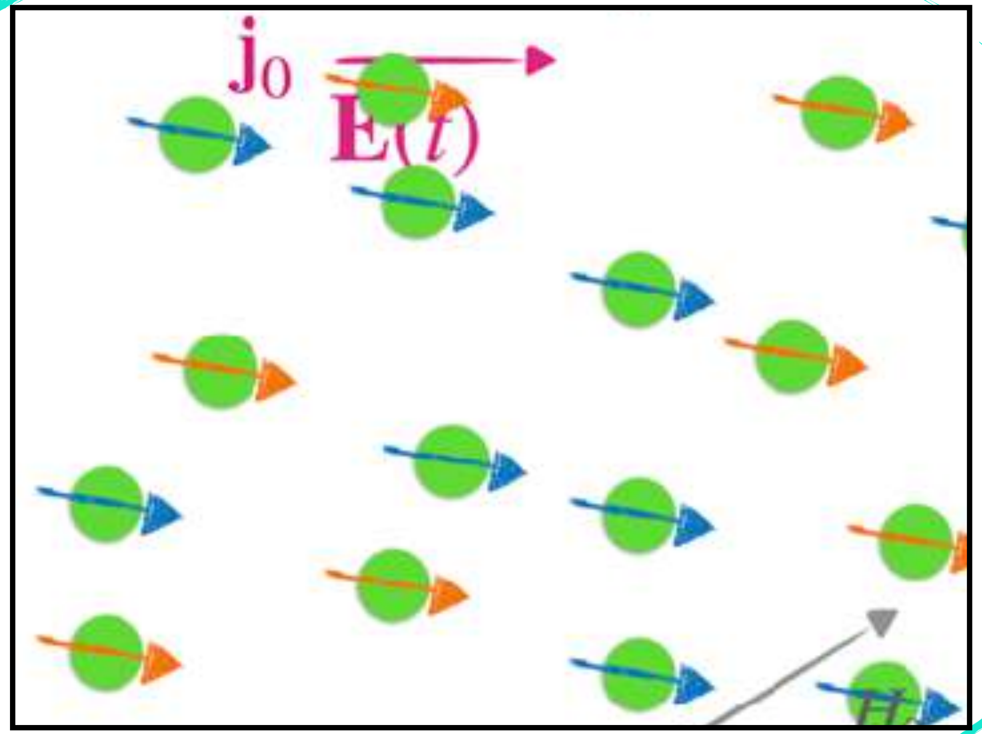

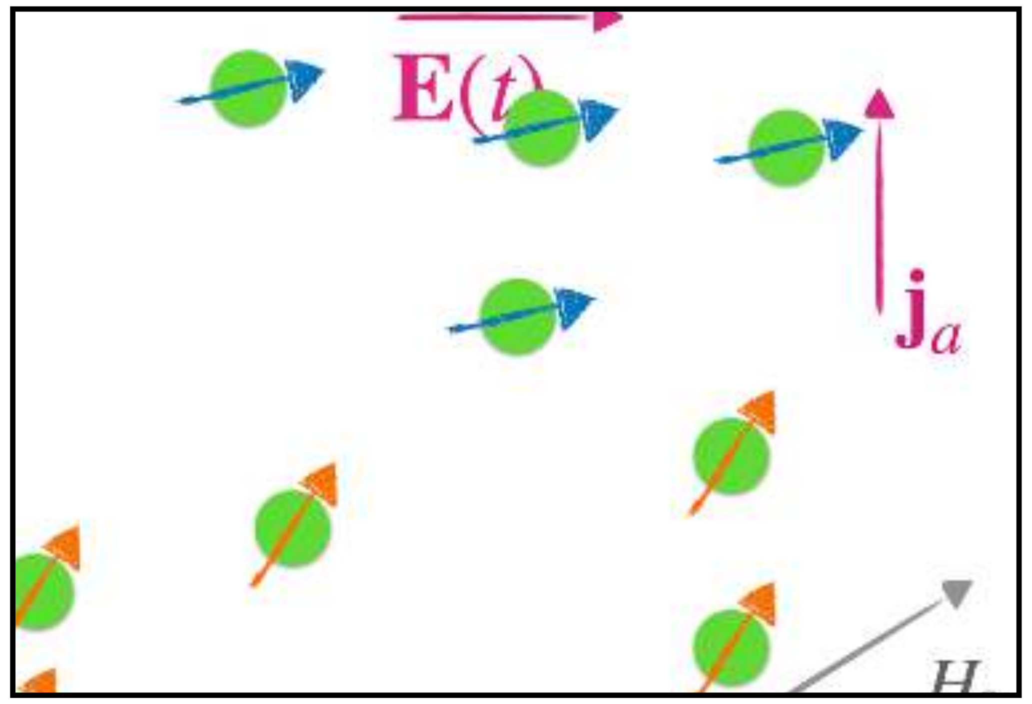

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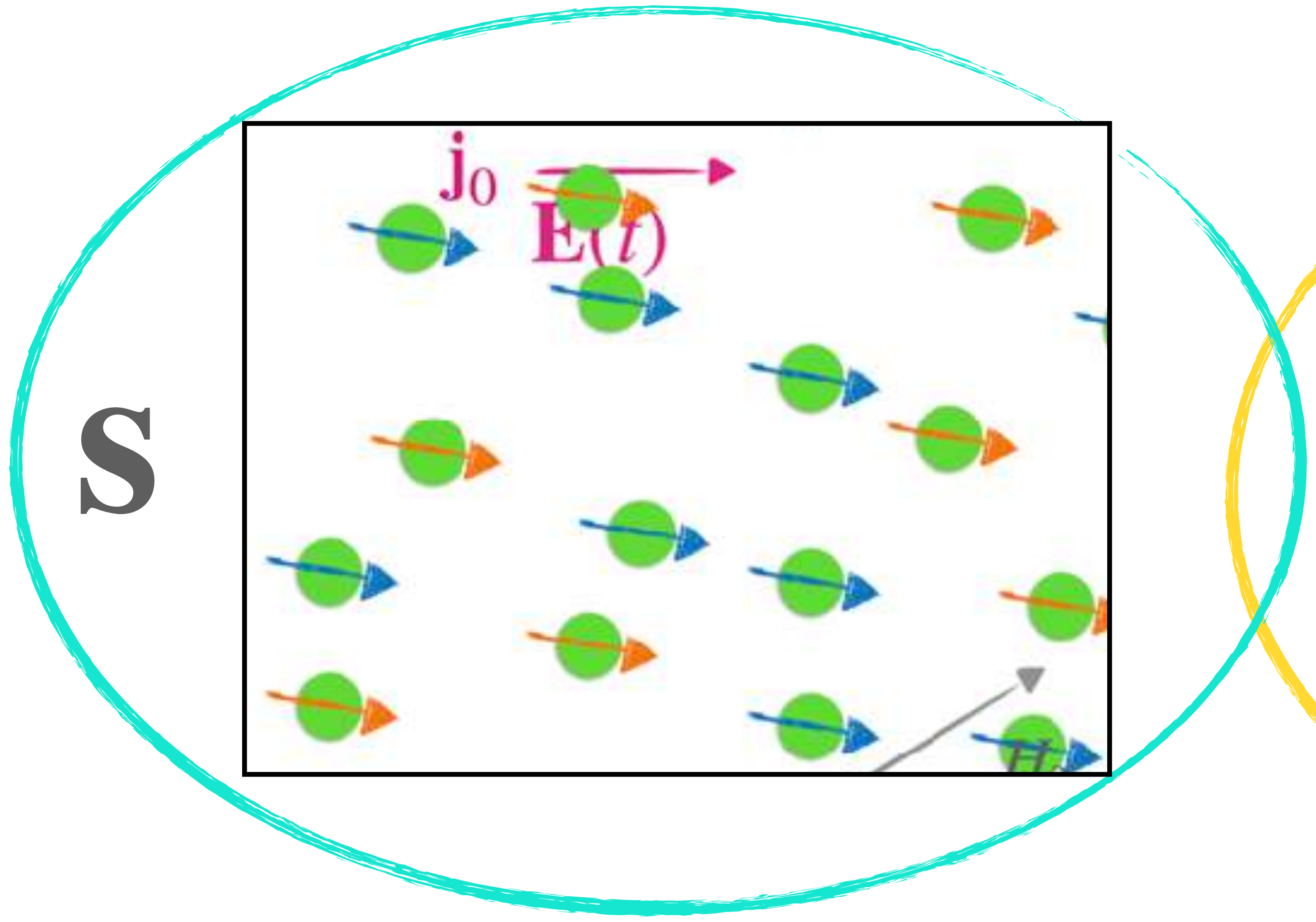

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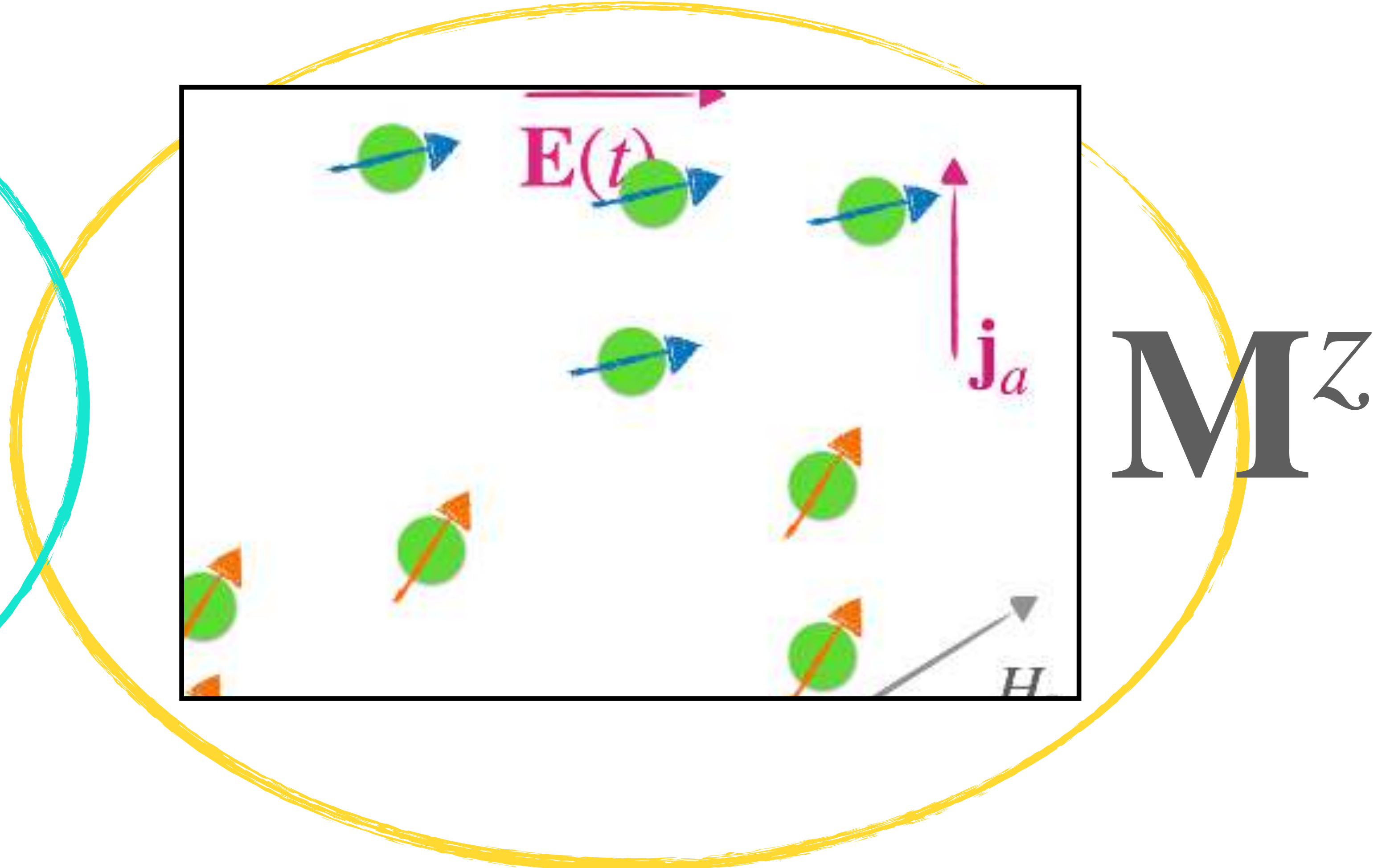

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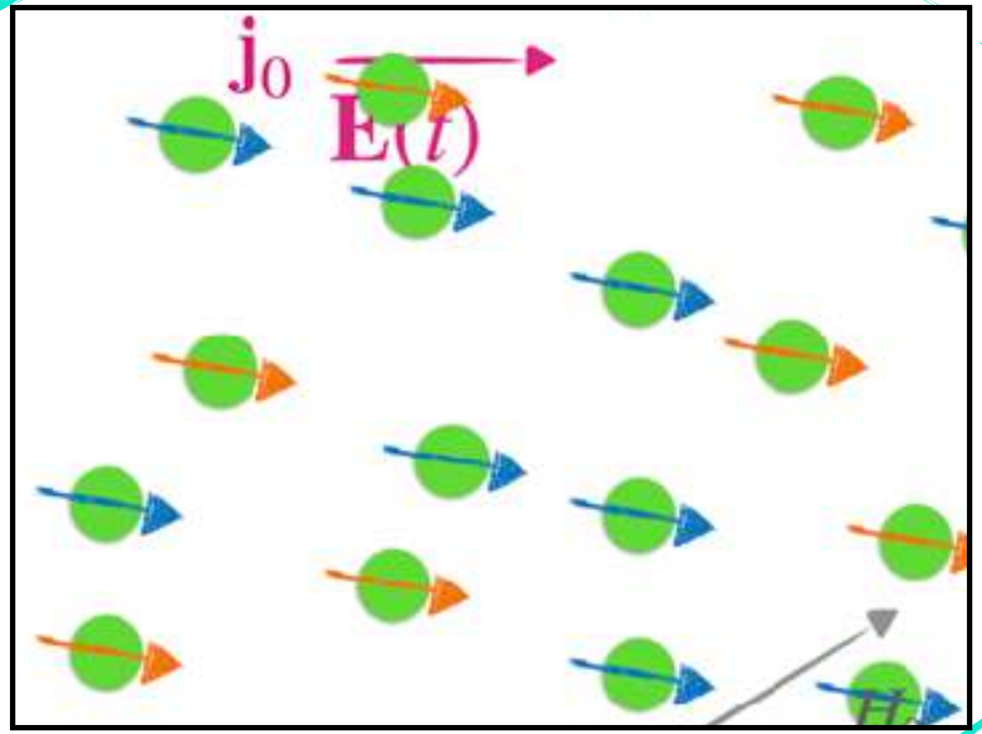

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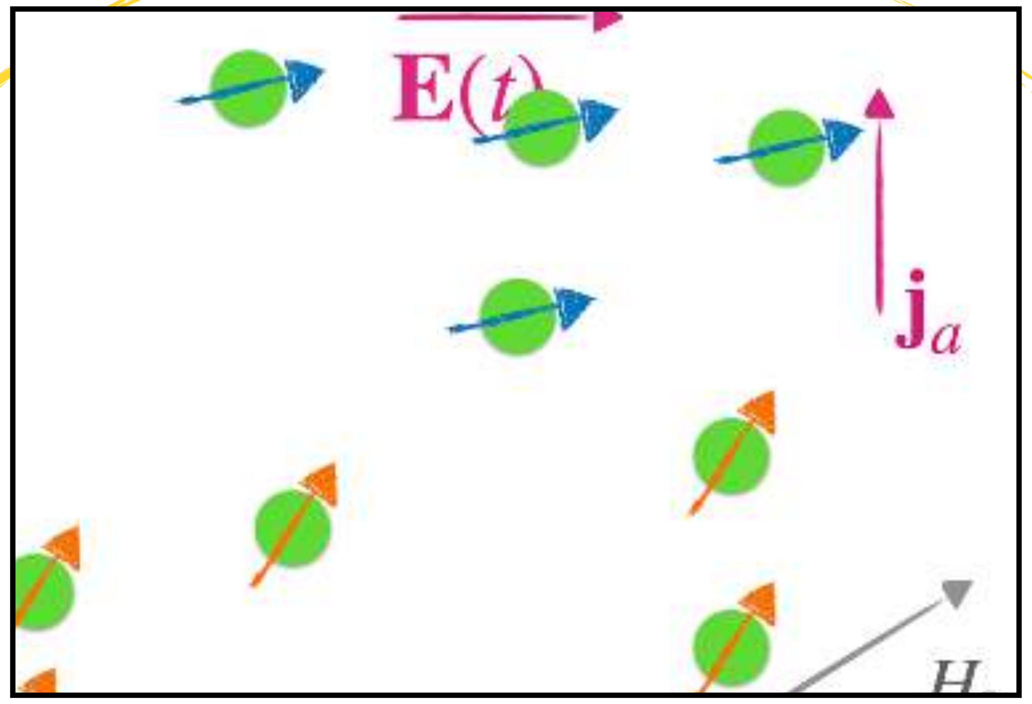

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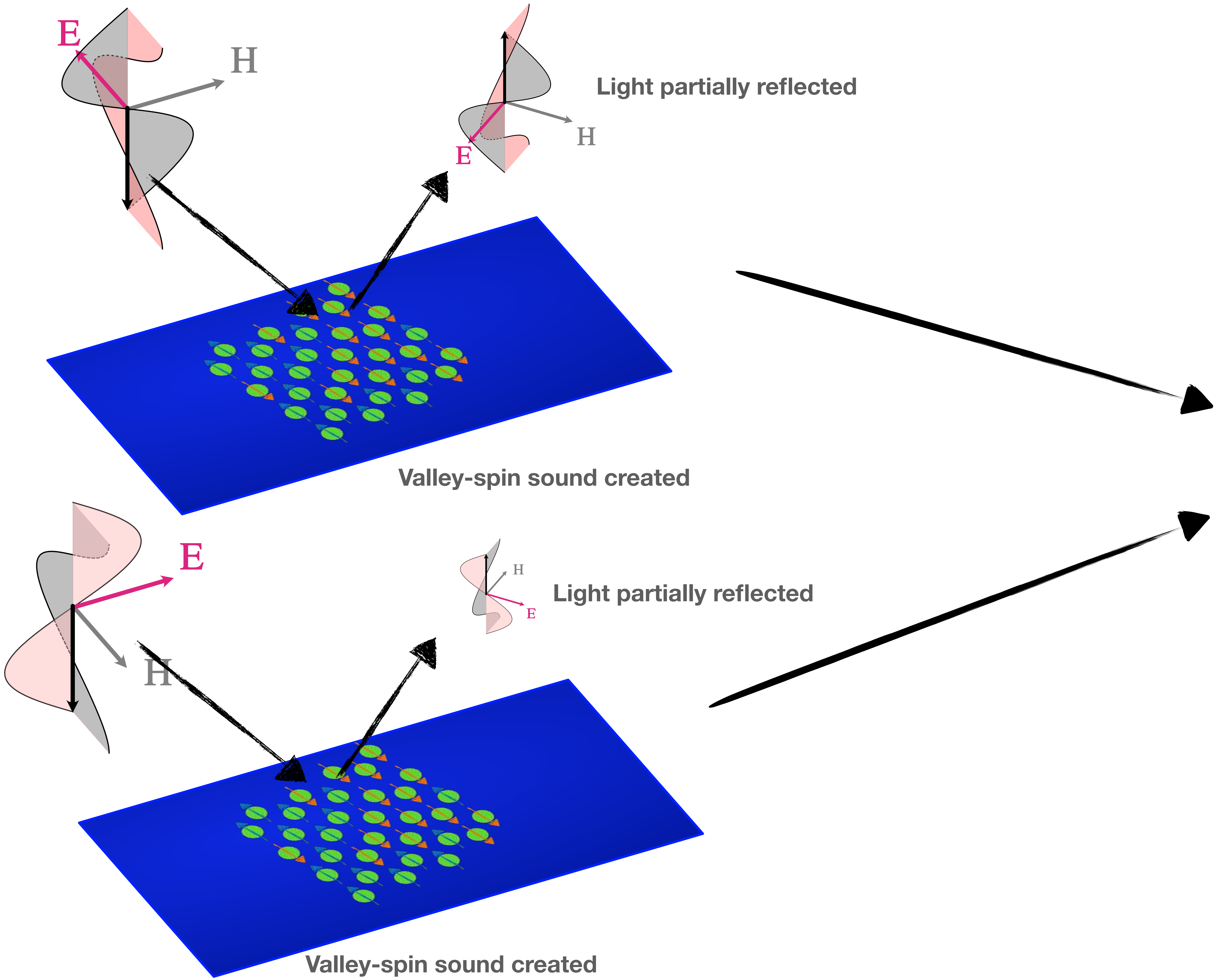
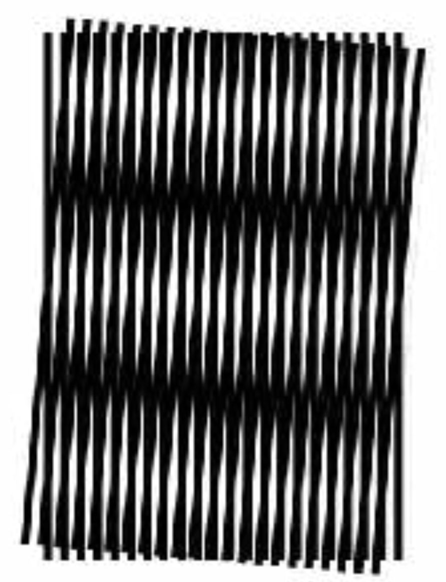


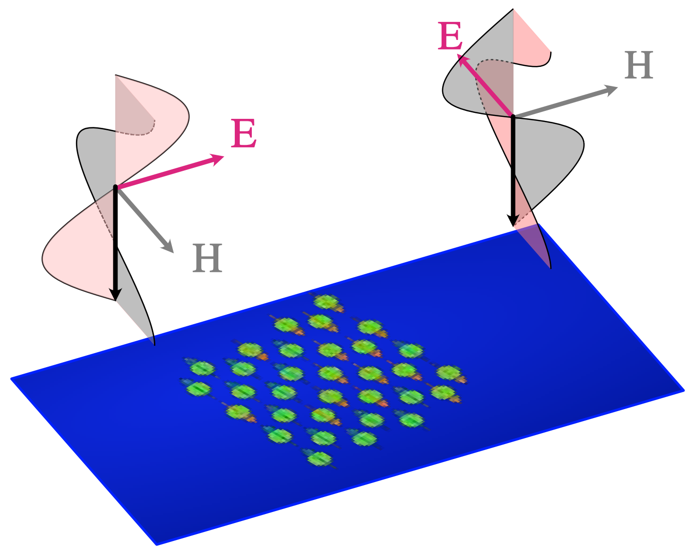
Ratio of absorptions tells us about the quantum geometry

Future directions

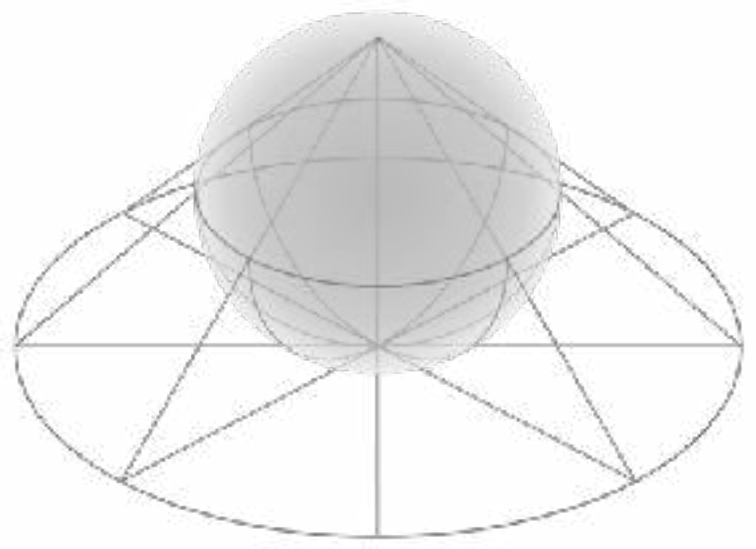
- Extend these techniques to systems other than metals: superconductors in particular



- Apply these techniques to twisted n-layer (moiré) systems



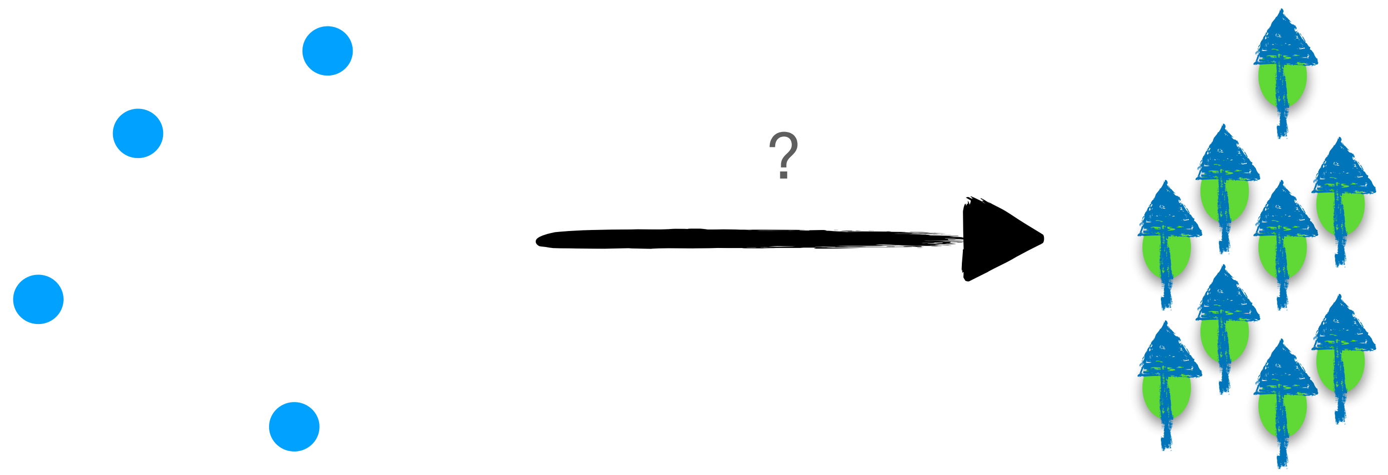
- Search for other experimental probes of quantum geometry



- Relating geometric formulations of quantum mechanics and semi-classical quasiparticle descriptions

Future directions

- **Extend these techniques to systems other than metals: superconductors in particular**
  - We can have spontaneous symmetry breaking, e.g. magnetism, superconductivity, …
  - A modified Fermi-liquid-like description still applies



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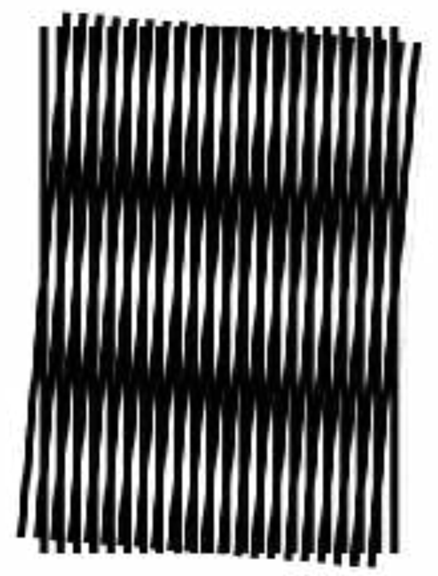

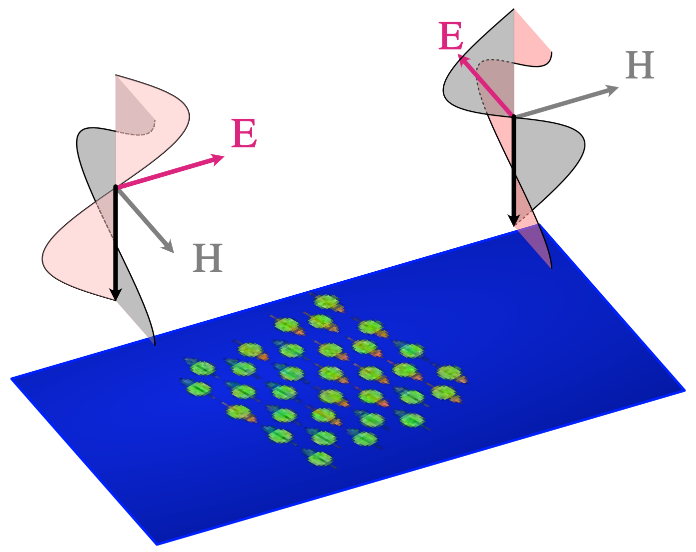

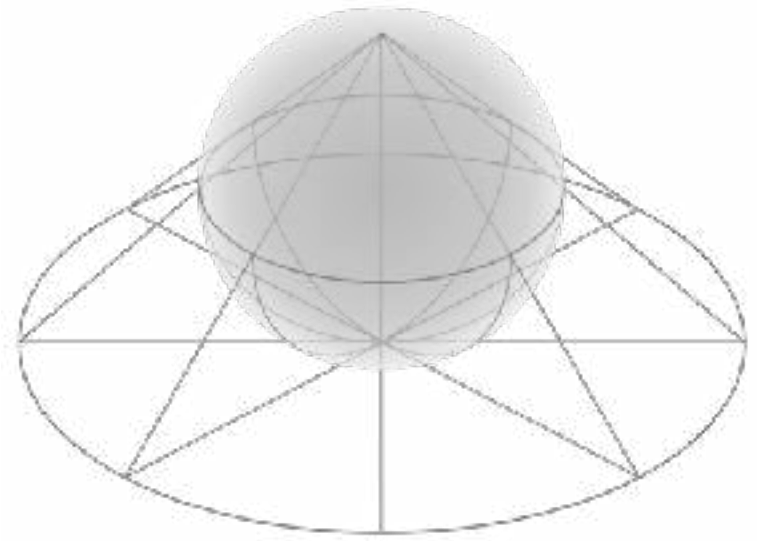

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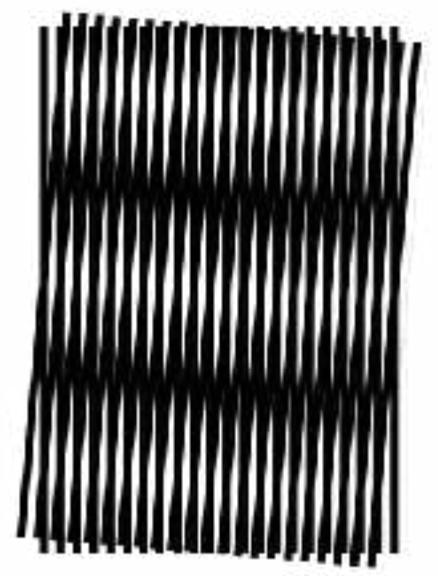

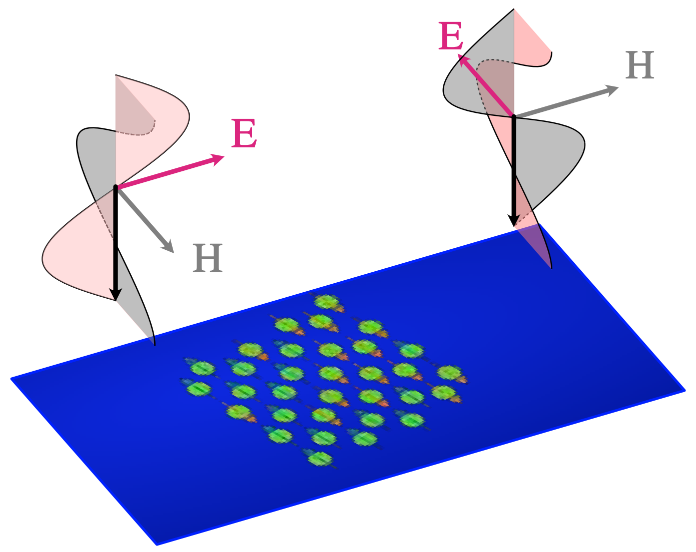

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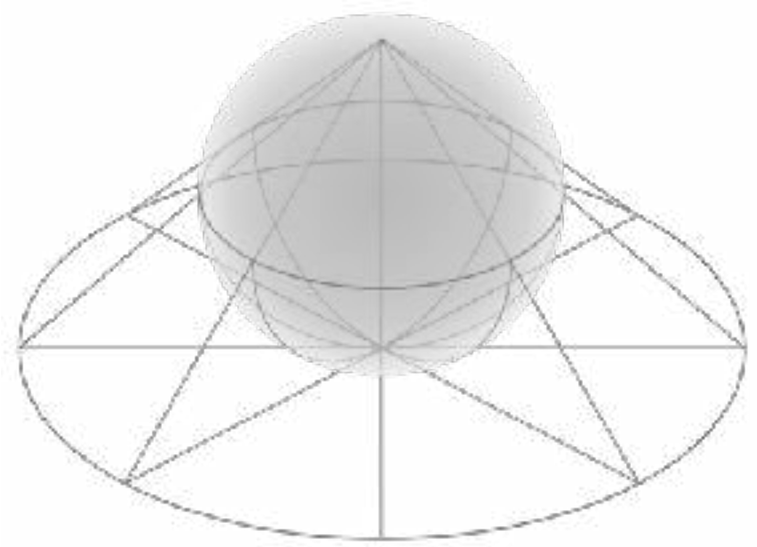

Summary

- Symmetry dictated Fermi liquid theory of graphene
- Neutral zero sound and first sound in graphene are overdamped for all spin-valley channels.
- Transport of spin-valley quantum numbers is generically diffusive

ZMR, Fal'ko, Glazman
PRB **103**, 075422 (2021)
10.1103/PhysRevB.103.075422

- External Zeeman and/or extrinsic SOC promote diffusive spin-valley excitations of graphene to well-defined oscillatory modes
- Both contribute absorption peaks to the optical conductivity
- Absorption measurement allows probing Berry curvature of system

ZMR, Maslov, Glazman
Under Review w/ PRL
arXiv:2107.02819

Summary

- Symmetry dictated Fermi liquid theory of graphene
- Neutral zero sound and first sound in graphene are overdamped for all spin-valley channels.
- Transport of spin-valley quantum numbers is generically diffusive

***ZMR*, Fal'ko, Glazman**
PRB **103**, 075422 (2021)
10.1103/PhysRevB.103.075422

- External Zeeman and/or extrinsic SOC promote diffusive spin-valley excitations of graphene to well-defined oscillatory modes
- Both contribute absorption peaks to the optical conductivity
- Absorption measurement allows probing Berry curvature of system

***ZMR*, Maslov, Glazman**
Under Review w/ PRL
arXiv:2107.02819

Thank you for your attention!

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