

# Hybridization of Higgs modes in a bond-density-wave state in cuprates

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# Outline

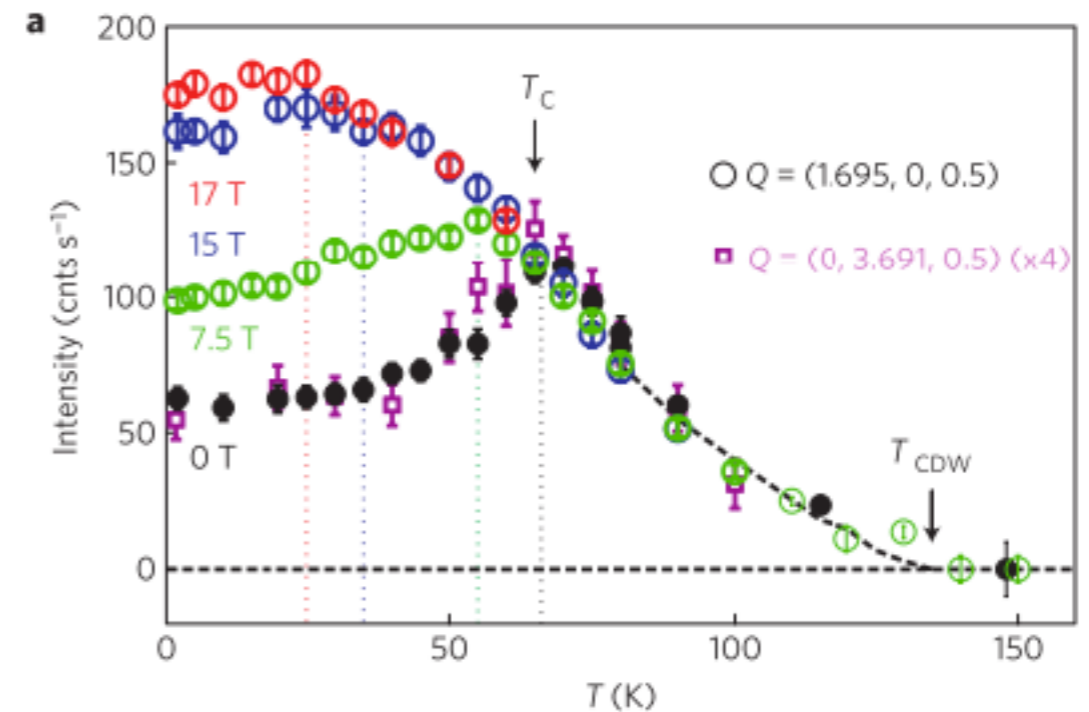
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- Competing charge order and superconductivity in cuprates - *A quick survey*
- Observation of collective modes via time-resolved reflectivity - *Background and motivation*
- Hybridization of Higgs modes - *Model and Results*
- Summary

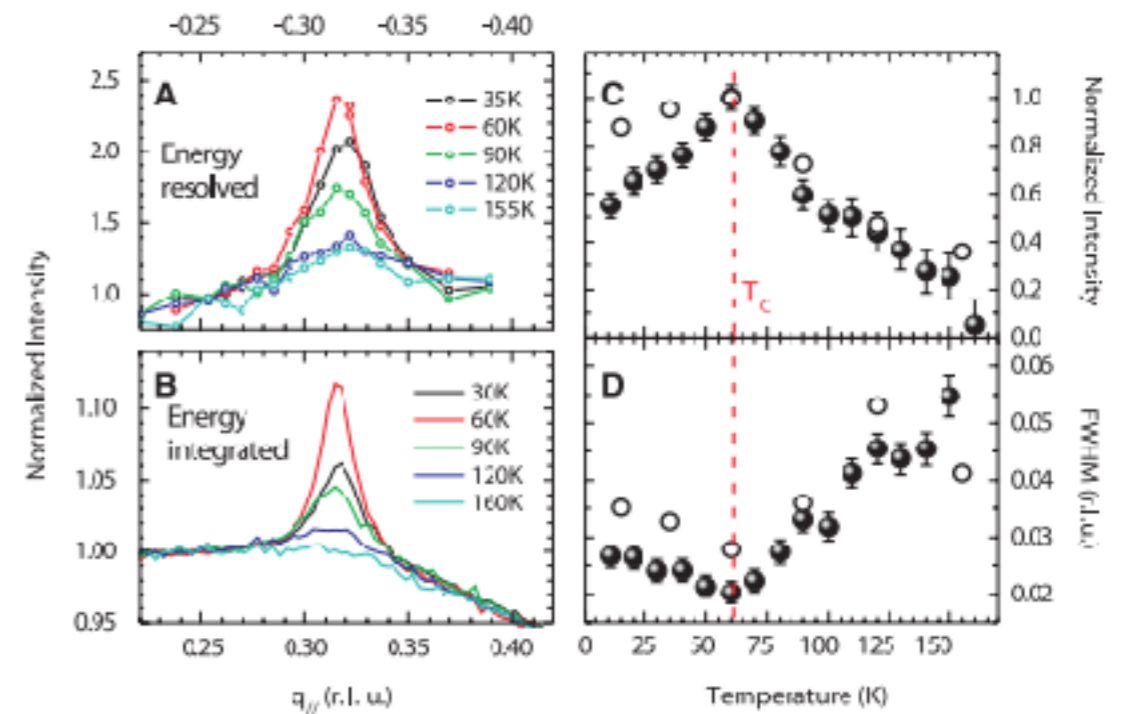
# Competition of superconductivity and charge order

Charge order is experimentally seen to compete with superconductivity in several cuprate families:

- Charge order suppressed below  $T_c$
- Charge order restored when SC is destroyed by magnetic field



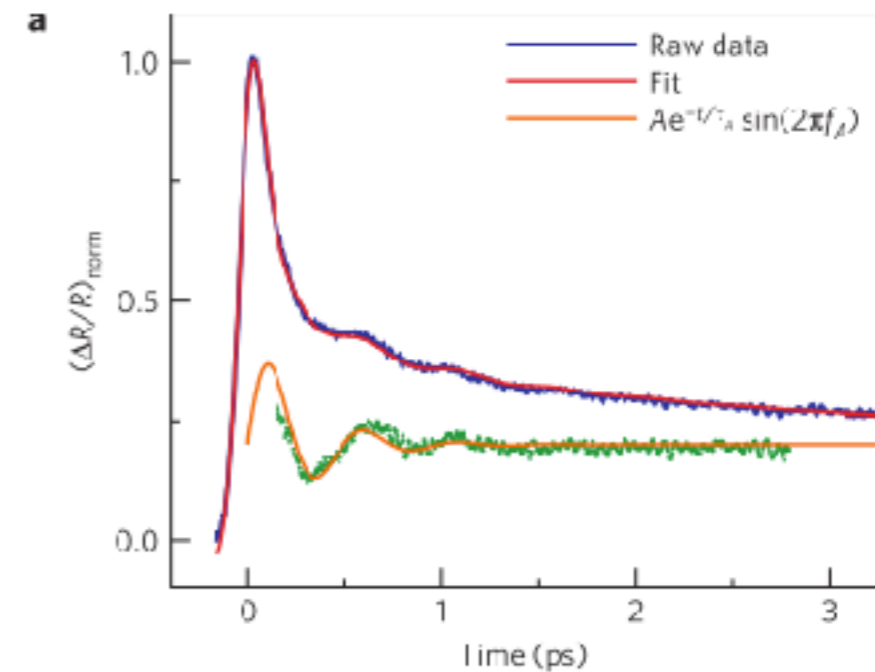
*J. Chang et al., Nat. Phys. 8, 871 (2012).*



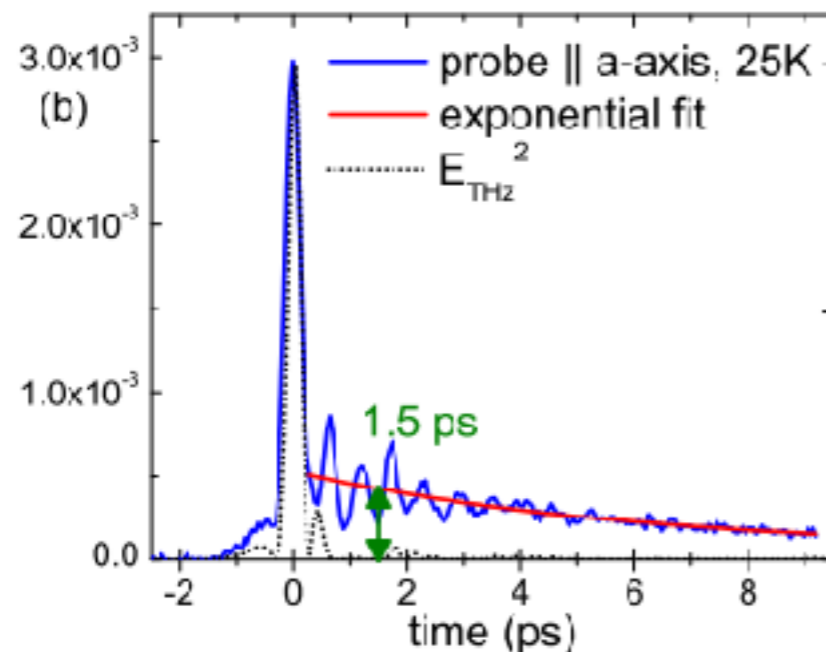
*G. Ghiringhelli, et al., Science 337, 821 (2012).*

# Observation of collective modes via time-resolved reflectivity

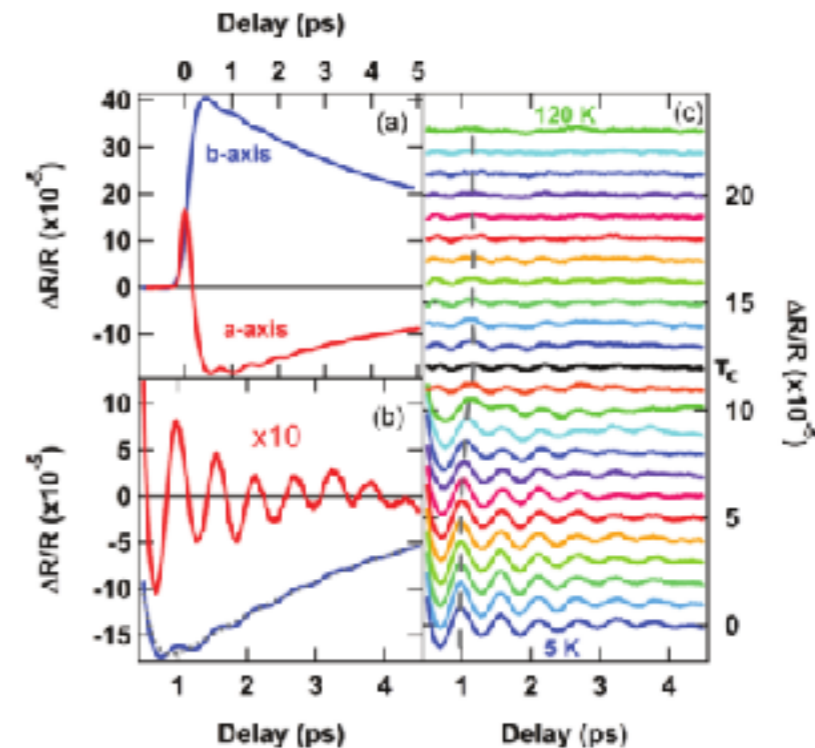
- Time domain reflectivity has been employed to study the collective modes of cuprates
- Multiple groups have observed signals associated with the collective modes of the charge order present in cuprates



*D.H. Torchinsky et al., Nat. Mater. 12, 387 (2013).*



*G.L. Dakovski, et al., Phys. Rev. B 91, 220506 (2015).*



*J.P. Hinton, et al., Phys. Rev. B 88, 60508 (2013).*

# A model of competing orders

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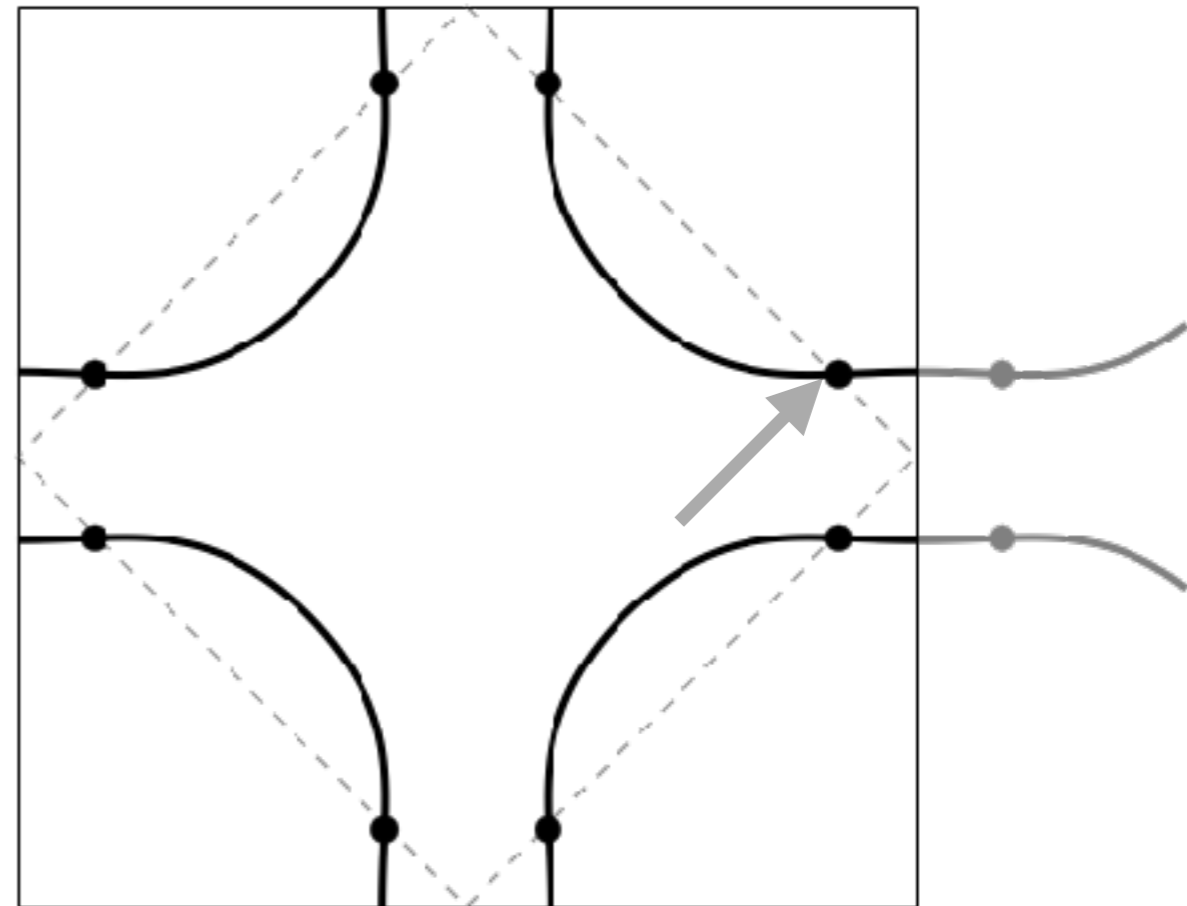
e.g.

*M.A. Metlitski and S. Sachdev, Phys. Rev. B 82, 075128 (2010)*

*J. D. Sau and S. Sachdev, Phys. Rev. B 89, 075129 (2014)*

*Y. Wang and A. V Chubukov, Phys. Rev. B 90, 035149 (2014)*

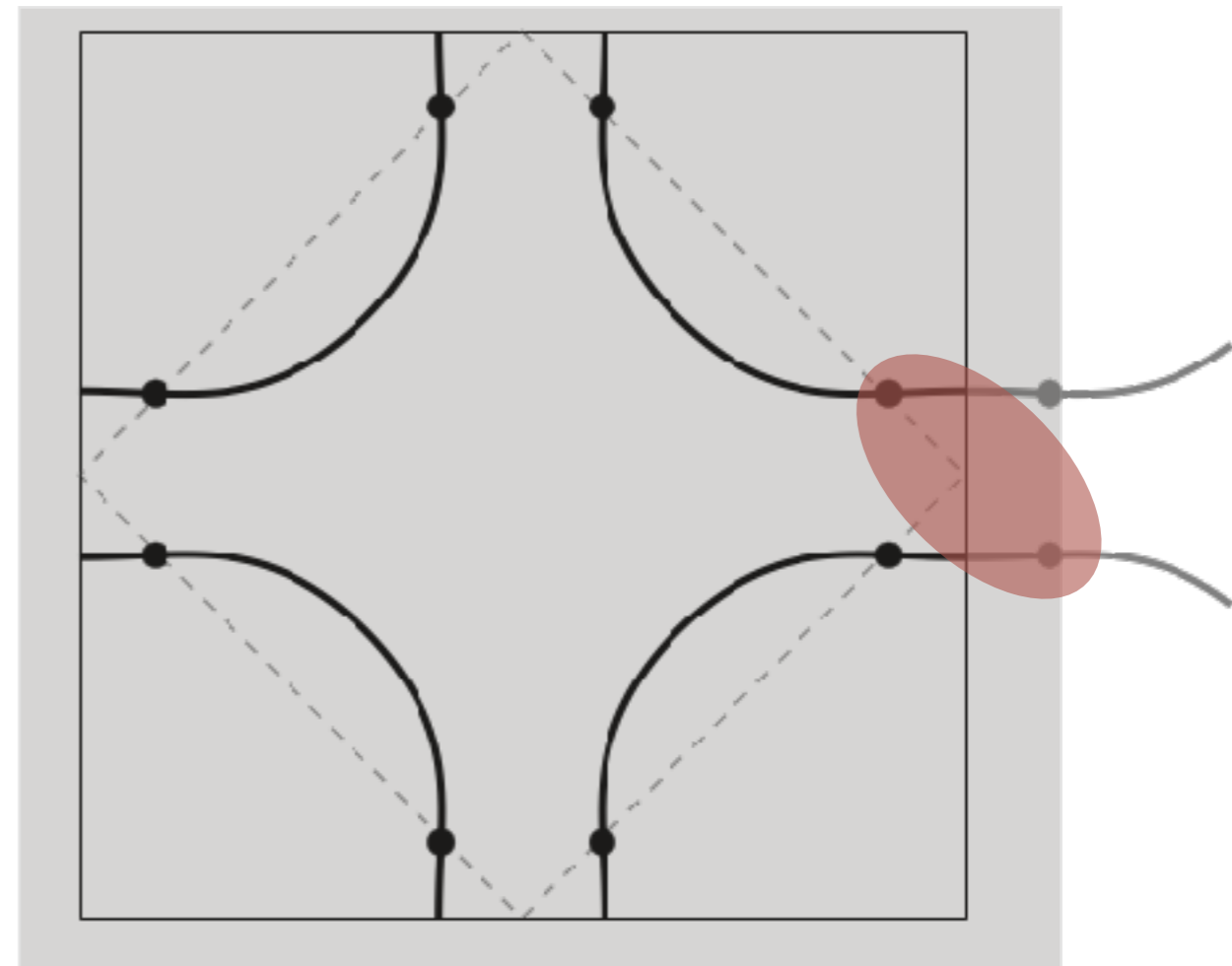
- We consider a **hotspot model** with interactions in the superconducting and bond-density-wave channels
- This can be obtained from a lattice model (e.g. t-J-V model)



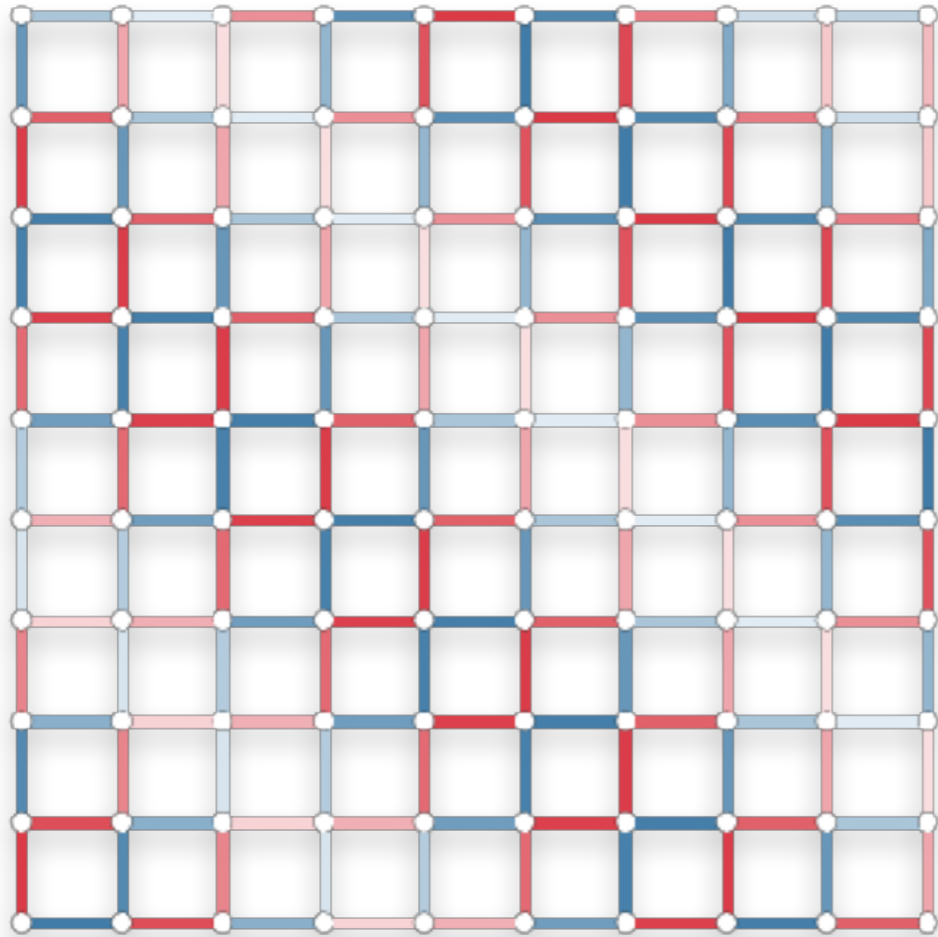
# Mean field phases

- The mean field theory contains **d-wave superconductivity (SC)  $\Delta$**  and **d-form-factor density wave (dFF-DW)  $\phi$**
- The symmetry of the orders allows us to restrict our attention to the vicinity of two hotspots

d-wave form factor +  
time reversal symmetry



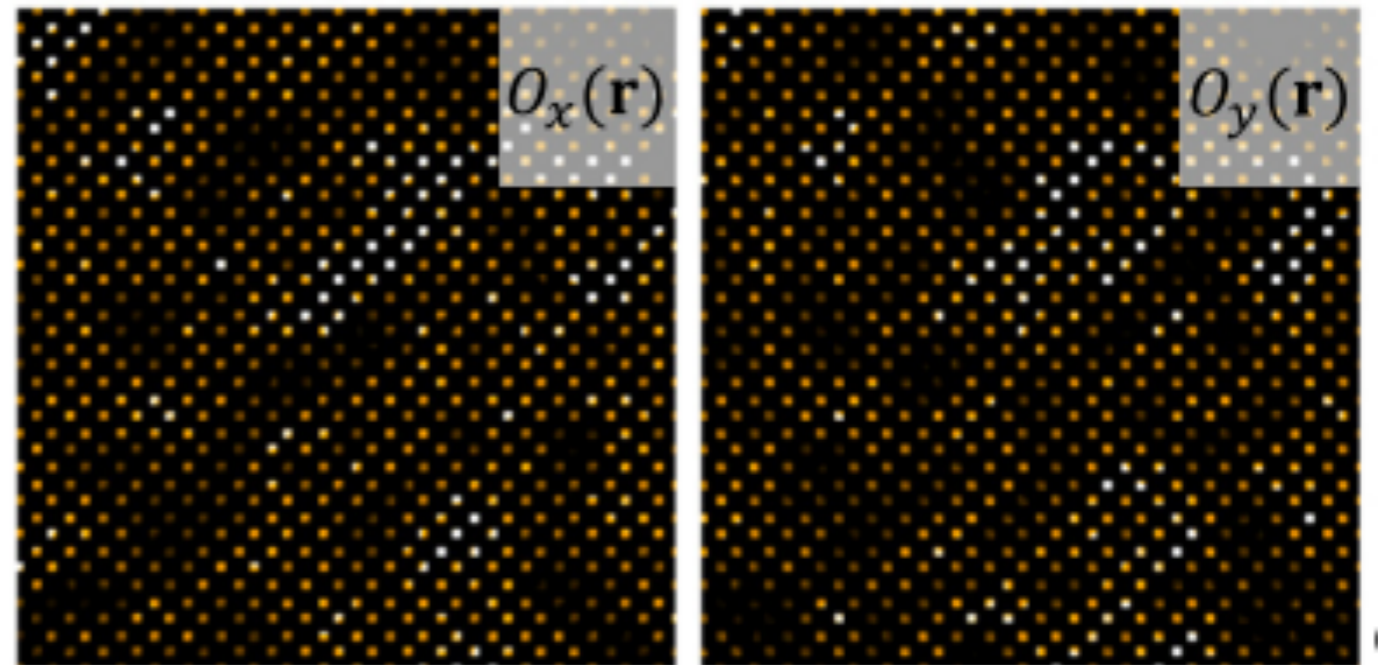
# d-form-factor density wave



Bond-density-wave

$$H_{\phi, \text{MF}} = \sum_{ij} \phi_{ij} c_{i\sigma}^\dagger c_{j\sigma}$$

$$\phi_{ij} \sim \cos \left( \vec{Q} \cdot \frac{\vec{r}_i + \vec{r}_j}{2} \right) (\text{NN}_{x_{ij}} - \text{NN}_{y_{ij}})$$



*K. Fujita et al., Proc. Natl. Acad. Sci. 111, E3026 (2014).*

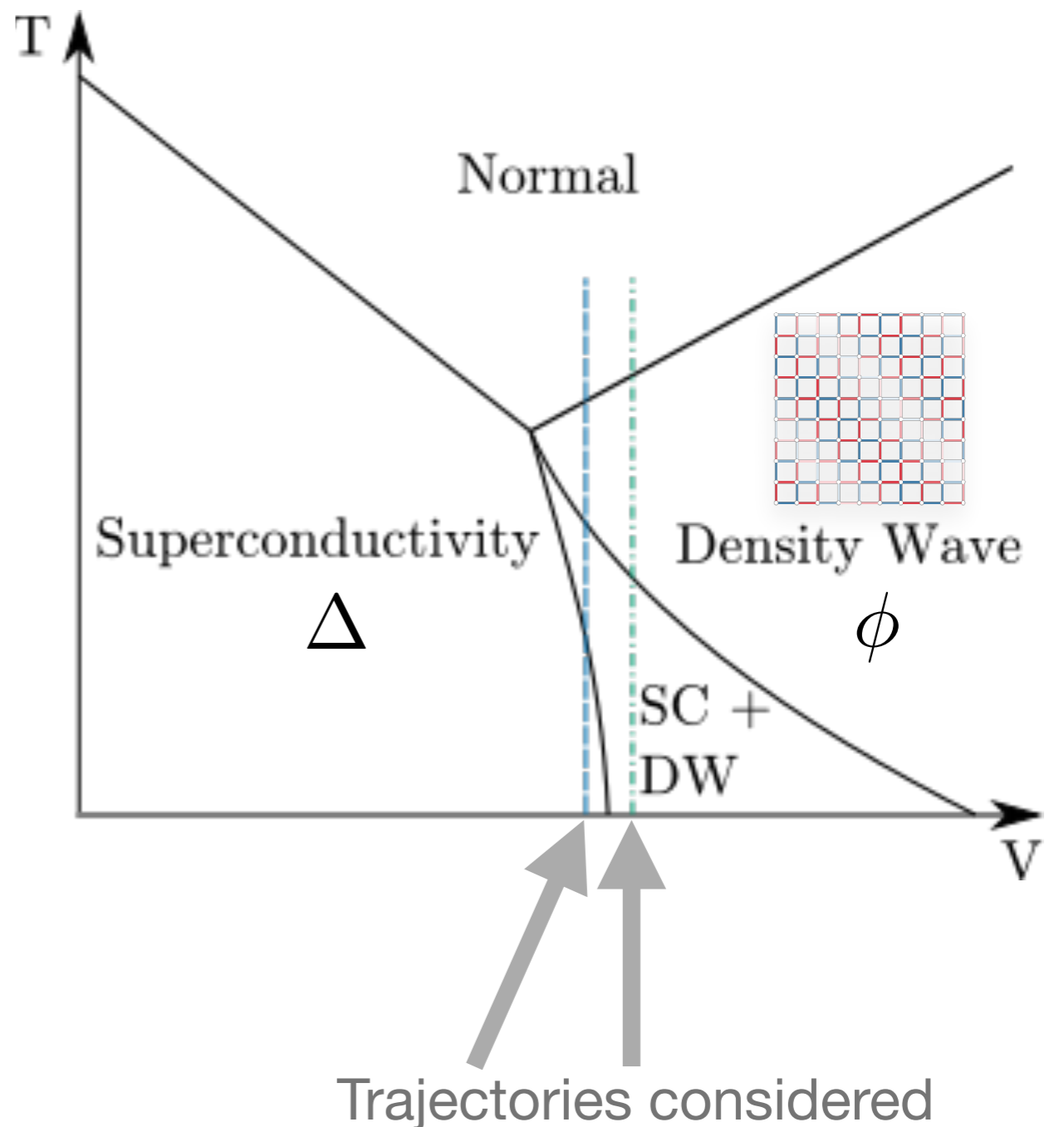
*J. D. Sau and S. Sachdev, Phys. Rev. B 89, 075129 (2014).*

*K.B. Efetov et al., Nat. Phys. 9, 442 (2013).*

*Y. Wang and A. V Chubukov, Phys. Rev. B 90, 035149 (2014).*

# The problem considered

- We consider four phases: *normal*, *density wave*, *superconductivity*, and **coexistent**
- We wish to study the evolution of collective modes with temperature as we enter the coexistent phase





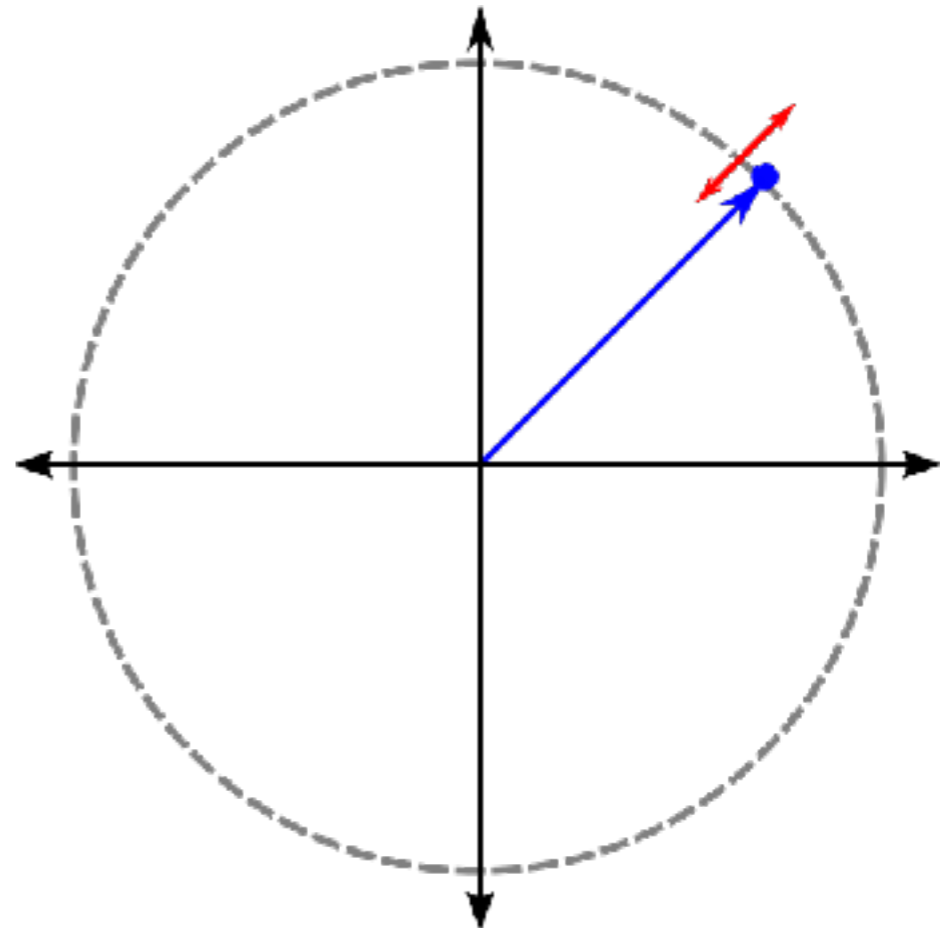
# Collective modes: Higgs Modes

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- Fluctuations of the order parameters can occur on top of the mean field state
- Here we consider the amplitude modes of the superconducting and density wave order parameters

$$\Delta_0 + \Delta(\omega, \mathbf{q})$$

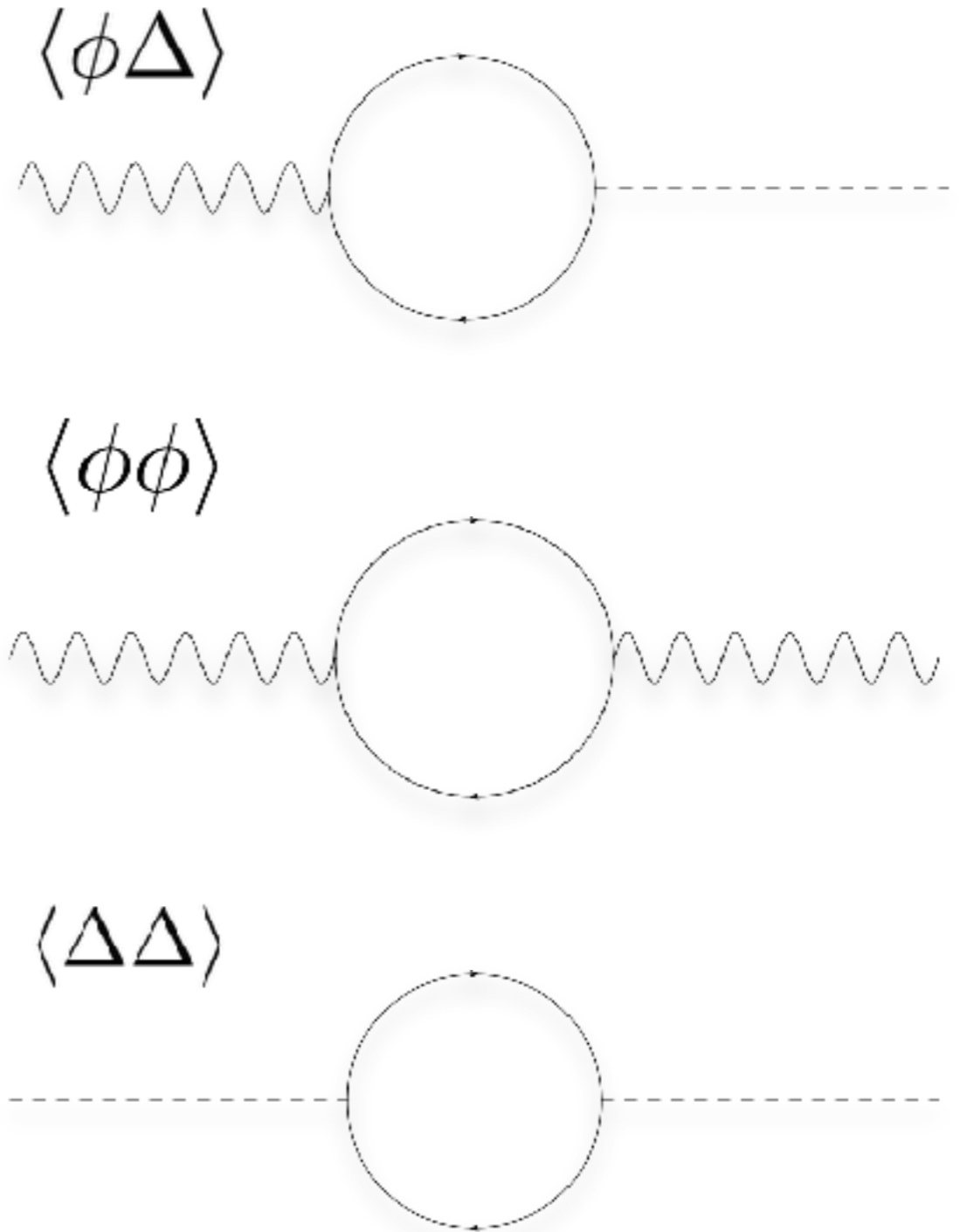
$$\phi_0 + \phi(\omega, \mathbf{q})$$



# Hybridization of Higgs (amplitude) modes

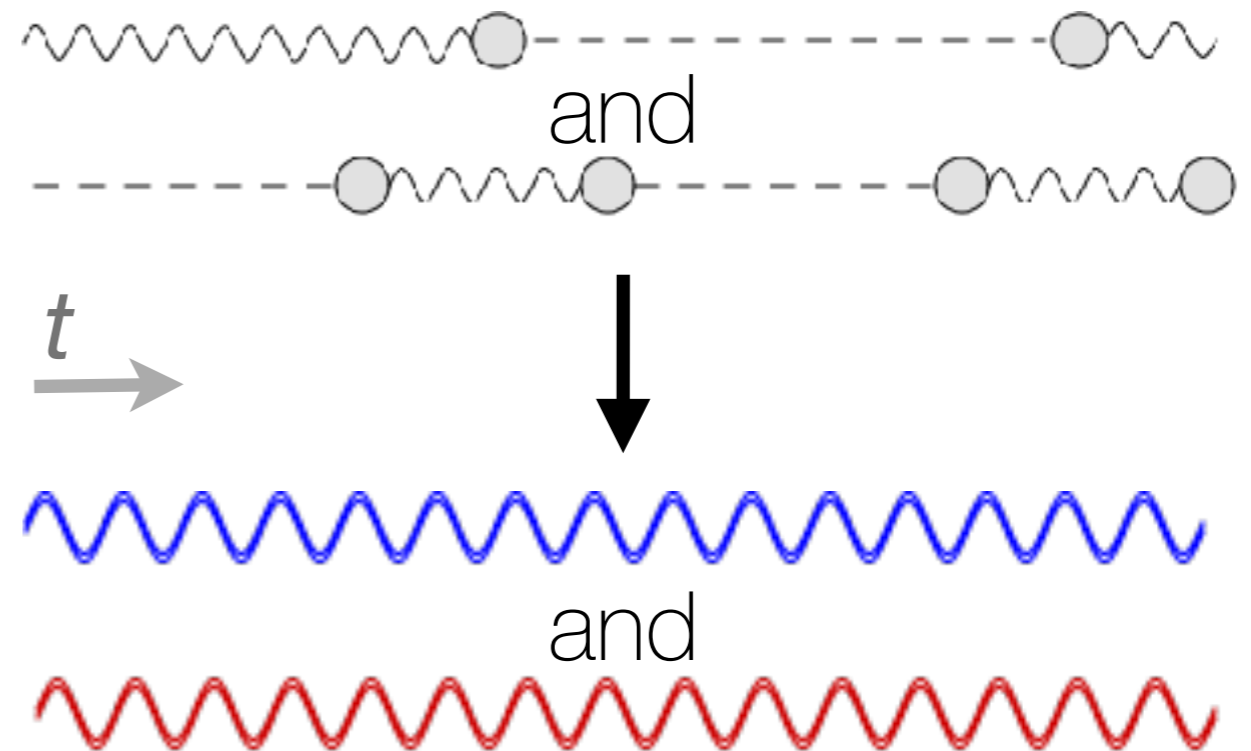
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- Collective modes are related to response functions of order parameters
- Disturbing one order parameter can cause fluctuations in another



# Hybrid Higgs modes

- Because of the coupling between the two modes, we look at the **eigenmodes** of the amplitude response
- These eigenmodes have a sharp frequency response corresponding to the dispersion of the collective modes
  - As a bonus, these modes are *more visible experimentally* due to their mixed nature



$$\langle O_i(q) O_j(-q) \rangle = D_q^{(i)} \delta_{ij}$$

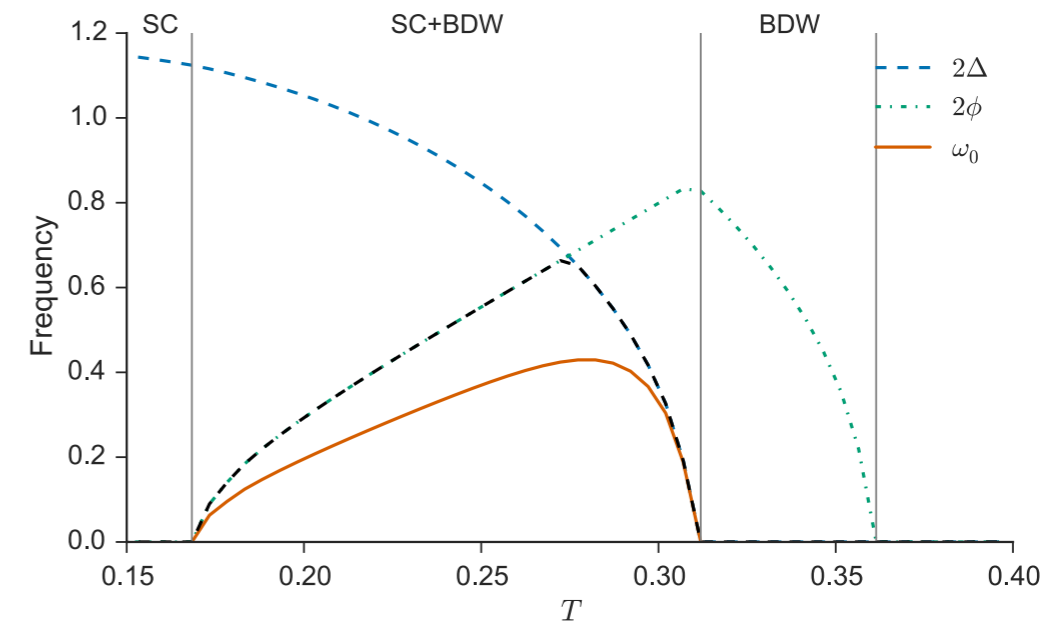
Eigenmodes

$$(D_i^R(\omega_i(\mathbf{q}), \mathbf{q}))^{-1} = 0$$

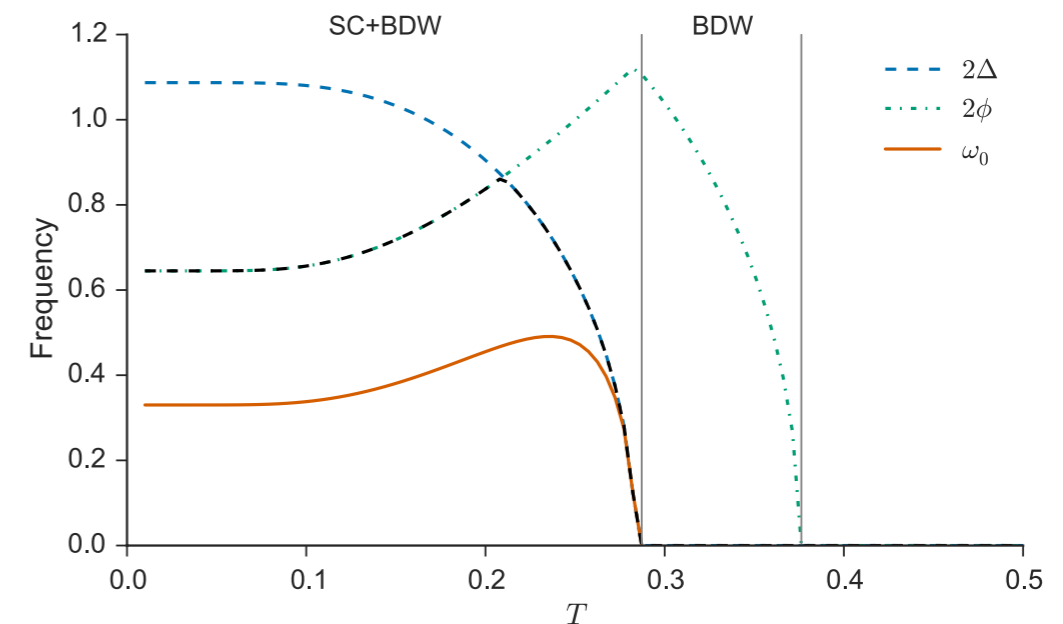
Mode dispersion

# Higgs mass

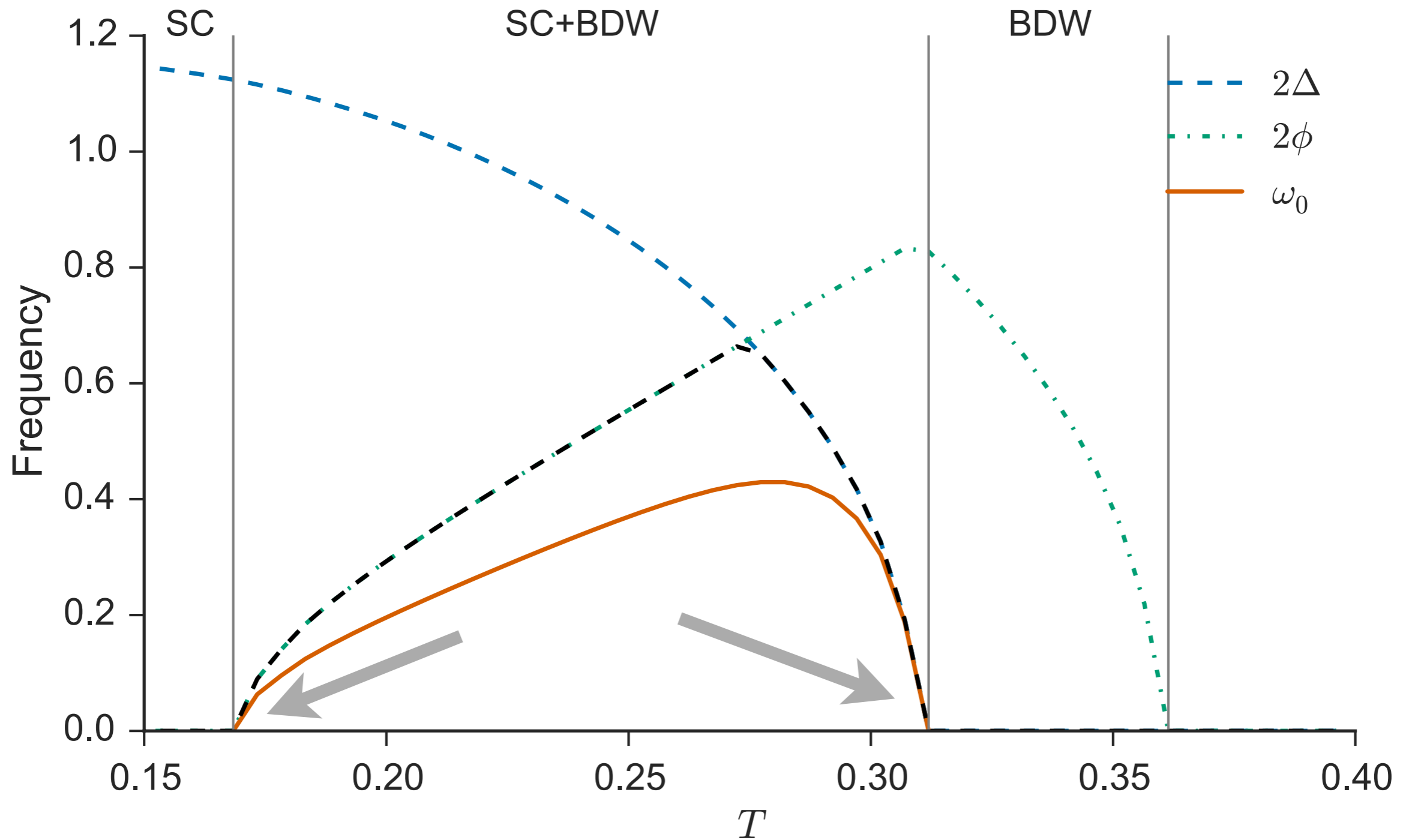
- The **mass** of the Higgs mode is the energy gap for creating a collective excitation
- We focus here on modes with frequencies at or below the pair creation energy
  - These should be protected from decaying to particle-hole pairs



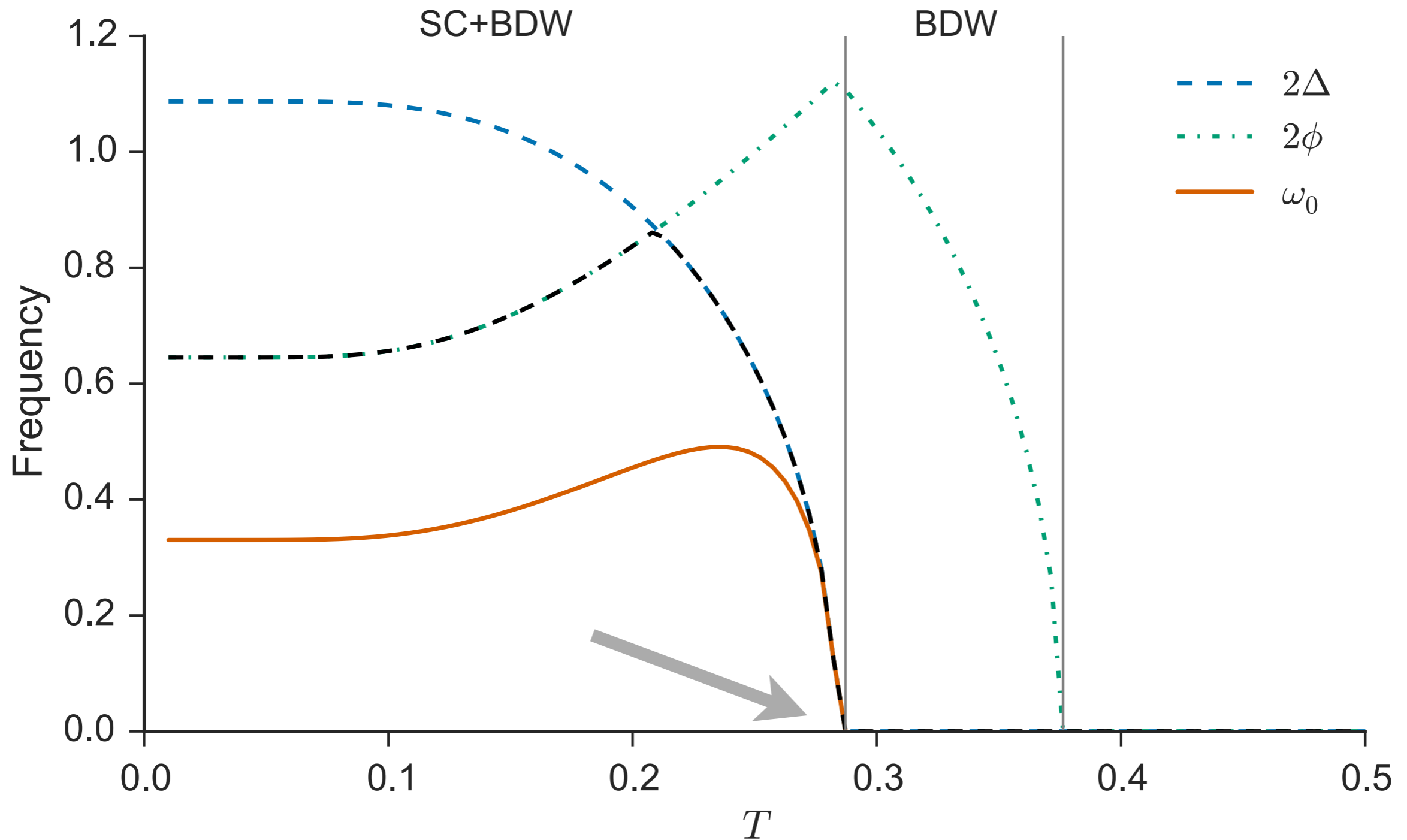
$$\omega_0 \equiv \text{Re} [\omega(\mathbf{q} \rightarrow \mathbf{0})]$$



# Higgs mass

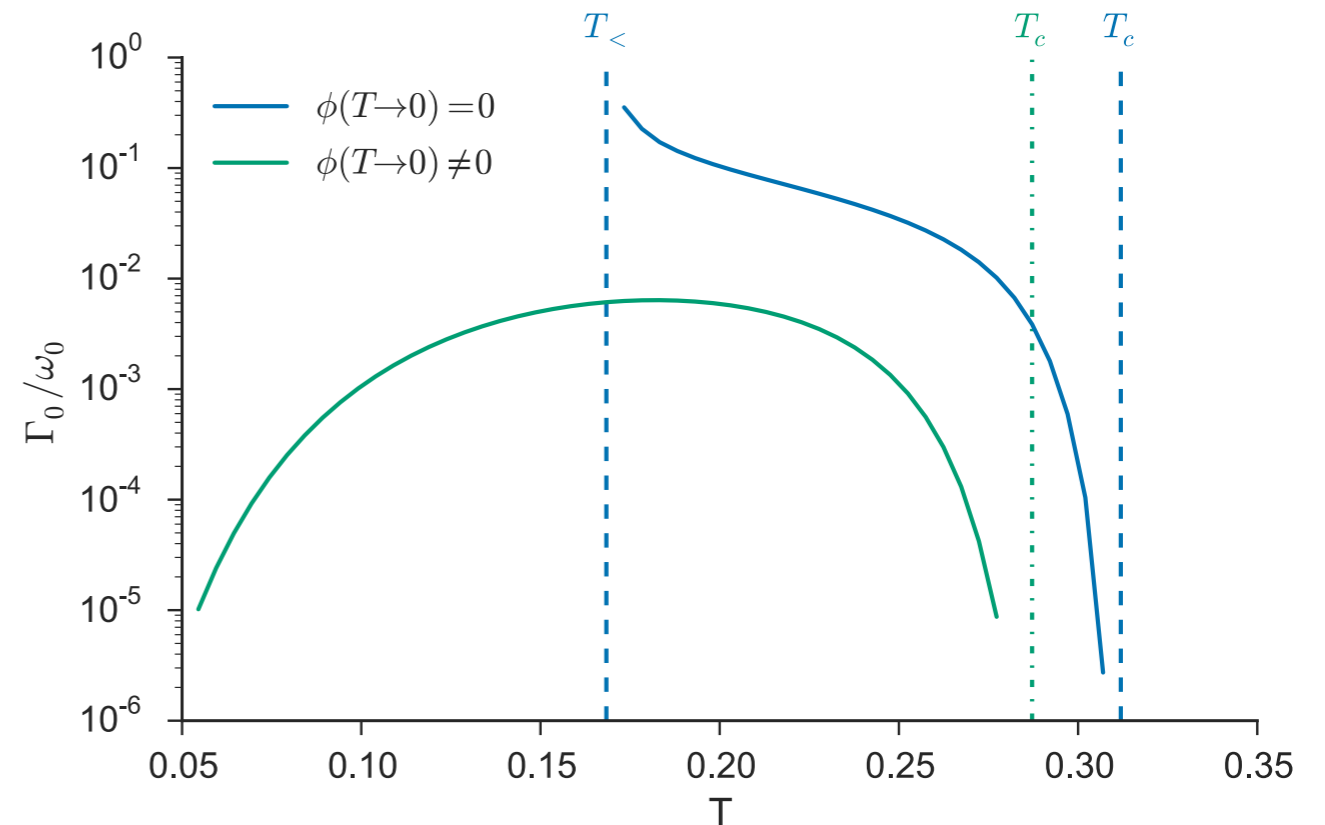


# Higgs mass

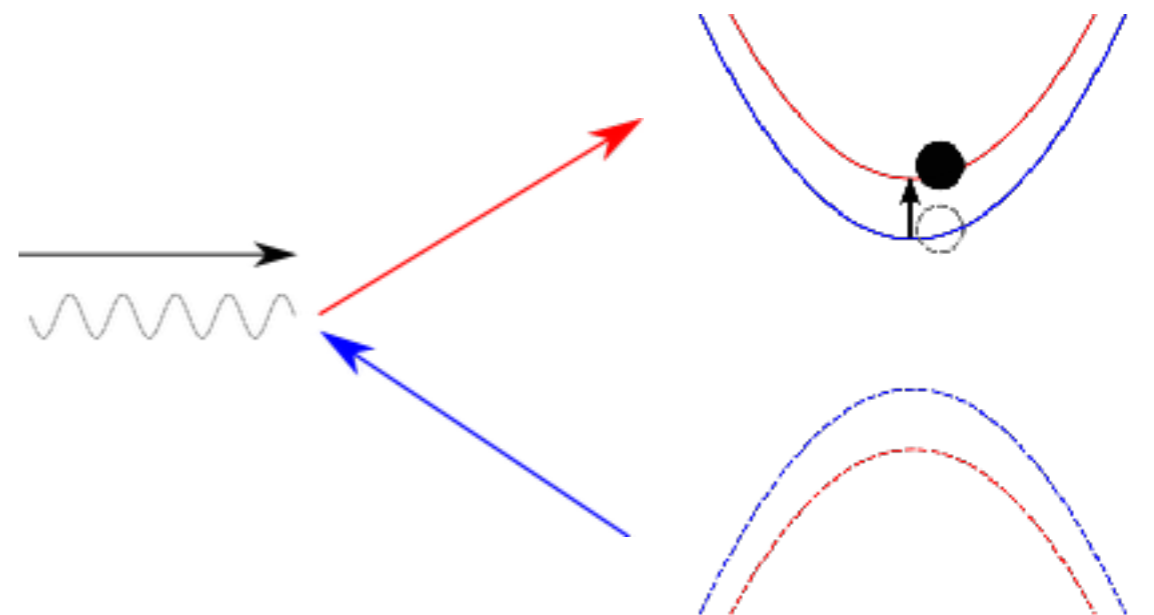


# Damping of collective modes

- The in-gap mode is still damped.
- Damping is due to scattering of thermally excited quasiparticles
- Notably the strength of damping initially increases inside the coexistent phase

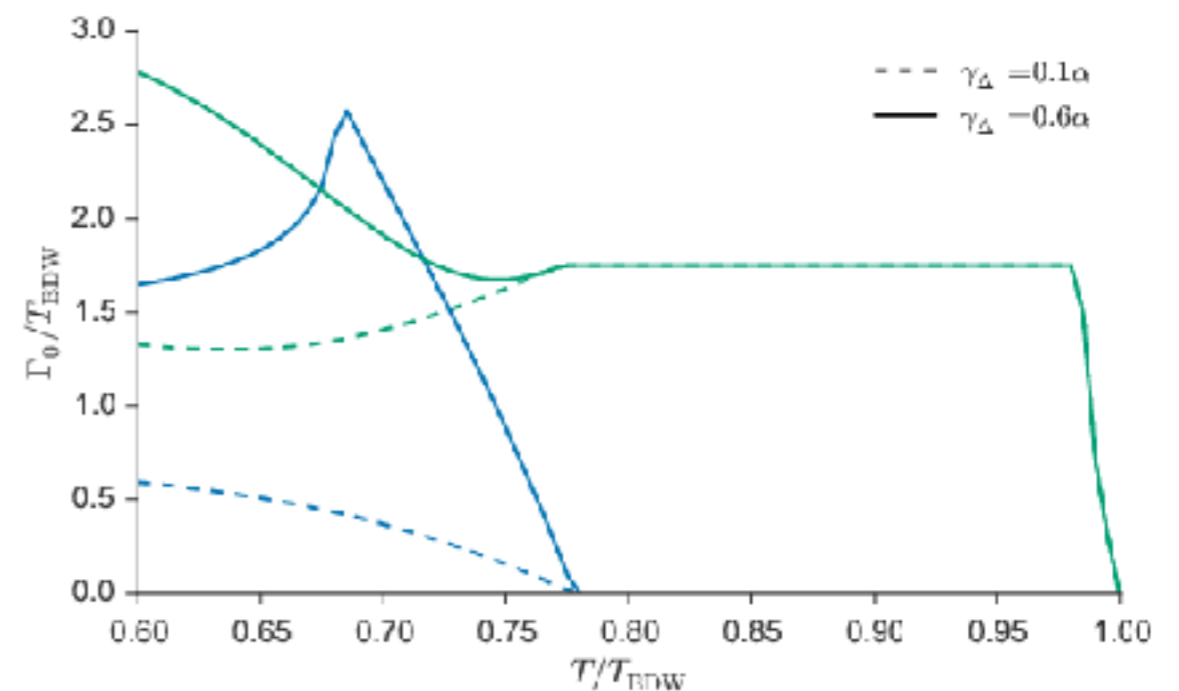
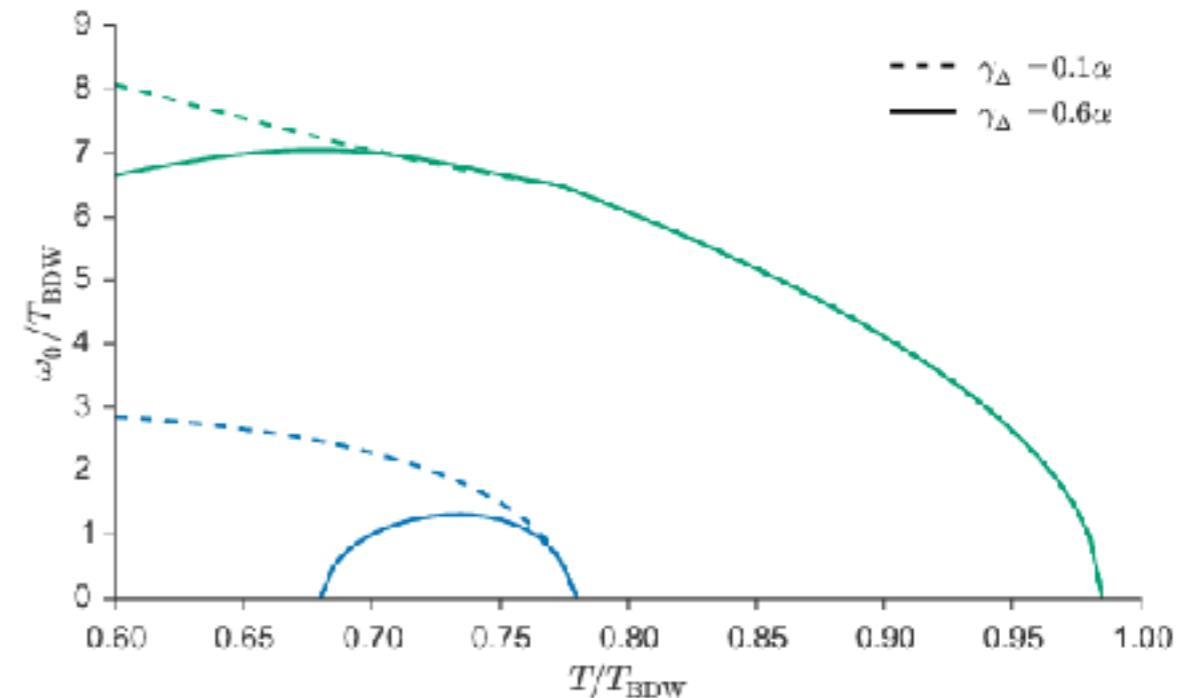


$$\Gamma_0 \equiv -\text{Im} [\omega(\mathbf{q} \rightarrow 0)]$$



# Effect of damping from the nodal regions

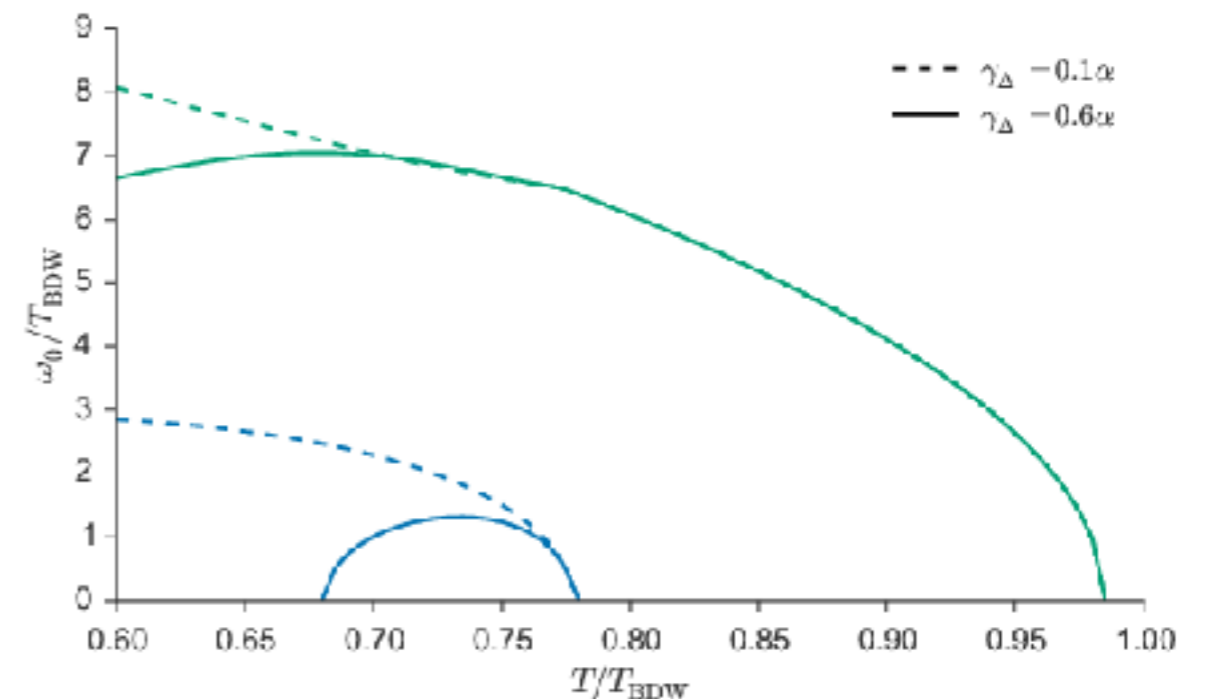
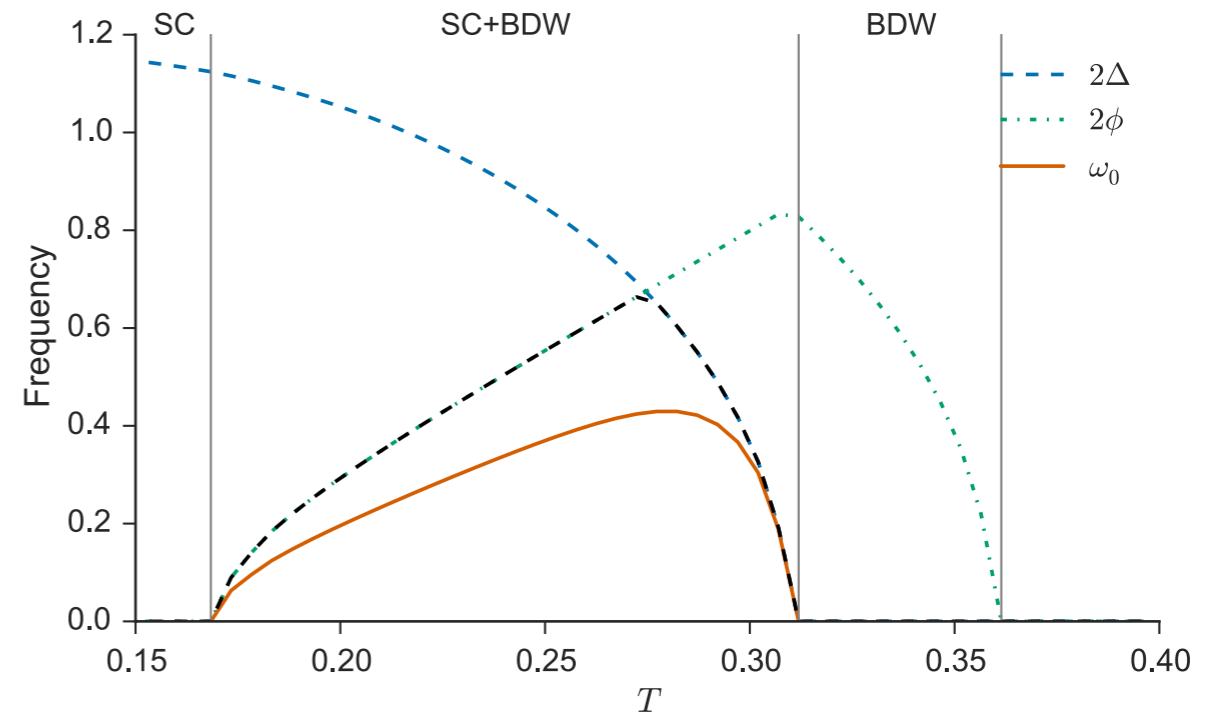
- To consider the effect of nodal pair-production we employed a time-dependent Ginzburg-Landau treatment
- Large enough damping can restrict the in-gap hybrid mode to a small vicinity of  $T_c$





# Summary

- We studied the **mixed amplitude modes** of *SC* and *dFF-DW* in a hotspot model and found
  - A slow mode inside the gap, which is nonetheless weakly damped
  - A fast mode outside the gap
- For sufficiently strong nodal damping the slow mode is restricted to a small vicinity of  $T_c$



**Thank you**

# Extra Slides

# Hamiltonian and mean field theory

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$$\begin{aligned}
 \check{H}_{\text{MF}} &= \check{H}_0 + \Delta \check{V}_\Delta + \phi \check{V}_\phi, & \Delta &= \frac{g_s}{4} T \sum_k \text{tr} \check{V}_\Delta \check{G}_k, \\
 \check{H}_0 &= \text{diag}(\xi_1, \xi_2) \otimes \hat{\tau}_z, \\
 \check{V}_\Delta &= \hat{\rho}_0 \otimes \hat{\tau}_1, \quad \check{V}_\phi = \hat{\rho}_1 \otimes \hat{\tau}_3, & \phi &= \frac{g_c}{4} T \sum_k \text{tr} \check{V}_\phi \check{G}_k,
 \end{aligned}$$

$$\mathcal{H}_{\text{int}}^\Delta = \frac{g_s}{4} \sum_{k,p,q} \Psi_{k+q,a}^\dagger \check{V}_\Delta \Psi_{k,a} \Psi_{p-q,b}^\dagger \check{V}_\Delta \Psi_{p,b}$$

$$\mathcal{H}_{\text{int}}^\phi = \frac{g_c}{4} \sum_{k,p,q} \Psi_{k+q,a}^\dagger \check{V}_\phi \Psi_{k,a} \Psi_{p-q,b}^\dagger \check{V}_\phi \Psi_{p,b}$$

# Time dependent Ginzburg-Landau

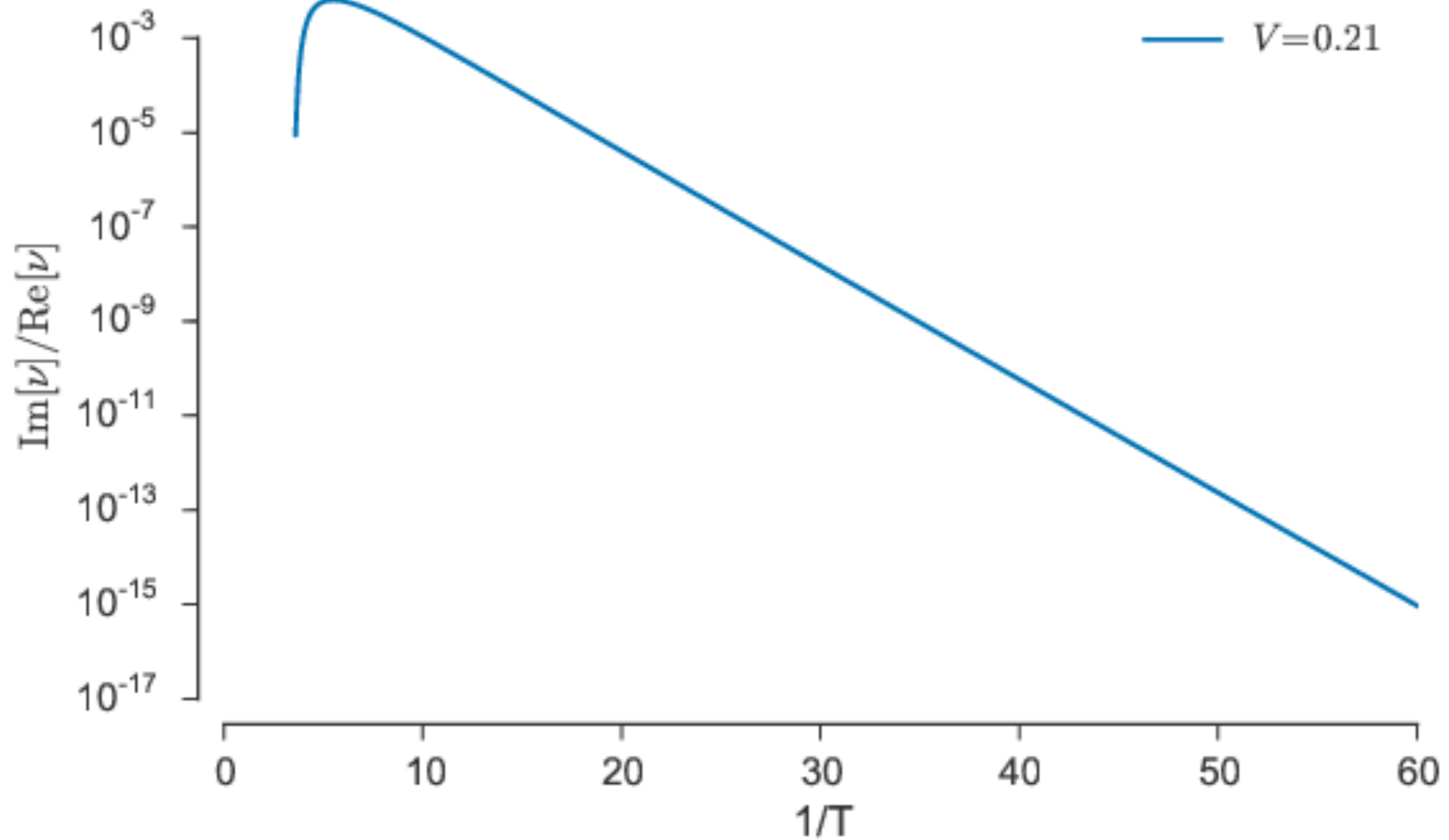
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$$-\frac{\partial^2 \Delta}{\partial t^2} - \gamma_{\Delta} \frac{\partial \Delta}{\partial t} = \frac{\partial \mathcal{F}_{GL}}{\partial \Delta^*},$$
$$-\frac{\partial^2 \phi}{\partial t^2} - \gamma_{\phi} \frac{\partial \phi}{\partial t} = \frac{\partial \mathcal{F}_{GL}}{\partial \phi^*},$$

$$\mathcal{F}_{GL} = \alpha_{\phi} |\phi|^2 + \alpha_{\Delta} |\Delta|^2 + \beta_{\phi} |\phi|^4 + \beta_{\Delta} |\Delta|^4 + u |\phi|^2 |\Delta|^2.$$

# Exponential suppression of damping for $T \rightarrow 0$

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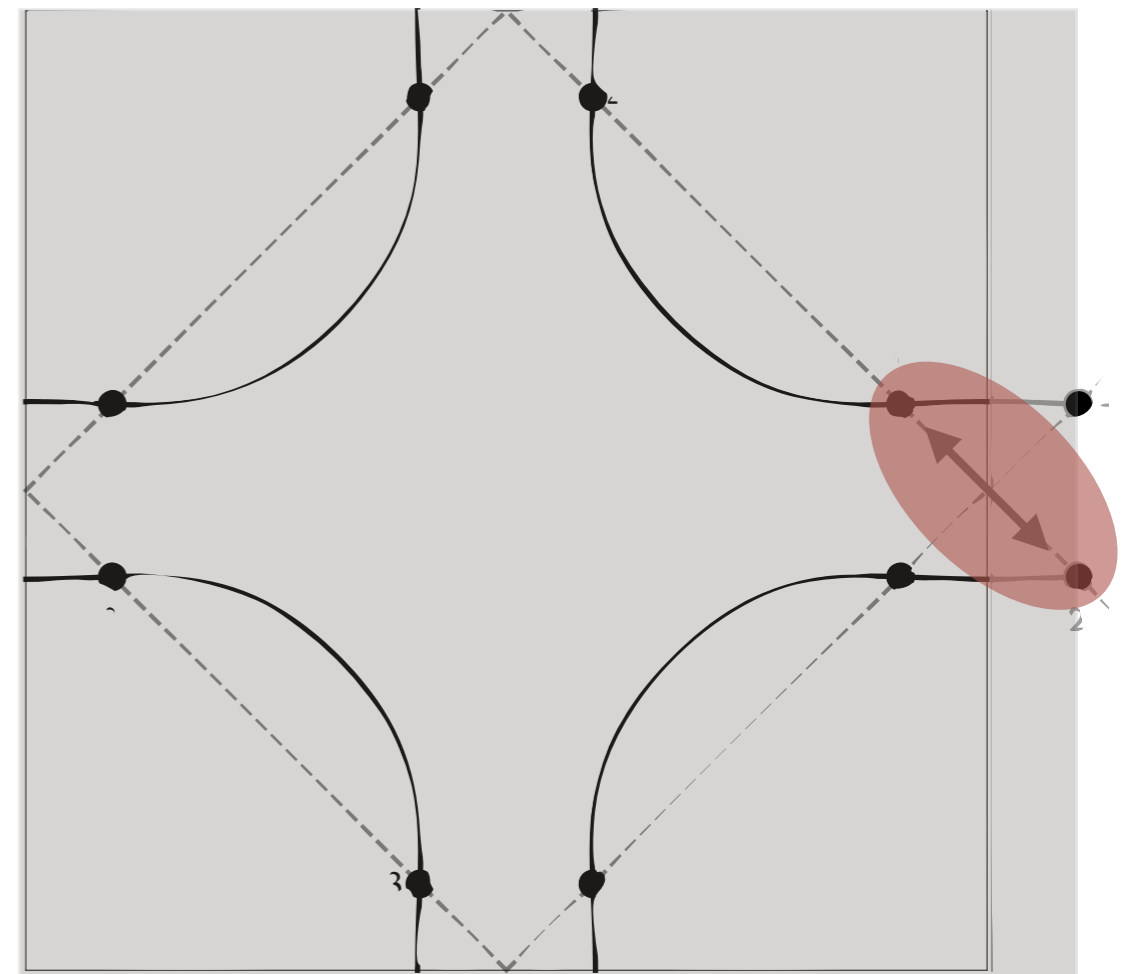
# Mean-field theory

$$H_{MF} = \begin{array}{c} \begin{array}{|c|c|} \hline \text{Particle} & \\ \hline \end{array} & \begin{array}{|c|c|} \hline \text{Hole} & \\ \hline \end{array} \\ \left[ \begin{array}{cc|cc} \xi_1(k) & \bar{\phi} & 0 & \Delta \\ \phi & \xi_2(k) & \Delta & 0 \\ \hline 0 & \bar{\Delta} & -\xi_1(-k) & -\bar{\phi} \\ \Delta & 0 & -\phi & -\xi_2(-k) \end{array} \right] \end{array}$$

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- Projecting onto channels of interest
- Restrict to vicinity of where Fermi surface intersects Magnetic Brillouin zone

d-wave form factor + time reversal symmetry



# A model of CuO planes

$$H = \sum_{i,j} t_{ij} c_{\sigma,i}^{\dagger} c_{\sigma,j} + \frac{1}{2} \sum_{\langle i,j \rangle} J \hat{S}_i \cdot \hat{S}_j + \frac{1}{2} \sum_{\langle i,j \rangle} V \hat{n}_i \hat{n}_j$$

The Hamiltonian consists of hopping on square lattice, nearest neighbor exchange, and nearest neighbor Coulomb repulsion.

e.g. J. D. Sau and S. Sachdev, *Phys. Rev. B* 89, 075129 (2014).

