Cavity superconductor Higgs-polaritons

Zachary Raines, Andrew Allocca, Victor Galitski

APS March Meeting

Mar. 5, 2019

Collaborators

Victor Galitski

Andrew Allocca

A brief outline

- *Context* - Exciton-Polaritons (but also c.f. previous talk)
- *Construction* - Higgs Photon coupling
- *Results* - Cavity Higgs-Polaritons

Cavity exciton-polaritons

Letter | Published: 29 August 2010

Spontaneous formation and optical manipulation of extended polariton condensates

E. Wertz, L. Ferrier, D. D. Solnyshkov, R. Johne, D. Sanvitto, A. Lemaître, I. Sagnes, R. Grousson, A. V. Kavokin, P. Senellart, G. Malpuech & J. Bloch

Nature Physics **6**, 860–864 (2010) | Download Citation

Vol 443|28 September 2006|**doi:10.1038/nature05131** nature

ARTICLES

Bose–Einstein condensation of exciton polaritons

J. Kasprzak[1], M. Richard[2], S. Kundermann[2], A. Baas[2], P. Jeambrun[2], J. M. J. Keeling[3], F. M. Marchetti[4], M. H. Szymańska[5], R. André[1], J. L. Staehli[2], V. Savona[2], P. B. Littlewood[4], B. Deveaud[2] & Le Si Dang[1]

PRL **110**, 196406 (2013) PHYSICAL REVIEW LETTERS week ending 10 MAY 2013

From Excitonic to Photonic Polariton Condensate in a ZnO-Based Microcavity

Feng Li,[1,2] L. Orosz,[3] O. Kamoun,[4] S. Bouchoule,[5] C. Brimont,[4] P. Disseix,[3] T. Guillet,[4] X. Lafosse,[5] M. Leroux,[1] J. Leymarie,[3] M. Mexis,[4] M. Mihailovic,[3] G. Patriarche,[5] F. Réveret,[3] D. Solnyshkov,[3] J. Zuniga-Perez,[1] and G. Malpuech[3]
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[3]*Institut Pascal, PHOTON-N2, Clermont Université, CNRS and Université Blaise Pascal, 24 Avenue des Landais, 63177 Aubière cedex, France*
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(Received 20 December 2012; revised manuscript received 1 February 2013; published 10 May 2013)

VOLUME 69, NUMBER 23 PHYSICAL REVIEW LETTERS 7 DECEMBER 1992

Observation of the Coupled Exciton-Photon Mode Splitting in a Semiconductor Quantum Microcavity

C. Weisbuch,[(a)] M. Nishioka,[(b)] A. Ishikawa, and Y. Arakawa
Research Center for Advanced Science and Technology, University of Tokyo, 4-6-1 Meguro-ku, Tokyo 153, Japan
(Received 12 May 1992)

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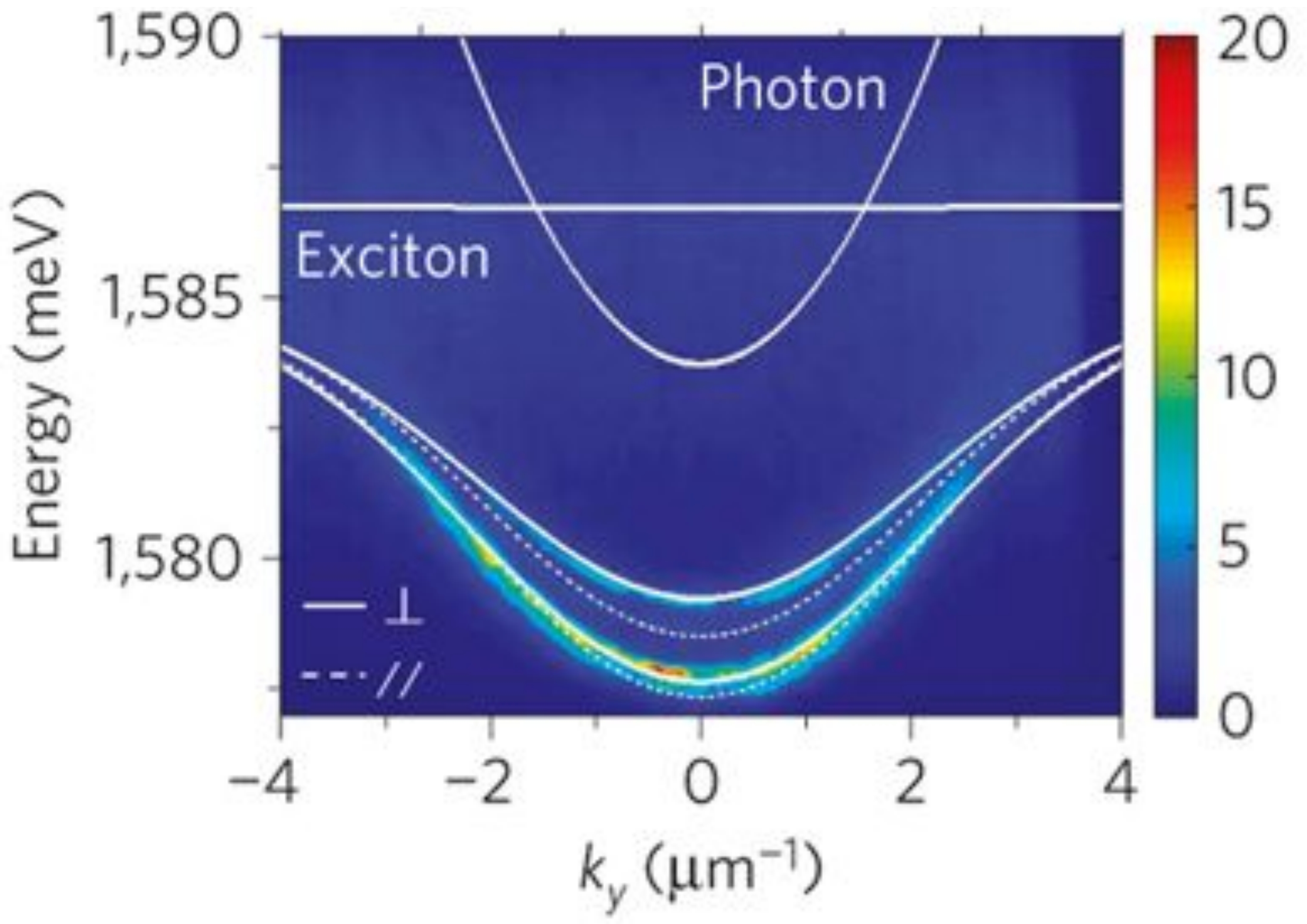

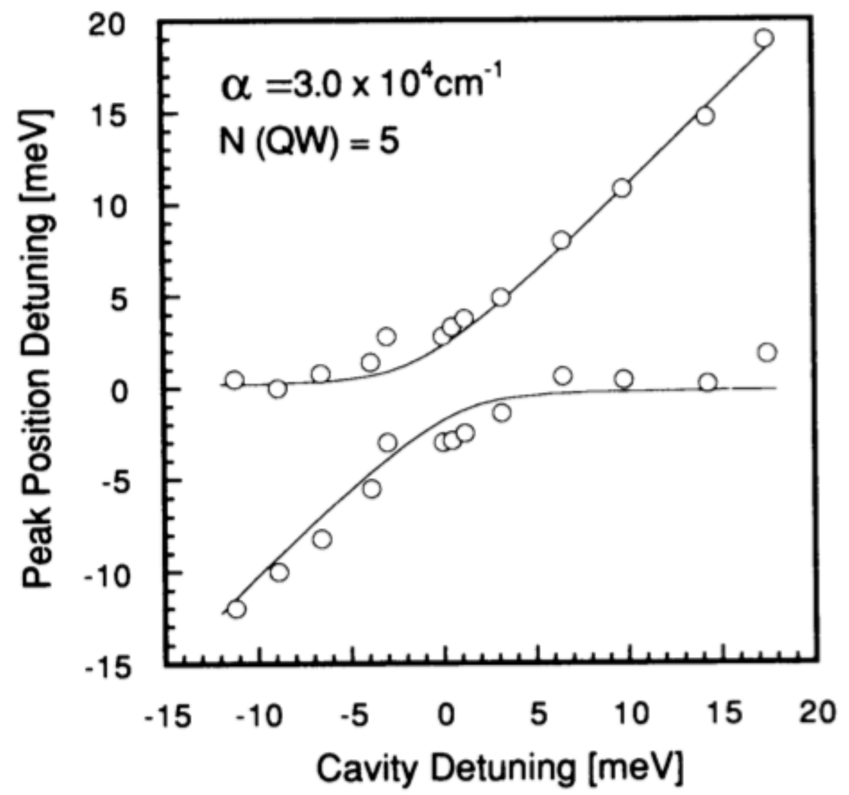

FIG. 3. Reflectivity peak positions as a function of cavity detuning for a five-quantum-well sample at $T=5$ K. The theoretical fit is obtained through a standard multiple-interference analysis of the DBR–Fabry-Pérot–quantum-well structure.

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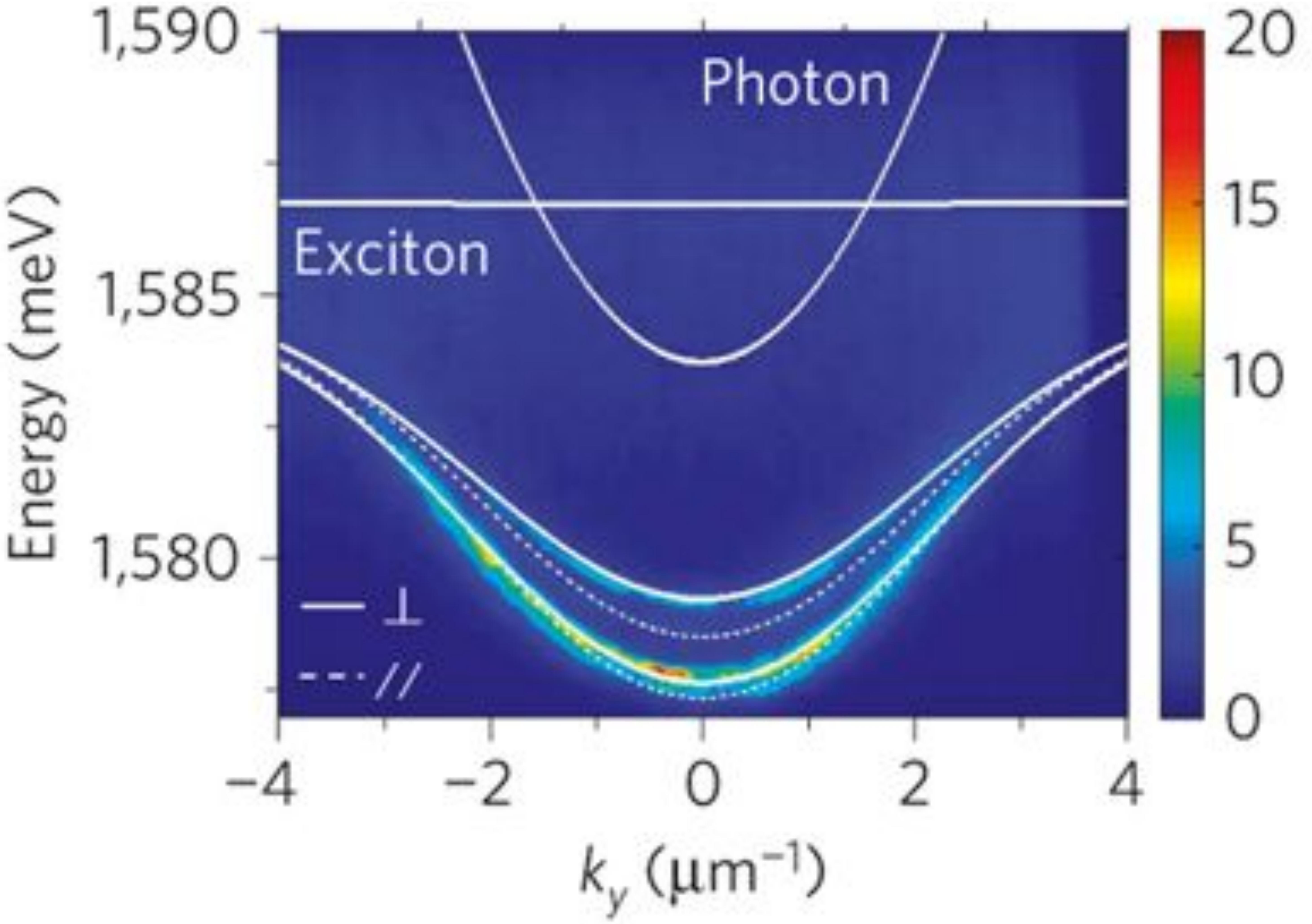

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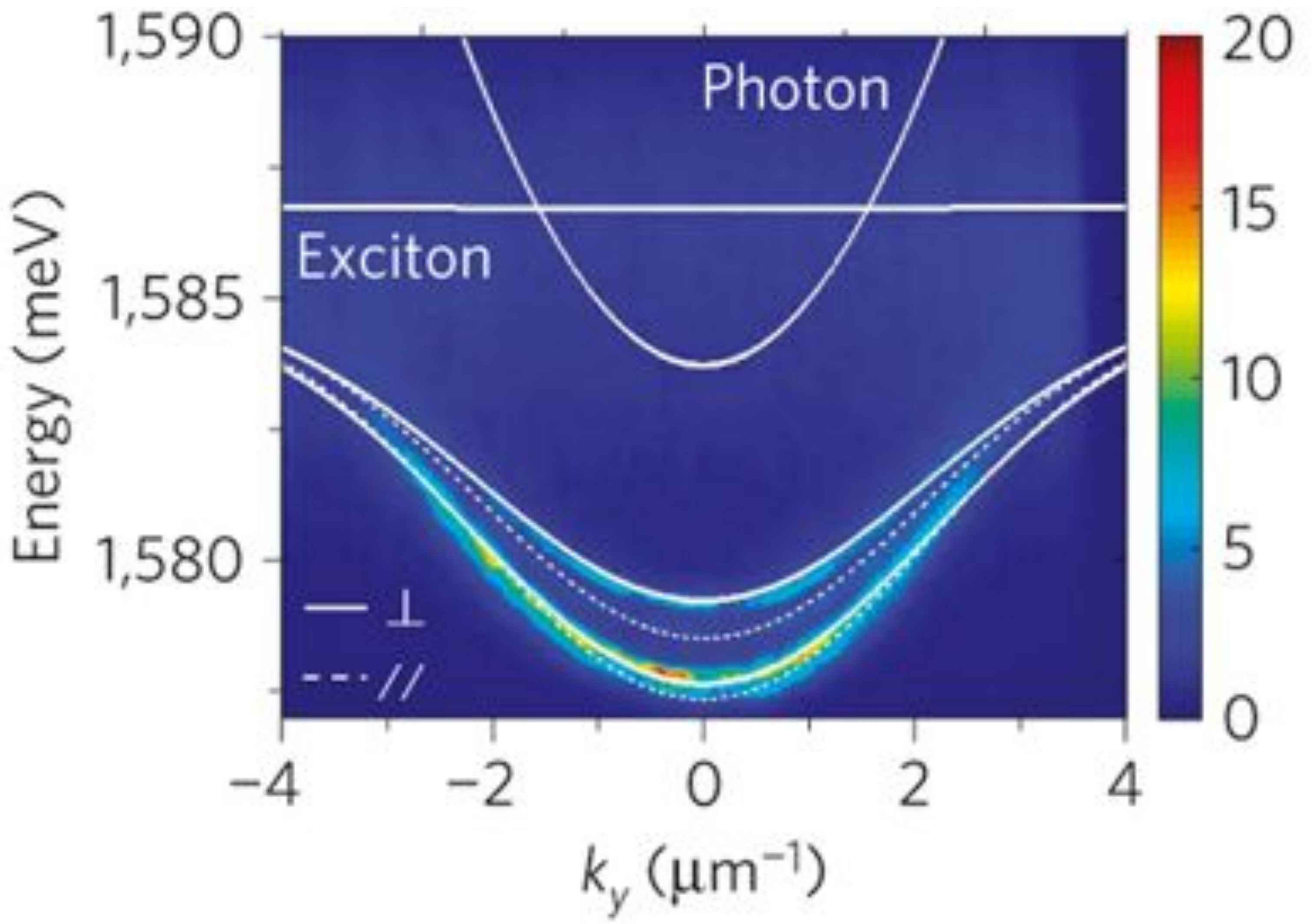


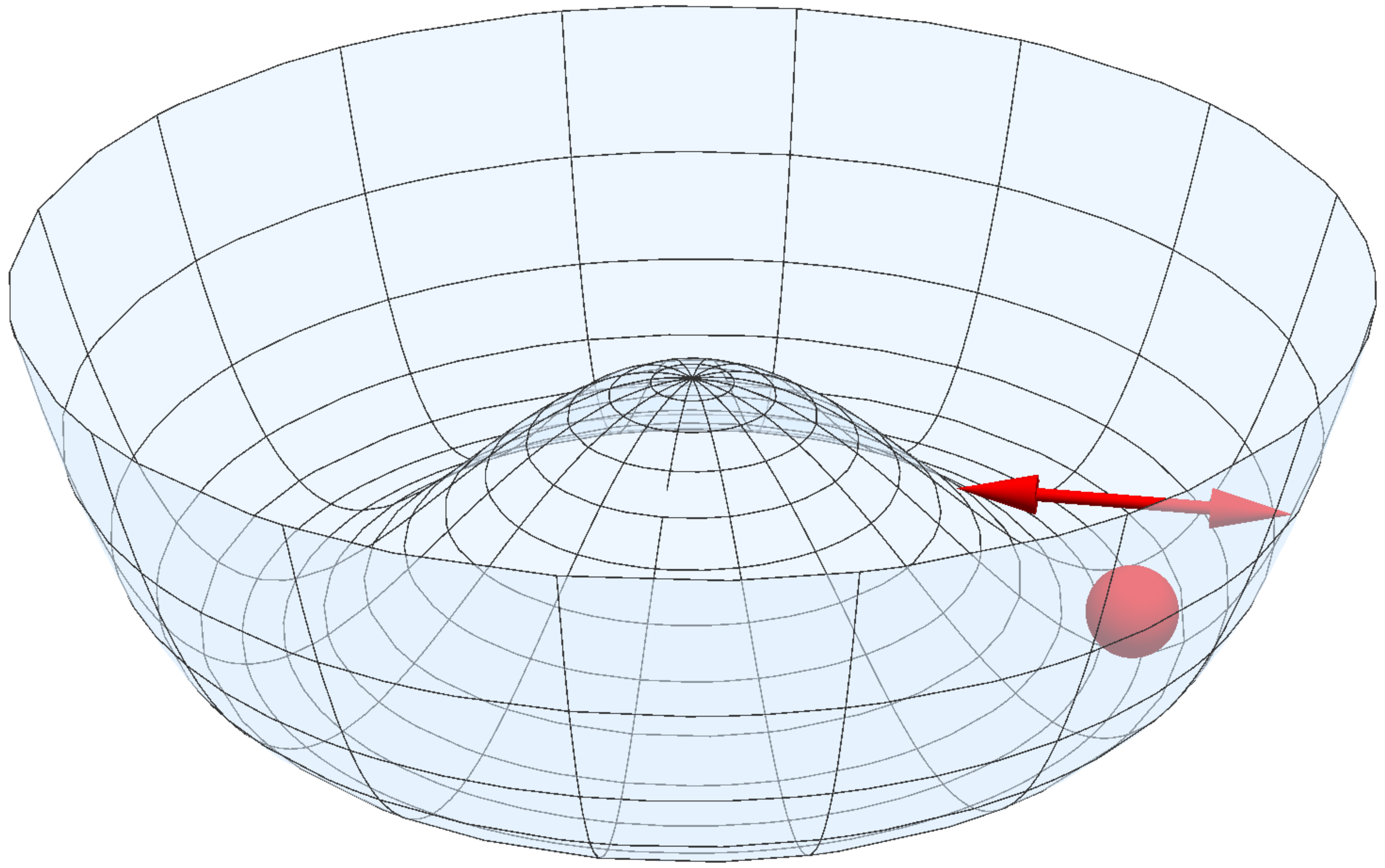
$$H = \begin{pmatrix} \Omega_{\text{exc}} & g \\ g & \omega_{\mathbf{q}} \end{pmatrix}$$

- Exciton-polaritons have been studied for 60 years — Hopfield *Phys. Rev.* **112**, 1555-1567 (1958).
- The physics can be qualitative well described by a theory of coupled bosons
- Not only can polaritonic states be formed, but condensation is observed up to room temperature in some cases — Plumhof et al. *Nature Materials* **13**, 247-252 (2014).

Coupling the Higgs mode to light

$$|\Delta(q,\Omega)| = \Delta_0 + \delta\Delta(q,\Omega)$$

$$\Omega_{\text{Higgs}} = 2\Delta_0$$



- The Higgs mode is the amplitude mode of the superconducting order parameter
- In general, photons do not couple to the Higgs mode of a superconductor as it has no charge or dipole moments.

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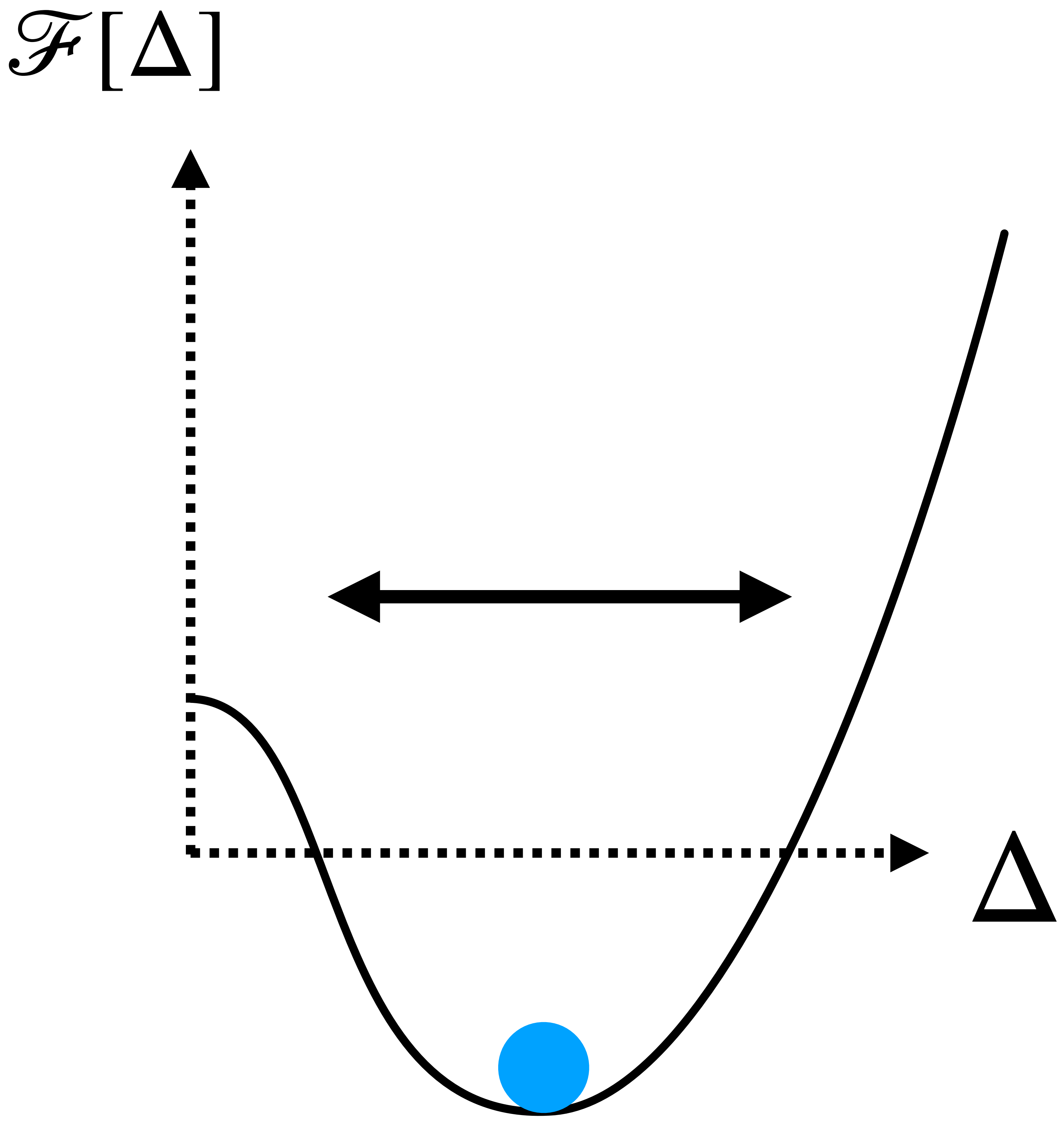
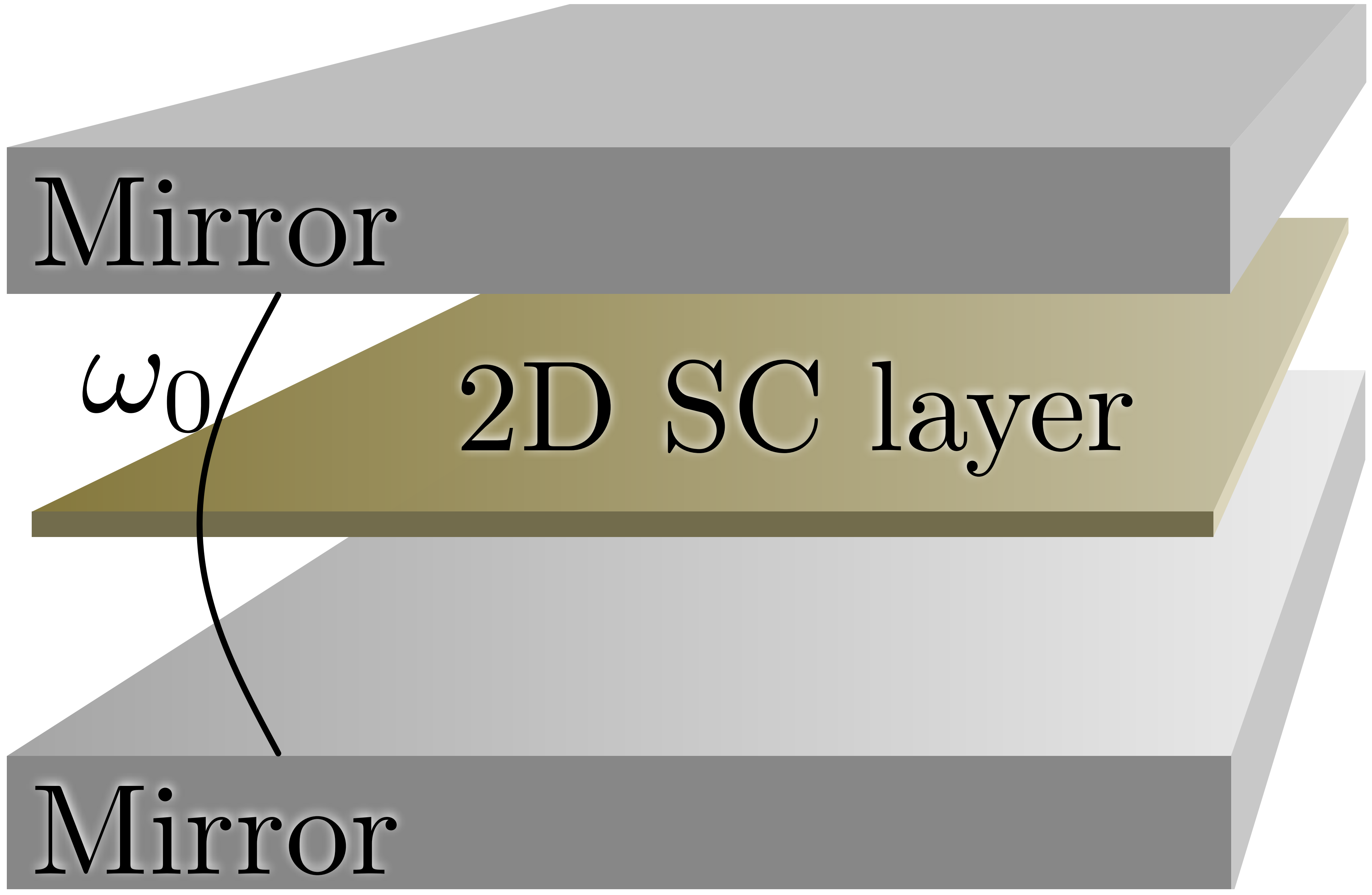


Photonic cavities: important properties for us

- Gapped photons due to finite size quantization
- This allows photon energies to be tuned into resonance with gapped solid state processes
- Tailoring of the wave functions allows for stronger light-matter coupling

$$\omega_q = \sqrt{c^2\left[q^2 + \left(\frac{n\pi}{L}\right)^2\right]} \sim mc^2 + \frac{q^2}{2m}$$



Linear coupling to the Higgs mode

System	Higgs-photon coupling?
Clean	$= 0$
Supercurrent	$= 0$
Disorder & Supercurrent	$\neq 0$

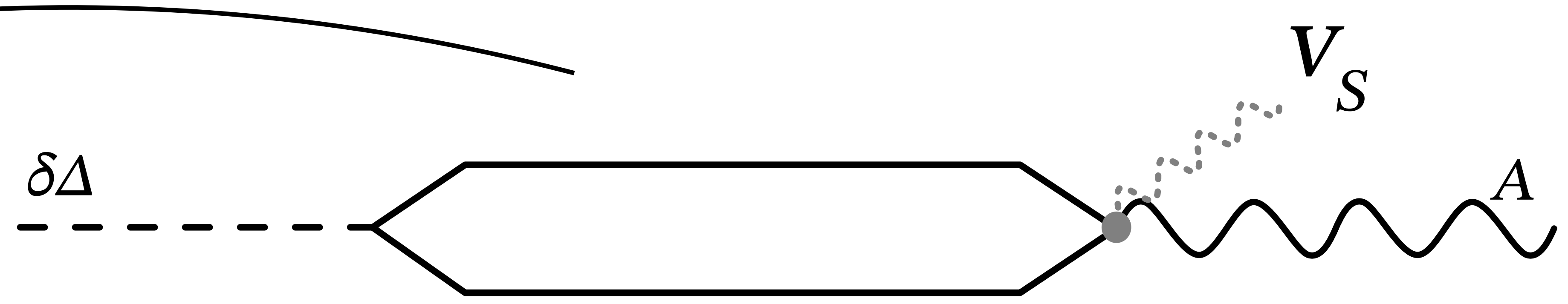
Moor, A., Volkov, A. F. & Efetov, K. B., Phys. Rev. Lett 118, 047001 (2017).
Nakamura, S. et al. Infrared activation of Higgs mode by supercurrent injection in a superconductor NbN. arXiv (2018).

Effective bosonic theory of coupled modes

Higgs

$$[\hat{G}^R]^{-1}(\omega, \mathbf{q}) = \begin{pmatrix} -\frac{2\nu}{\lambda} - \Pi_h^R(\omega, \mathbf{q}) & \mathbf{g}^R(\omega, \mathbf{q}) \\ \mathbf{g}^R(\omega, \mathbf{q}) & \hat{D}^{-1}(\omega, \mathbf{q}) - \hat{\Pi}_A^R(\omega, \mathbf{q}) \end{pmatrix}$$

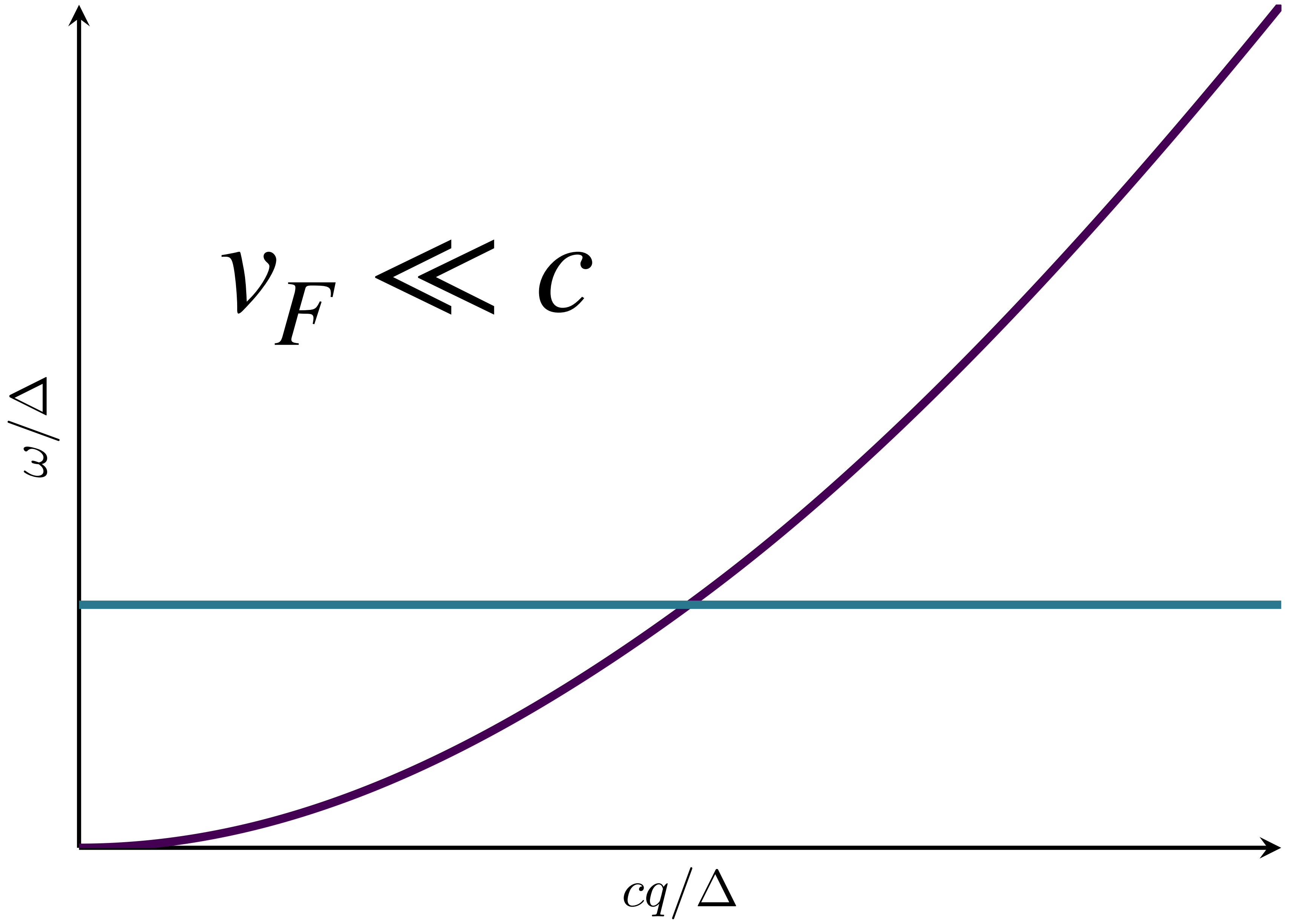
Photon



- We want to obtain the eigenmodes like in the exciton-polariton case

- Of particular importance is the hybridization term g

  - This is mediated by diffusive modes



Set q → 0 in the fermionic response

Effective bosonic theory of coupled modes

Higgs

$$[\hat{G}^R]^{-1}(\omega, \mathbf{q}) = \begin{pmatrix} -\frac{2\nu}{\lambda} - \Pi_h^R(\omega) & \mathbf{g}^R(\omega) \\ \mathbf{g}^R(\omega) & \hat{D}^{-1}(\omega, \mathbf{q}) - \hat{\Pi}_A^R(\omega) \end{pmatrix}.$$

Photon

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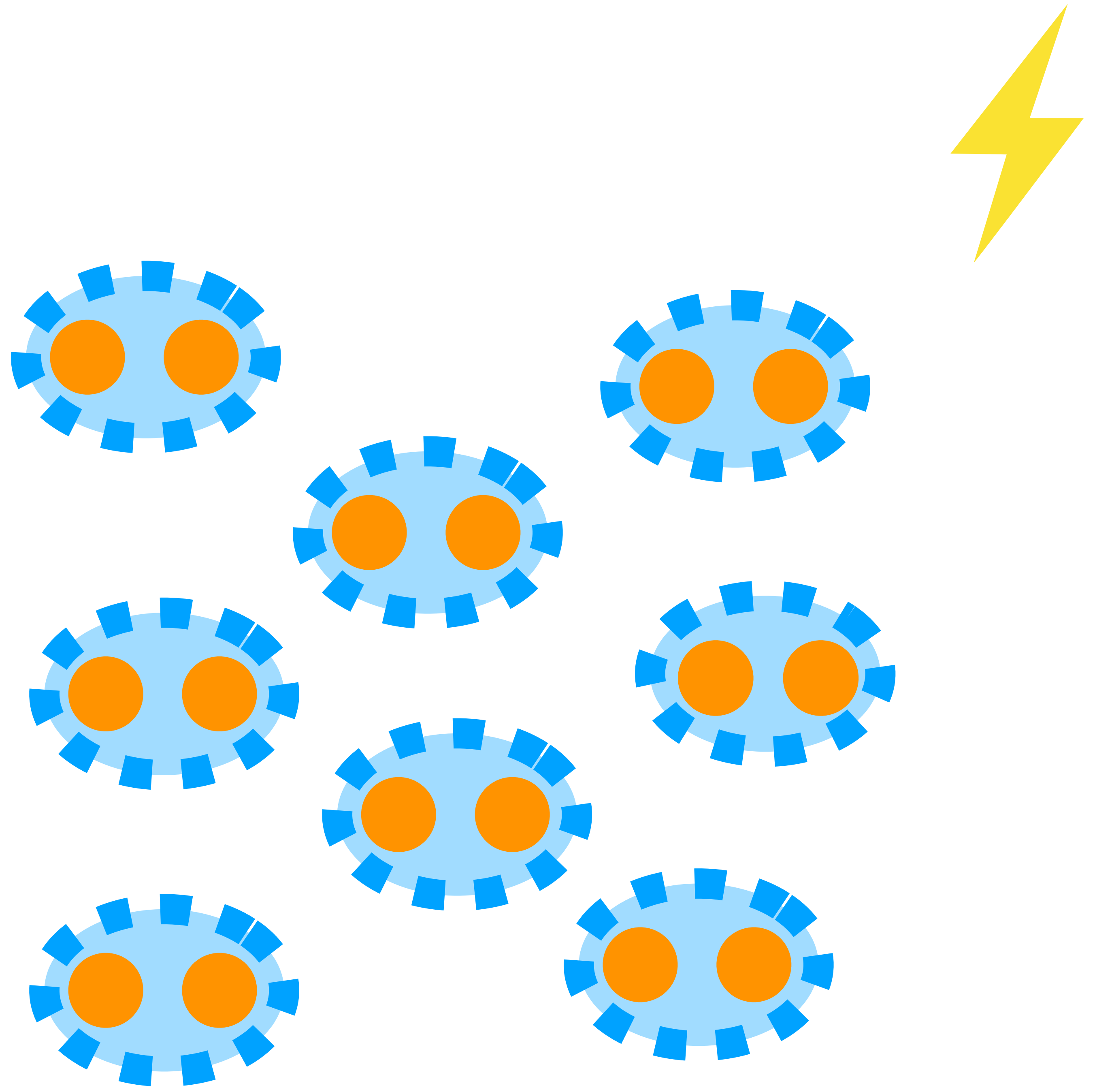
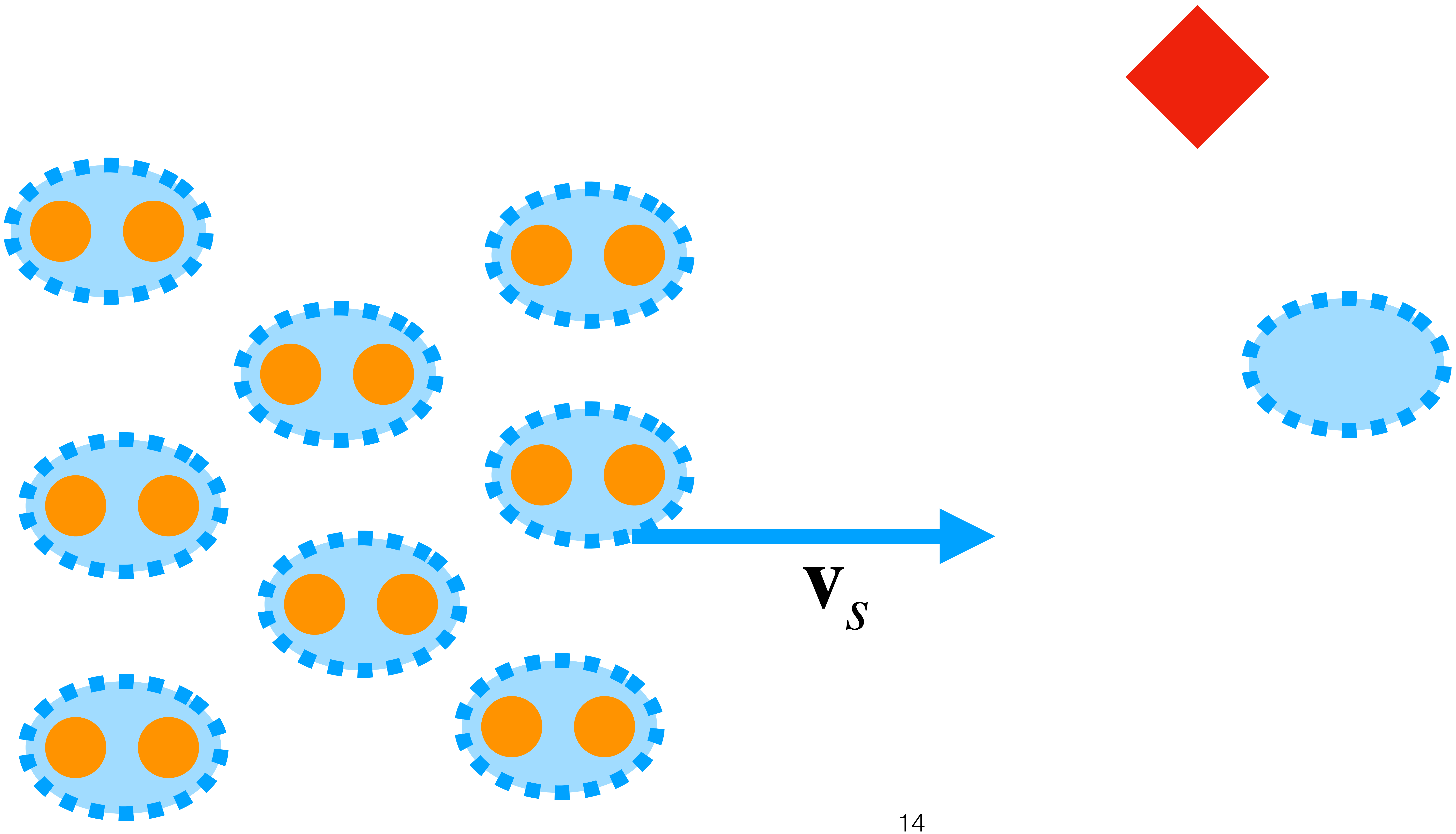
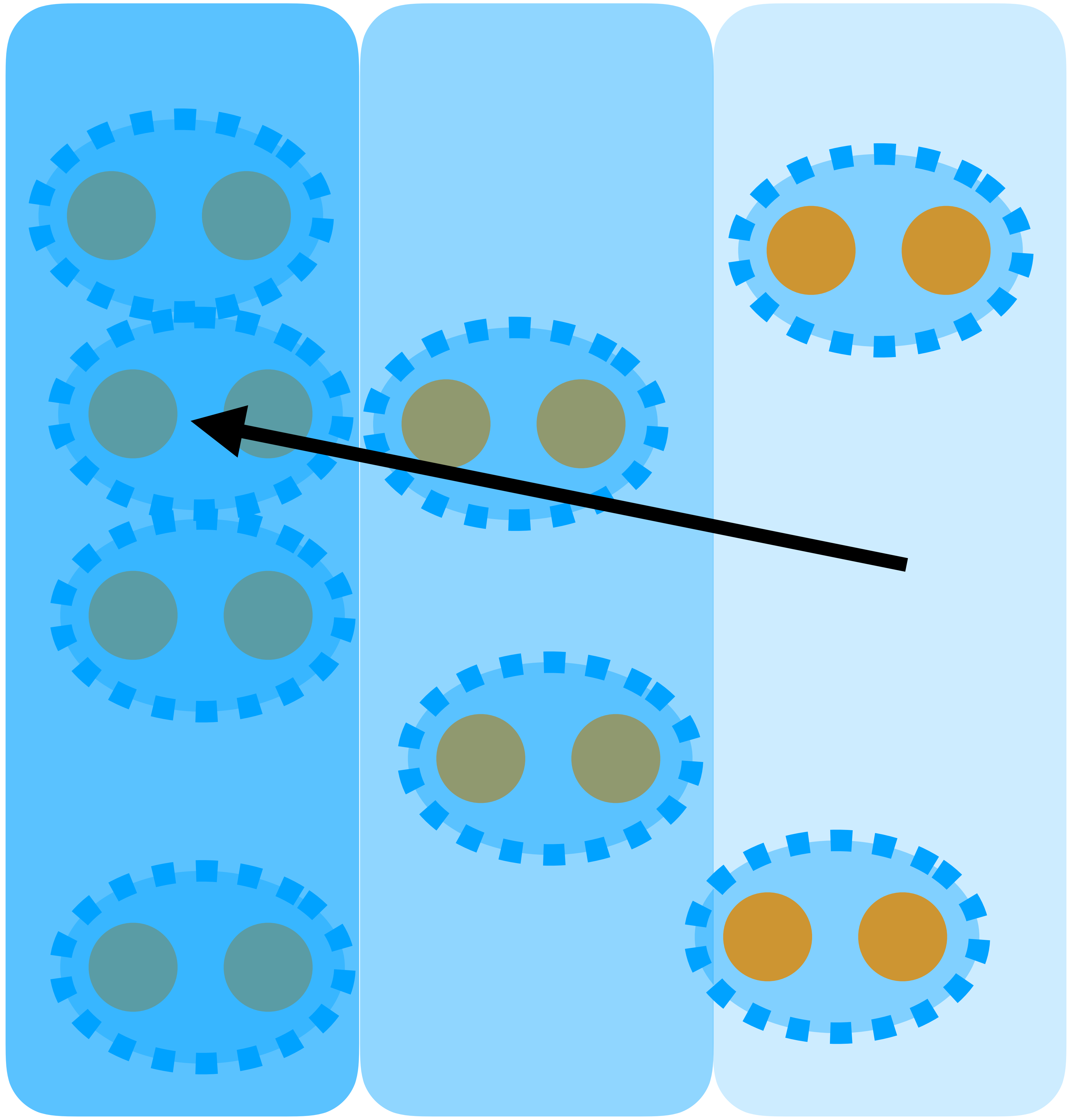



$\mathbf{v}_s$

Creating a Higgs from a photon



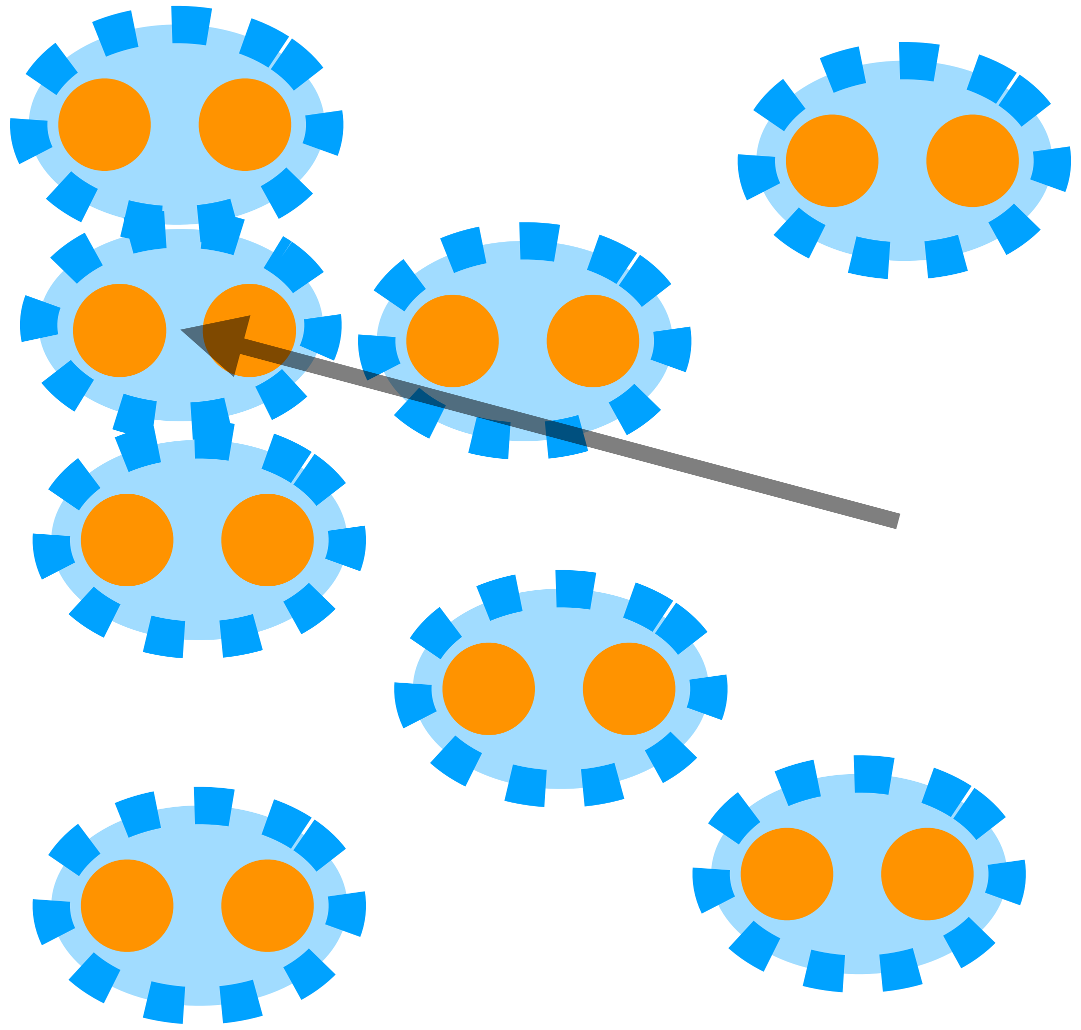
Effective bosonic theory of coupled modes

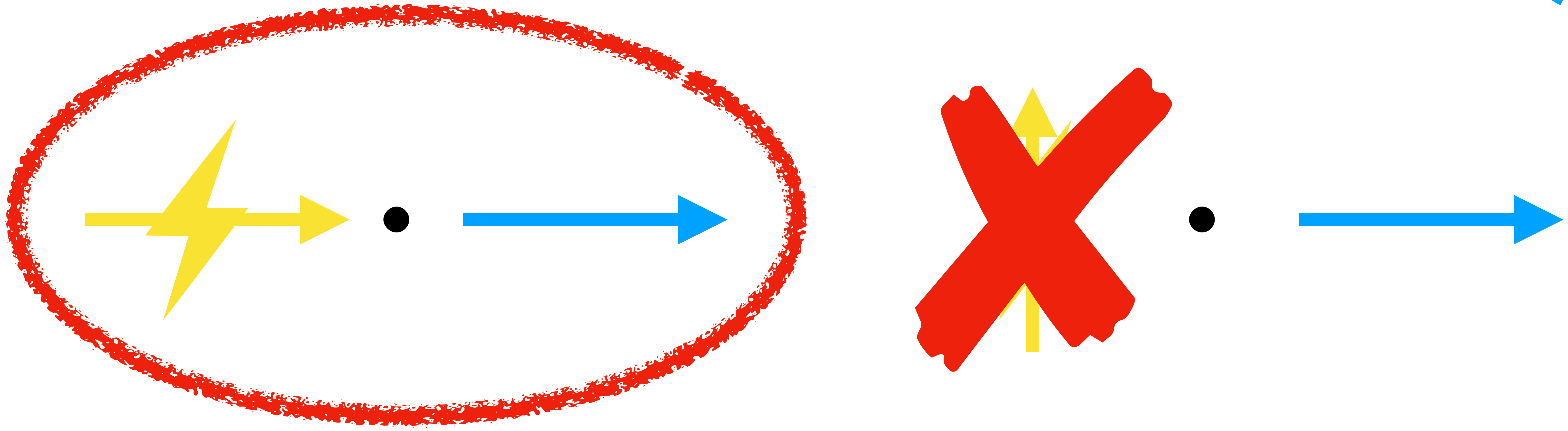
Higgs

$$[\hat{G}^R]^{-1}(\omega, \mathbf{q}) = \begin{pmatrix} -\frac{2\nu}{\lambda} - \Pi_h^R(\omega) & \mathbf{g}^R(\omega) \\ \mathbf{g}^R(\omega) & \hat{D}^{-1}(\omega, \mathbf{q}) - \hat{\Pi}_A^R(\omega) \end{pmatrix}$$

Photon

$$\mathbf{g}^R \propto \mathbf{v}_s$$





$= 0$

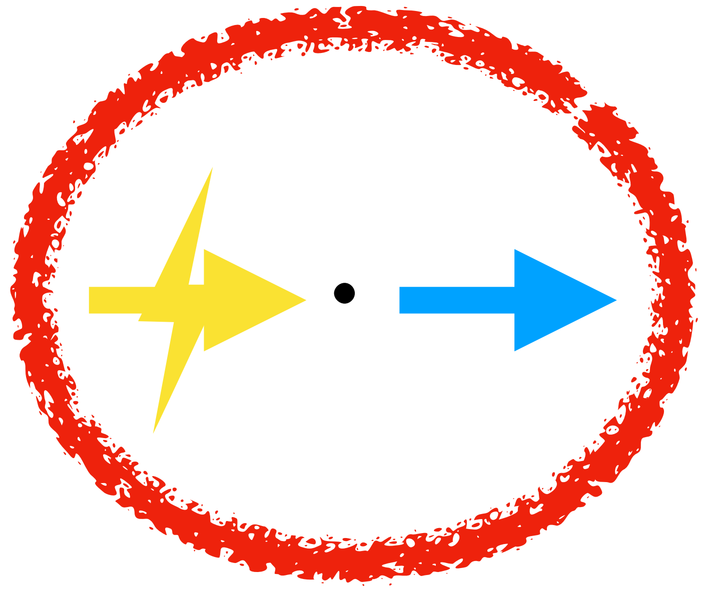
These form approximately decoupled polarizations

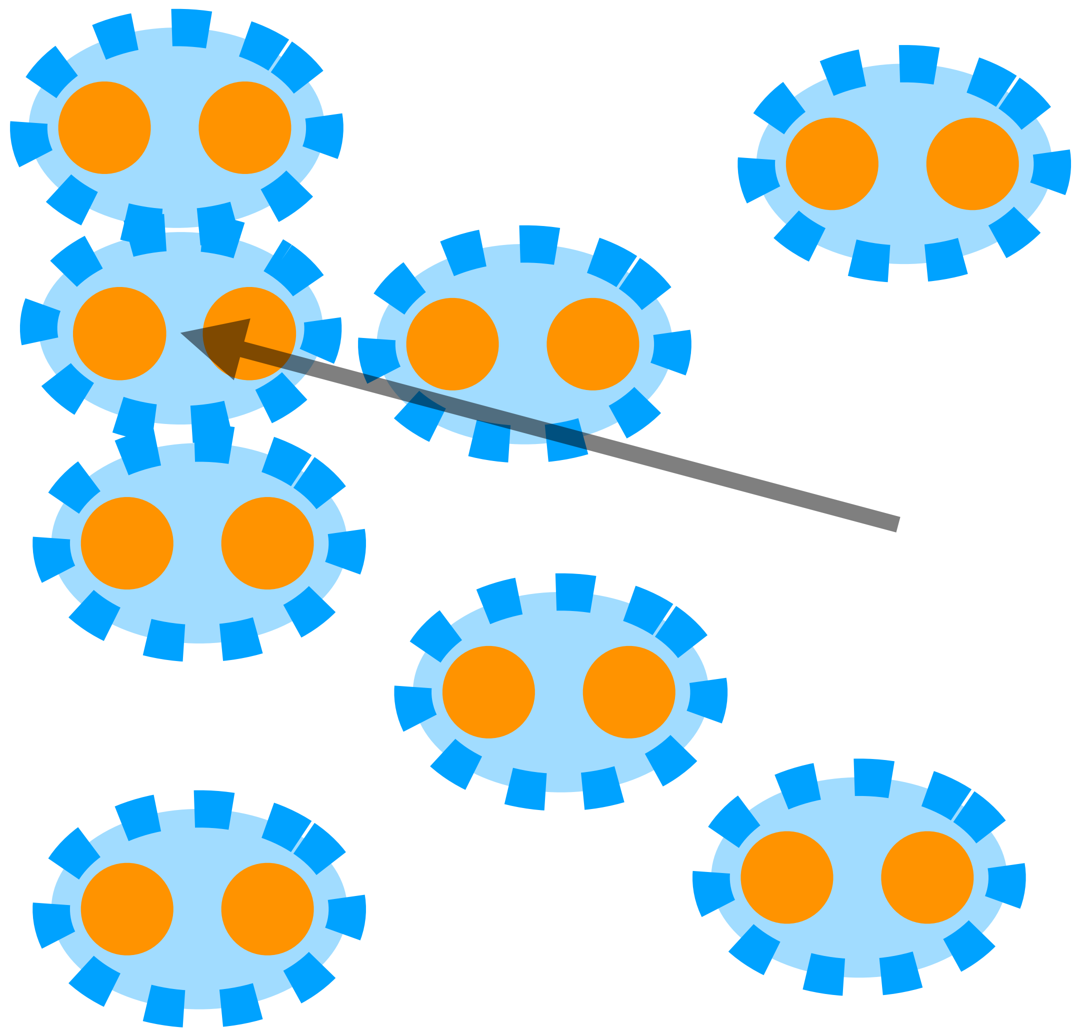
Effective bosonic theory of coupled modes

Higgs

$$[\hat{G}^R]^{-1}(\omega, \mathbf{q}) = \begin{pmatrix} -\frac{2\nu}{\lambda} - \Pi_h^R(\omega) & g^R(\omega) \\ g^R(\omega) & D^{-1}(\omega, \mathbf{q}) - \Pi_A^R(\omega) \end{pmatrix}$$

Photon





Hybrid Modes

$$\det\left\{[G^R]^{-1}(\omega_{\mathbf{q}}, \mathbf{q})\right\} = 0$$

Spectral Function

$$\mathscr{A}(\omega, \mathbf{q}) = -\frac{1}{2\pi i}\,\mathrm{tr}\left[\hat{G}^R(\omega, \mathbf{q}) - \hat{G}^A(\omega, \mathbf{q})\right]$$

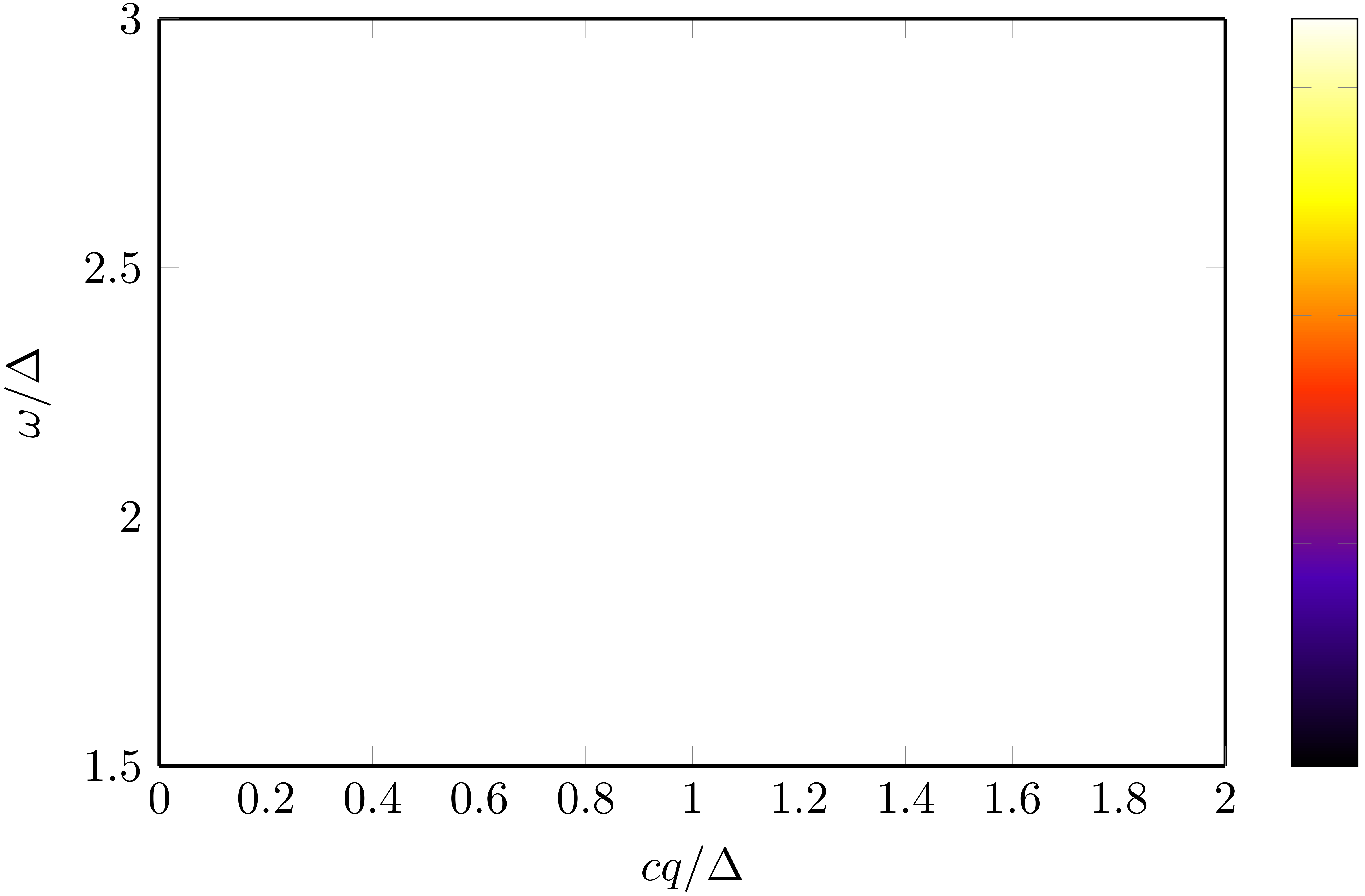
Effective bosonic theory of coupled modes

- Undamped excitation
  - δ - function at mode energy
- Damped excitation
  - Lorentzian at mode energy

Spectral Function

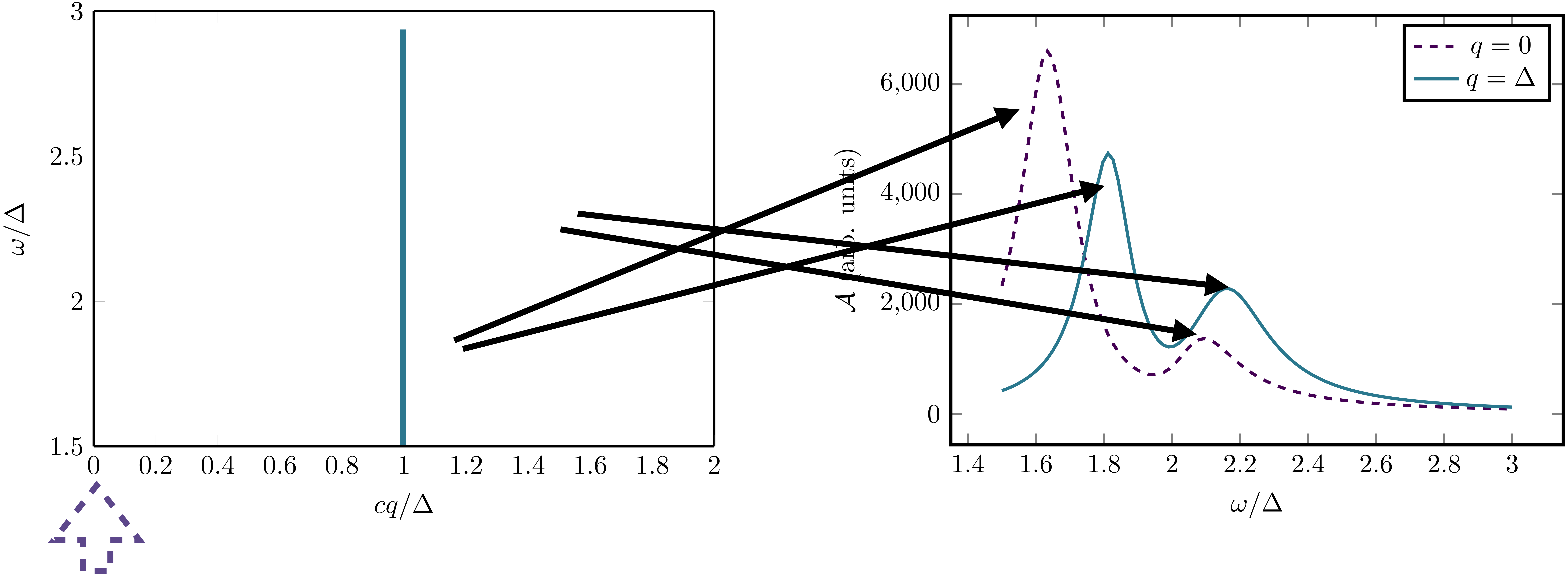
$$\mathscr{A}(\omega, \mathbf{q}) = -\frac{1}{2\pi i}\,\mathrm{tr}\left[\hat{G}^R(\omega, \mathbf{q}) - \hat{G}^A(\omega, \mathbf{q})\right]$$

Polariton Spectral function



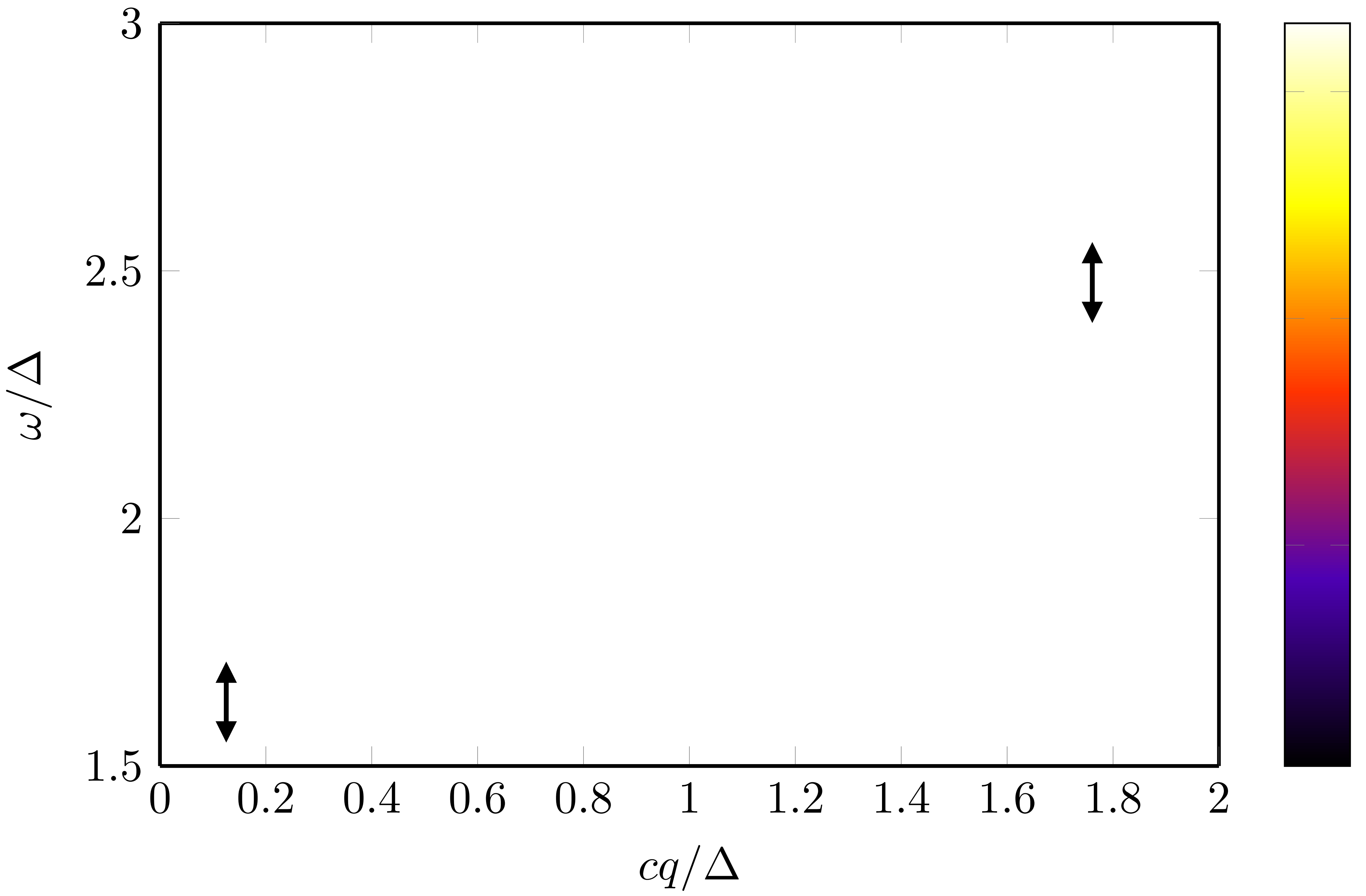
Polariton Spectral function

$$\mathscr{A} = -\frac{1}{2\pi i}\,\mathrm{tr}\left[G^R - G^A\right]$$



Damping of the Higgs Polariton

- There are two competing effects here
  - the Higgs mode is damped by its coupling to the two particle continuum
  - but, the hybridization pushes lower branch down into the gap



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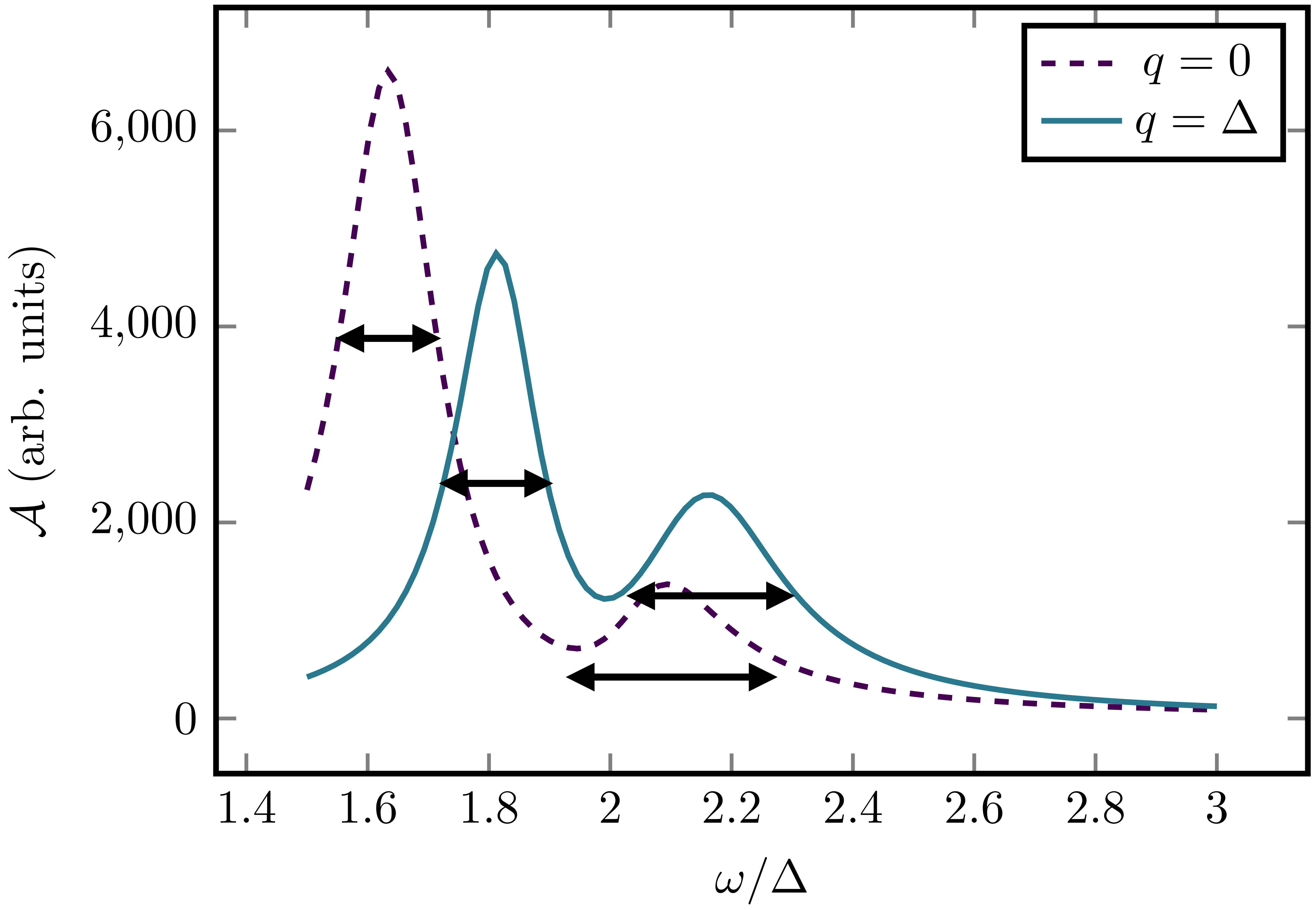
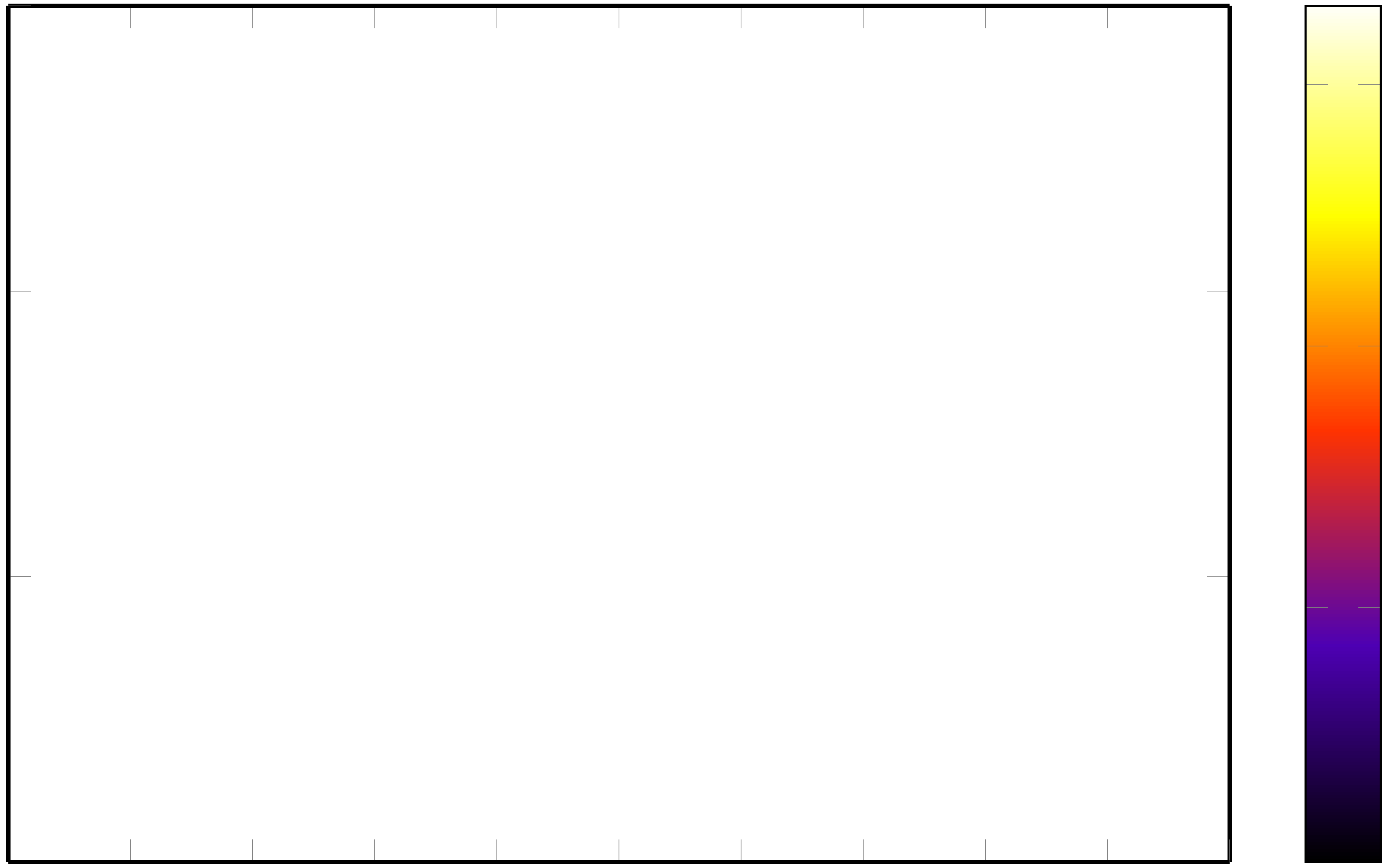


Conclusions / Outlook

- Cavity photons can hybridize with the Higgs mode of a superconductor to form well defined hybrid excitations - *cavity Higgs-polaritons*

- These could provide a new means of manipulation of the superconducting state

- A full treatment of condensation phenomena for these objects could lead to new phases



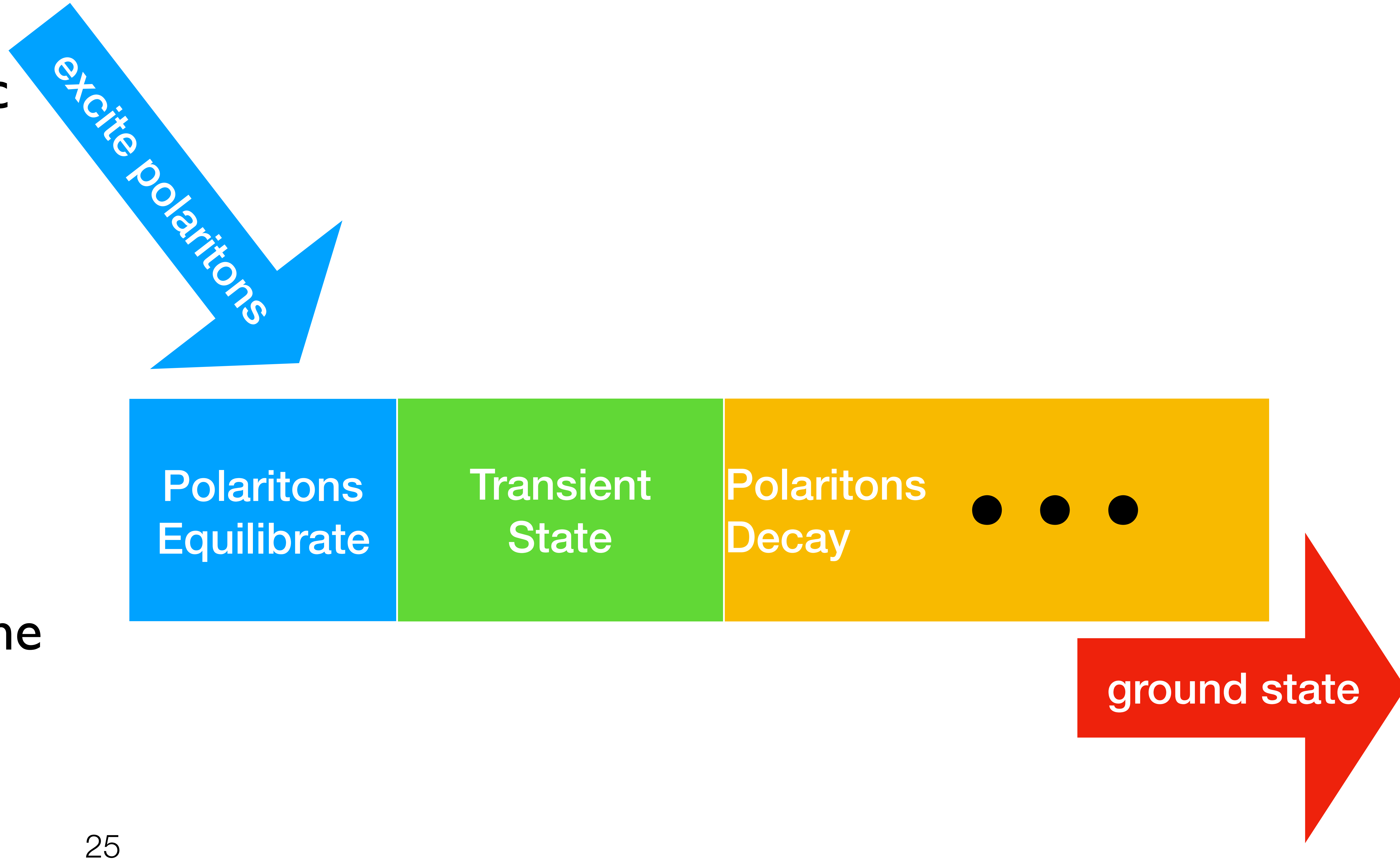
see also: **S08.00003 : Cavity Quantum Enhancement of Superconductivity**
Jonathan Curtis
11:39 AM–11:51 AM Thursday, March 7
BCEC Room: 150

Thank you!

Extra Slides

Condensation

- In analogy with exciton-polariton condensation, we conjecture that it may be possible for Higgs-polaritons to condense
    - Residual interaction arise from quartic terms
    - Thermalization time from physics of order Δ
    - Decay time of order cavity decay rate
- Requires full self-consistent solution of the coupled problem



$$iS = \pi\nu \int_{\epsilon,\epsilon',\mathbf{q}} \left(\frac{1}{4} \left[\vec{d}_{\epsilon'\epsilon} \hat{\mathcal{D}}^{-1}_{\epsilon\epsilon'} \vec{d}_{\epsilon\epsilon'} + \vec{c}_{\epsilon'\epsilon} \hat{\mathcal{C}}^{-1}_{\epsilon\epsilon'} \vec{c}_{\epsilon\epsilon'} \right] \right.$$
$$+ \left[\vec{c}_{\epsilon'\epsilon} \hat{s}^c_{\epsilon\epsilon'} + \vec{d}_{\epsilon'\epsilon} \hat{\sigma}_1 \hat{s}^d_{\epsilon\epsilon'} \right] \vec{h}(\epsilon - \epsilon')$$
$$\left. + \frac{e}{c} D \left[\vec{c}_{\epsilon'\epsilon} \hat{r}^c_{\epsilon\epsilon'} + \vec{d}_{\epsilon'\epsilon} \hat{\sigma}_1 \hat{r}^d_{\epsilon\epsilon'} \right] \mathbf{p}_s \cdot \vec{\mathbf{A}}(\epsilon - \epsilon') \right)$$

$$\mathcal{F}[\omega, \hat{x}, \hat{y}] = -i\nu \int d\epsilon \left(\left[\hat{x}^c_{\epsilon_-\epsilon_+} \right]^T \hat{\mathcal{C}}_{\epsilon_+\epsilon_-} \hat{y}^c_{\epsilon_+\epsilon_-} \right.$$
$$\left. + \left[\hat{x}^d_{\epsilon_-\epsilon_+} \right]^T \hat{\sigma}_1 \hat{\mathcal{D}}_{\epsilon_+\epsilon_-} \hat{\sigma}_1 \hat{y}^d_{\epsilon_+\epsilon_-} \right),$$

$$\hat{\Pi}^h(\omega) = \hat{\mathcal{F}}(\omega, \hat{s}, \hat{s})$$
$$\hat{\Pi}^A_{ij}(\omega) = \frac{e^2}{c^2} D^2 p^i_S p^j_S \hat{\mathcal{F}}(\omega, \hat{r}, \hat{r}) + \hat{\Pi}_{\mathrm{MB};ij}$$
$$\hat{\mathbf{g}}(\omega) = \frac{e}{c} D \mathbf{p}_S \hat{\mathcal{F}}(\omega, \hat{s}, \hat{r}),$$

Moor, A., Volkov, A. F. & Efetov, K. B. Phys. Rev. Lett **118**, 047001 (2017).

Disorder and supercurrent: the Keldysh nonlinear-sigma model

$$iS_{NLSM} = -\frac{\pi\nu}{8}\mathrm{Tr}\left[D\left(\hat{\partial}\check{Q}\right)^2 + 4i\left(i\hat{\tau}_3\partial_t + \check{\Delta}\right)\check{Q}\right] - i\frac{\nu}{4\lambda}\mathrm{Tr}\,\check{\Delta}^{\dagger}\hat{\gamma}^{q}\check{\Delta}$$

$$\hat{\partial}\check{X} = \nabla\check{X} + i[\mathbf{p}_s\tau_3 - \frac{e\eta}{c}\check{A}, \check{X}]$$

$$\check{Q} = \check{Q}_{sp} + \eta\check{Q}_1 + \frac{1}{2}\eta^2\check{Q}_2$$

$$\check{\Delta} = (\Delta_0 + \eta\delta\hat{\Delta})i\hat{\tau}_2$$

- We make the Gaussian approximation — going to second order in the formal expansion parameter η
- Due to the expansion about the saddle point all terms of order η^1 vanish
- The terms of order η^0 do not depend on our fluctuation fields

Yes!

Disorder and supercurrent

$$iS_{\eta^2} = -\frac{\pi\nu\eta^2}{8}\,\mathrm{Tr}\Big[$$

$$+2D\frac{e}{c}[\hat{\tau}_3, \check{Q}_1][\mathbf{p}_s \cdot \check{\mathbf{A}}, \check{Q}_{\rm sp}] - \qquad 4i(i\tau_2\delta\hat{\Delta})\check{Q}_1\Big]$$

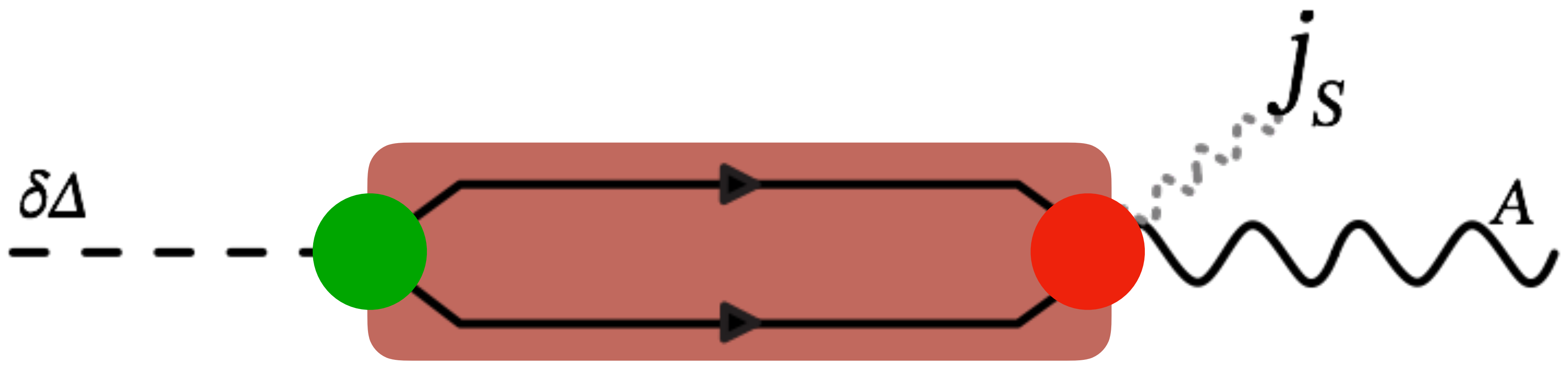
$$\hat{\partial}_0\check{X} = \nabla\check{X} + i[\mathbf{p}_s\tau_3, \check{X}]$$

$$\check{Q}_n = \check{U}\check{V}^{-1}\hat{\sigma}_3\hat{\tau}_3\check{W}^n\check{V}\check{U}$$

$$\{\hat{W}, \hat{\sigma}_3\hat{\tau}_3\} = 0$$

- Coupling of diffusion modes to
  - to photons
  - to the Higgs mode

- Photon-Higgs coupling is mediated by diffuson and Cooperon modes



Aside: solving the usadel equation in the presence of a uniform supercurrent

Retarded Quasiclassical Green's function

$$\hat{g}^R(\epsilon) = u_\epsilon \tau_3 + i v_\epsilon \tau_2$$

$$u_\epsilon^2 - v_\epsilon^2 = 1$$

Usadel Equation

$$\Delta u_\epsilon - \epsilon v_\epsilon = i\Gamma u_\epsilon v_\epsilon$$

Ricatti Parametrization

$$u_\epsilon = \frac{1+\gamma_\epsilon^2}{1-\gamma_\epsilon^2}, \qquad v_\epsilon = \frac{2\gamma_\epsilon}{1-\gamma_\epsilon^2}$$

$$\gamma^4 + 2\frac{\epsilon + i\Gamma}{\Delta}\gamma^3 - 2\frac{\epsilon - i\Gamma}{\Delta}\gamma - 1 = 0$$

$$p = \frac{\Gamma^2}{\Delta^2} + \frac{\epsilon^2}{\Delta^2} - 1$$

$$q = 2\frac{\Gamma\epsilon}{\Delta^2}$$

$$y = \begin{cases} -2\sqrt{\frac{-p}{3}}\,\operatorname{sgn} q \cosh\left(\frac{1}{3}\cosh^{-1}\left(\frac{-3|q|}{2p}\sqrt{\frac{-p}{3}}\right)\right), & 4p^3 + 27q^2 > 0 \cap p < 0 \\ 2\sqrt{\frac{p}{3}}\sinh\left(\frac{1}{3}\sinh^{-1}\left(\frac{3q}{2p}\sqrt{\frac{p}{3}}\right)\right), & 4p^3 + 27q^2 > 0 \cap p > 0 \\ 2\sqrt{\frac{-p}{3}}\cos\left(\frac{1}{3}\cos^{-1}\left(\frac{3q}{2p}\sqrt{\frac{-p}{3}}\right) - \frac{4\pi}{3}\right), & 4p^3 + 27q^2 \le 0. \end{cases}$$

$$\rho = \frac{\epsilon\cos\phi + \Gamma\sin\phi}{\Delta\cos 2\phi}$$

$$\gamma_\epsilon = e^{i\phi_\epsilon}\left(\rho - \sqrt{(\rho + i0)^2 - 1}\right)$$

- The equation can be solved analytically but it is much messier than the Γ=0 case

Non perturbative in the super current

$$\mathscr{D}^{-1}_{\epsilon_+\epsilon_-} = -Dq^2 + i\zeta_R(\epsilon_+)\cosh\phi_+ + i\zeta_A(\epsilon_-)\cosh\phi_-^*$$
$$-\frac{\Gamma}{\zeta_R(\epsilon_+)^2\zeta_A(\epsilon_-)^2}\left[\zeta_R(\epsilon_+)\zeta_A(\epsilon_-) + (z_+z_-' - \Delta_0^2)\cosh(\phi_+ - \phi_-^*) - \Delta_0(\omega + 2i\gamma)\sinh(\phi_+ - \phi_-^*)\right]$$
$$\times\left[(z_+z_-' + \Delta_0^2)\cosh(\phi_+ + \phi_-^*) + 2\Delta_0\epsilon\sinh(\phi_+ + \phi_-^*)\right]$$

$$\left[\mathscr{C}^{(R/A)}_{\epsilon_+\epsilon_-}\right]^{-1} = -Dq^2 + i\zeta_R(\epsilon_+)\cosh\phi_+ + i\zeta_R(\epsilon_-)\cosh\phi_-$$
$$-\frac{\Gamma}{\zeta_R(\epsilon_+)^2\zeta_R(\epsilon_-)^2}\left[\zeta_R(\epsilon_+)\zeta_A(\epsilon_-) + (z_+z_- - \Delta_0^2)\cosh(\phi_+ - \phi_-) - \Delta_0\omega\sinh(\phi_+ - \phi_-)\right]$$
$$\times\left[(z_+z_- + \Delta_0^2)\cosh(\phi_+ + \phi_-) + 2\Delta_0 z\sinh(\phi_+ + \phi_-)\right]$$

Diffusons and Cooperons

$$\check{Q} = \check{U}\check{V}^{-1}e^{-\check{W}/2}\hat{\sigma}_3\hat{\tau}_3 e^{\check{W}/2}\check{V}\check{U} \longrightarrow \check{Q}^2 = 1$$

Spectral angle representation

$$\check{V} = \begin{pmatrix} e^{\hat{\tau}_1\theta_\epsilon/2} & 0 \\ 0 & e^{\hat{\tau}_1\theta_\epsilon^*/2} \end{pmatrix}$$

$$\epsilon \cosh\theta_\epsilon - \Delta \sinh\theta_\epsilon = 0$$

Usadel Equation

$$\check{U} = \begin{pmatrix} 1 & F(\epsilon) \\ 0 & -1 \end{pmatrix}$$

Thermal Rotation

$$\check{Q}_{\text{sp}} = \check{U}\check{V}^{-1}\hat{\sigma}_3\hat{\tau}_3\check{V}\check{U}$$

Saddlepoint solution

$$\{\hat{W}, \hat{\sigma}_3\hat{\tau}_3\} = 0$$

Algebra of target manifold

In the presence of a supercurrent?

$$\sum_{\mathbf{k},q}\sum_{\alpha,\alpha'}\frac{n_F(\alpha E_{\mathbf{k}})-n_F(\alpha' E_{\mathbf{k}})}{i\Omega_m-(\alpha E_{\mathbf{k}}-\alpha' E_{\mathbf{k}})}\left(-e\mathbf{v}_{\mathbf{k}}\cdot\mathbf{A}_q\hat{\tau}_0\right)_{\alpha,\alpha'}\left(\frac{\Delta}{E_{\mathbf{k}}}\hat{\tau}_3+\frac{\xi_{\mathbf{k}}}{E_{\mathbf{k}}}\hat{\tau}_1\right)_{\alpha',\alpha}=0$$

$$\sum_{\mathbf{k},q}\sum_{\alpha,\alpha'}\frac{n_F(E^{\alpha}_{\mathbf{k}})-n_F(E^{\alpha'}_{\mathbf{k}})}{i\Omega_m-(E^{\alpha}_{\mathbf{k}}-E^{\alpha'}_{\mathbf{k}})}\mathbf{v}_S\cdot\mathbf{A}_q\left(\frac{\xi_{\mathbf{k}}}{\lambda_{\mathbf{k}}}\hat{\tau}_3-\frac{\Delta}{\lambda_{\mathbf{k}}}\hat{\tau}_1\right)_{\alpha,\alpha'}\left(\frac{\Delta}{\lambda_{\mathbf{k}}}\hat{\tau}_3+\frac{\xi_{\mathbf{k}}}{\lambda_{\mathbf{k}}}\hat{\tau}_1\right)_{\alpha',\alpha}=0$$

Particle-Hole symmetry **No**

c.f. ξ-approximation

In the presence of disorder?

$$iS_{NLSM} = -\frac{\pi\nu}{8}\,\mathrm{Tr}\left[D\left(\hat{\partial}\check{Q}\right)^2 + 4i\left(i\hat{\tau}_3\partial_t + (\Delta_0 + \delta\hat{\Delta})i\hat{\tau}_2\right)\check{Q}\right] - i\frac{\nu}{2\lambda}\,\mathrm{Tr}\,\check{\Delta}^\dagger\hat{\gamma}^q\check{\Delta}$$

$$\hat{\partial}\check{X} = \nabla\check{X} - i[\tfrac{e}{c}\check{\mathbf{A}}, \check{X}]$$

$$\check{Q} = \check{U}\check{V}^{-1}e^{-\check{W}/2}\hat{\sigma}_3\hat{\tau}_3 e^{\check{W}/2}\check{V}\check{U}$$

$$\{\hat{W}, \hat{\sigma}_3\hat{\tau}_3\} = 0$$

- In the Coulomb gauge, there is no linear coupling between photons and diffusion modes